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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635

For first teaching in 2017

Y410/01 Summer 2024 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y410/01 series overview

This paper proved to be a straightforward test, with a mean mark of 40 out of 60. There were plenty of accessible questions for all candidates, though the last two questions proved to be more testing. There were many well-presented scripts and there was no evidence of candidates running out of time.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• scored marks on all questions, including Questions 3, 4, 7, 8 and 9 • showed sound knowledge of complex numbers • applied complex numbers to solve polynomial equations • understood the rules of matrix arithmetic and applied matrices and determinants to transformations • were able to relate loci in an Argand diagram to cartesian geometry • used algebra with fluency and accuracy • understood sigma notation and applied this to the summation of series.	• scored marks in Questions 1, 2, 5 and 6 • made some algebraic slips and errors • did not solve more complex, multi-stage problems, such as Questions 8 (b) and 9.

Question 1 (a)

1 The quadratic equation $x^2 + ax + b = 0$, where a and b are real constants, has a root $2 - 3\text{i}$.

(a) Write down the other root. **[1]**

Virtually all candidates wrote down the complex conjugate here and picked up this first mark.

Question 1 (b)

(b) Hence or otherwise determine the values of a and b. **[3]**

This question was also very well answered, using a variety of methods. Most candidates used the sum and product of the roots to find *a* and *b*, but a substantial number also multiplied out the factors $x - 2 + 3\text{i}$ and $x - 2 - 3\text{i}$.

Question 2 (a)

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & a \\ -1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$, where a is a constant.

(a) By multiplying out the matrices on both sides of the equation, verify that $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$. **[4]**

This proved to be a straightforward test of matrix multiplication and as with the previous question, virtually all candidates gained full marks.

Question 2 (b)

(b) State the property of matrix multiplication illustrated by this result. **[1]**

This question was responded to less well, with many candidates confusing associativity with commutativity.

Question 3 (a)

3 **(a)** Using standard summation formulae, write down an expression in terms of n for $\sum_{r=1}^{2n} r^3$. **[1]**

Most managed to substitute 2n for n in the formula for Σr^3, though occasionally the brackets in the $(2n)^2$ term was omitted.

Question 3 (b)

(b) Hence show that $\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2(an+b)(cn+d)$, where a, b, c and d are integers to be determined. **[5]**

These marks relied on candidates recognising that they needed to subtract the sum of the first n terms from the sum of the first 2n terms. The subsequent factorisation was usually correct, although many candidates multiplied out the terms, simplified and then re-factorised.

Question 4 (a)

4 **In this question you must show detailed reasoning.**

The roots of the cubic equation $x^3 - 3x^2 + 19x - 17 = 0$ are α, β and γ.

(a) Find a cubic equation with integer coefficients whose roots are $\frac{1}{2}(\alpha - 1)$, $\frac{1}{2}(\beta - 1)$ and $\frac{1}{2}(\gamma - 1)$. **[4]**

This question was most easily solved using the standard method of substituting $u = \frac{1}{2}(x - 1)$, replacing x with $2u + 1$ in the cubic, and then simplifying. However, quite a few candidates found the sum and product of roots for the original cubic and constructed the corresponding sums and products for the transformed equation (see the alternative mark scheme); this required some careful work and was generally less successful.

Question 4 (b)

(b) Hence or otherwise solve the equation $x^3 - 3x^2 + 19x - 17 = 0$. **[3]**

The 'hence' here required a correct answer to part (a) and quite a few candidates used the 'otherwise' method of using the factor theorem to find the $x = 1$ root and then polynomial division to find the two complex roots. As this question required 'detailed reasoning', we did require some mention of the factor theorem in determining the $x = 1$ root. Very occasionally we came across scripts where candidates found these roots in part (a), converted them to the new roots and then constructed the transformed cubic from them, which was allowed full marks if done correctly.

Question 5 (a)

5 (a) Find the volume scale factor of the transformation with associated matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix}$. **[2]**

For this paper candidates are not required to know how to calculate the determinant of a 3 × 3 matrix by calculation and most used their calculator correctly to find the correct answer. Most then recognised this as giving the volume scale factor.

Question 5 (b) (i)

(b) The transformations S and T of the plane have associated 2×2 matrices **P** and **Q** respectively.

(i) Write down an expression for the associated matrix of the combined transformation S followed by T. **[1]**

Most students responded with **QP** here, but there were a few **PQ** responses.

Question 5 (b) (ii)

The determinant of **P** is 3 and $\mathbf{Q} = \begin{pmatrix} k & 3 \\ -1 & 2 \end{pmatrix}$, where k is a constant.

(ii) Given that this combined transformation preserves both orientation and area, determine the value of k. **[3]**

This question was generally well done, though a determinant $2k - 3$ was seen occasionally.

Question 6 (a)

6 You are given that $\mathbf{M} = \begin{pmatrix} 4 & -9 \\ 1 & -2 \end{pmatrix}$.

(a) Prove that $\mathbf{M}^n = \begin{pmatrix} 1+3n & -9n \\ n & 1-3n \end{pmatrix}$ for all positive integers n. **[6]**

This proved to be a straightforward induction question, with many responses determining a target expression for $\mathbf{M}^{k+1}$ before calculating it. The final statement was usually accurate but needs to be clear.

Assessment for learning

Proved true for $n = 1$.

If assumed true for $n = k$, **then** proved true for $n = k + 1$.

Therefore true for all positive integers n.

