Qualification Accredited

Oxford Cambridge and RSA

AS LEVEL

Examiners' report

FURTHER MATHEMATICS A

H235

For first teaching in 2017

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Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper 1 series overview

This is one of the optional modules in the AS Further Mathematics A specification, alongside the compulsory Pure Core, and the other optional modules in Statistics, Discrete Mathematics and Additional Pure Mathematics. This component complements the Mechanics content of the AS Mathematics course and requires candidates to solve practical problems using a variety of techniques including energy, power, momentum, connected particles, horizontal and vertical circles, and dimensional analysis.

To do well on this paper, candidates needed to be able to work systematically and accurately with the standard principles of Mechanics. They will often need to make sense of more complex situations, for example, correctly combining several forces and/or types of energy in a variety of situations, such as two or more sources of gravitational potential and/or kinetic energy and resistance. The ability to tackle multi-stage problems is also required for some questions, together with mathematical modelling and application of a derived model.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• made good use of diagrams to help them understand a problem before solving it • were consistent in defining the directions of different velocities and/or forces before attempting to solve a problem • recognised the different forces that were acting and when they needed resolving • understood the different quantities in mechanics, such as work, kinetic and potential energy, momentum, impulse, power, angular velocity, and the correct formula required to work with each of them, together with the associated dimensions • were able to use Newton's Second Law (F = ma) confidently together with the relationship between Force and energy • were efficient and effective in applying their knowledge, always choosing the simplest possible route to the solution • understood the difference between a vector and a scalar.	• did not have an effective strategy for managing the different directions that objects might be travelling, especially in impulse/momentum and collisions questions, resulting, for example, in sign errors • did not identify all the relevant quantities required to solve a problem • did not recognise where a problem involved two separate stages in the same question • were not able to use a model to make predictions correctly • did not recognise modelling assumptions already addressed in the question • made avoidable errors in calculations and algebraic manipulation.

Question 1 (a)

1 A particle P of mass 2.5 kg is moving with a constant speed of $4\,\text{m s}^{-1}$ in a straight line on a smooth horizontal plane when it collides directly with a fixed vertical wall.

After the collision P moves away from the wall with a speed of $2.8\,\text{m s}^{-1}$.

(a) Calculate the coefficient of restitution between P and the wall. **[1]**

This question is a simple starter which the great majority of candidates answered correctly. Some candidates just divided the velocities, occasionally the wrong way round, to give a negative value. Candidates should be aware that any answer outside of the range 0 ≤ e ≤ 1, e.g. -0.7 or 1.2 (using 4 – 2.8), is impossible.

Question 1 (b) and (c)

(b) Find the magnitude and state the direction of the impulse exerted on P by the wall. **[3]**

(c) State the magnitude and direction of the impulse exerted on the wall by P. **[2]**

Impulse is a vector quantity by nature, and therefore this question required careful consideration of the direction of motion before and after the collision and the correct use of the formula mv – mu.

In part (b), most candidates assumed that motion towards the wall was positive, and away from the wall was negative. While the majority of candidates gained full marks, there were many who made sign errors or used magnitudes, e.g. |mv| - |mu|, usually ending up with ±3Ns instead of the required ±17Ns. A few candidates attempted to use inappropriate techniques such F = ma. Many candidates found the correct impulse but did not state the magnitude and therefore lost a mark if their impulse was negative.

This question does not give a diagram to work with, but only implies that the object is initially approaching the wall, and then moves away from the wall after the collision. A description such as 'away from the wall' or 'in the direction of P's motion after the collision', gained the third mark for stating the correct direction.

Alternatively, a diagram with the object moving right towards the wall and left away from the wall, together with the statement that the impulse was to the left, or an indication that the positive direction was towards the wall and the impulse was in the negative direction, was also acceptable. However, some candidates did not adequately describe the direction of the impulse on the object, and some made no attempt to state any direction.

Part (c) then addresses candidates' knowledge of Newton's third law, as applied to impulse gained and lost in a collision. Again, most candidates gained full marks, usually giving an answer that had the same magnitude as before but in the opposite direction. However, a few were given no marks for this, with some even claiming that there was no impulse on the wall because it didn't move in the collision, clearly misunderstanding the nature of this law. Others attempted to recalculate the impulse, often using only the initial or final speed of P, or stated that as there was no impulse, there was also no direction. In some cases, a zero impulse was given, together with the statement that it was towards or away from the wall.

Exemplar 1

1(a)	$e = \frac{2.8}{4} = 0.7$
1(b)	$I = mv - mu$ $= (2.5 \times -2.8) - (2.5 \times 4)$ $= -25\text{ Ns}$ 25Ns to the left.
1(c)	25 Ns to the right.

In this exemplar, the candidate has not drawn any diagram in part (a) (included for reference). In part (b), the candidate has correctly applied the formula for the impulse (M1) but somehow calculated -25 instead of -17 and given the magnitude as 25 (A0). They have then stated that the impulse is 'to the left' but made no indication of where the wall is in relation to the ball. Therefore, we do not know if 'to the left' is towards the wall or away from the wall (B0).

In part (c), the magnitude is the same as in part (b), so this gains B1FT, and the direction is opposite to part (b), so this also gets B1.

Question 2 (a) (i) and (ii)

2 A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 1.8 m. The other end of the string is attached to a fixed point, O, on a smooth horizontal plane. Initially, P is moving with a constant speed of $12\,\text{m s}^{-1}$ in a horizontal circle with O as its centre.

(a) **(i)** Find the magnitude of the acceleration of P. **[1]**

(ii) State the direction of the acceleration of P. **[1]**

This pair of questions, which examine basic knowledge of centripetal acceleration, was answered correctly by nearly all candidates. A small minority showed misconceptions such as confusing acceleration with change of speed rather than change of velocity, or appeared not to know the formula, used the wrong values in the formula, or calculated the wrong value after using the formula correctly. A very small number of candidates used the wrong formula, e.g. $\frac{mv^2}{r}$, or tried to use $\omega^2 r$ instead of $\frac{v^2}{r}$, with varying degrees of success.

Most candidates stated the correct direction of acceleration; incorrect answers were mostly a variation on the acceleration being tangential, clockwise, or anti-clockwise. A few candidates stated that the acceleration was zero and therefore there was no associated direction, or that the direction was towards the centre, despite stating that the magnitude was zero.

Question 2 (b) (i)

A force is now applied to P in such a way that its angular velocity increases. At the instant that the angular velocity reaches $8\,\text{rad s}^{-1}$, the string breaks.

(b) **(i)** Find the speed with which P is moving at the instant that the string breaks. **[1]**

This question examined candidates' knowledge of the relationship between speed and angular speed or velocity, requiring a simple application of the standard formula $v = r\omega$. Again, nearly all responses were correct, with the few incorrect answers usually caused by using the wrong formula, e.g. $\omega = r\dot{\theta}^2$ together with $v = r\dot{\theta}$, or using $r\omega^2 = \frac{v^2}{r}$ or even $r\omega^2 = \frac{v}{r}$, rather than using the standard formula. A very small number of candidates even used values from part a), e.g. $v = \frac{a\text{ (from (a)(i))}}{\omega}$.

Question 2 (b) (ii)

(ii) Find the tension in the string at the instant that the string breaks. **[2]**

The second part then tested candidates' knowledge of another key formula, this time using acceleration and Newton's second law to find the tension generated in circular motion. Again, most candidates handled this without issue, but a somewhat larger minority made mistakes that almost always prevented them from gaining any marks. For example, using their value of v instead of ω in the formula $a = r\omega^2$, reusing the acceleration from part (a), including weight in the calculation, attempting to resolve, or using