Exemplar 1

6(a)

$n = 1$

$M^1 = \begin{pmatrix} 4 & -9 \\ 1 & -2 \end{pmatrix} = M$

assume true for $n = k$

$M^k = \begin{pmatrix} 1+3k & -9k \\ k & 1-3k \end{pmatrix}$

6(a) (continued)

$n = k+1$

$= \begin{pmatrix} 1+3k & -9k \\ k & 1-3k \end{pmatrix}\begin{pmatrix} 4 & -9 \\ 1 & -2 \end{pmatrix}$

$= \begin{cases} 4+12k-9k = 3k+4 \\ 4k+1-3k = k+1 \end{cases}$

$-9 - 27k + 18k = -9k - 9$

$-9k - 2 + 6k = -3k - 2$

$= \begin{pmatrix} 3k+4 & -9k-9 \\ k+1 & -3k-2 \end{pmatrix}$

$= \begin{pmatrix} 1+3(k+1) & -9(k+1) \\ (k+1) & 1-3(k+1) \end{pmatrix}$

If true for $n = k$, true for $n = k+1$

If true for $n = 1$, true for all $n \in \mathbb{N}$

In Exemplar 1, there is insufficient working to gain the first B1 and the final statement is not precise enough.

Question 6 (b)

(b) A student thinks that this formula, when $n = 0$ and $n = -1$, gives the identity matrix and the inverse matrix $\mathbf{M}^{-1}$ respectively.

Determine whether the student is correct. **[3]**

Some candidates lost a mark here by offering no working to show that $\mathbf{M}^{-1}$ is indeed the inverse matrix, which could be done either by showing that $\mathbf{M}\,\mathbf{M}^{-1} = \mathbf{I}$ or showing the determinant is 1.

Question 7 (a)

7 Three planes have equations

$$x + 2y - 3z = 0,$$
$$-x + 3y - 2z = 0,$$
$$x - 2y + kz = k,$$

where k is a constant.

(a) For the case $k = 0$, the origin lies on all three planes.

Use a determinant to explain whether there are any other points that lie on all three planes in this case. **[2]**

Most candidates calculated the determinant of the matrix of coefficients correctly. To earn the second mark, it was necessary to point out that this is non-zero and therefore the origin is the unique point of intersection. There was a little confusion from candidates here about matrix and determinant, for example stating, 'the determinant is non-singular' rather than 'the matrix is non-singular'.

Question 7 (b) (i)

(b) You are now given that $k = 1$.

(i) Show that there are no points that lie on all three planes. **[3]**

Many candidates lost marks here as they assumed it is sufficient to say 'the determinant is zero' without showing that the equations are inconsistent, for example by eliminating one variable using two different pairs of equations. There were a surprising number of slips in the working here from those who attempted this.

Question 7 (b) (ii)

(ii) Describe the geometrical arrangement of the three planes. **[1]**

According to the specification the correct term is 'prismatic intersection', but the majority of responses used 'triangular prism' or even just 'prism' and these were both accepted.

Question 8 (a)

8 In an Argand diagram, the point P representing the complex number w lies on the locus defined by $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. You are given that $\mathrm{Re}(w) = 1$.

(a) Find w. **[2]**

Most errors here were associated with the argument $\frac{3}{4}\pi$, taking the gradient of $z - 7$ as 1 instead of -1.

Question 8 (b)

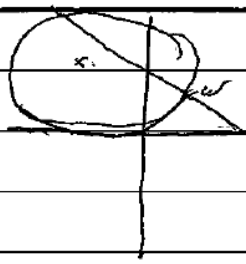
The point P also lies on the locus defined by $\{z : |z+3-9\mathrm{i}| = k\}$, where k is a constant.

(b) Find the complex number represented by the other point of intersection of the loci defined by $\{z : |z+3-9\mathrm{i}| = k\}$ and $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. **[7]**

Most solutions seemed to recognise the first locus as a circle, although there were quite a few errors seen in the circle equation (such as sign errors or k instead of k^2). The correct radius relied on part (a) being done correctly. When the circle and the radius were correct, solving simultaneously was often done well.

Exemplar 2

8(b)



$(x+3)^2+(y-9)^2=k$

$(1+3)^2+(6-9)^2=k$

$k=25$

~~$y=-x+7$~~

$y=-(x-7)$

$(x+3)^2+((-(x-7))-9)^2=25$

$(x+3)^2+(-x+7-9)^2=25$

$(x+3)^2+(-x-2)^2=25$

$x^2+6x+9+x^2+4x+4=25$

$2x^2+10x+13=25$

$2x^2+10x-12=0$

$x^2+5x-6=0$

$x=\frac{-5\pm\sqrt{5^2-4(-6)}}{2}$

$x=1$ or -6

$y=-(x-7)$

$=-(-6-7)$

$=13$

other complex number:

$-6+13i$

In Exemplar 2, the circle equation contains k and not k^2, but this is recovered later so only 1 mark is lost.

Question 9

9 In this question you must show detailed reasoning.

Find a vector **v** which has the following properties.

- It is a unit vector.
- It is parallel to the plane $2x+2y+z=10$.
- It makes an angle of 45° with the normal to the plane $x+z=5$. **[8]**

Out of the three conditions, it was a surprise that the first one (on the unity of the vector) was often lacking. The second condition was quite frequently misinterpreted as implying that the vector must be a scalar multiple of 2**i** + 2**j** + **k**, rather than normal to this vector. The angle formula often gained a method mark. Getting all three conditions expressed correctly and then solving the three equations accurately was seen relatively infrequently.

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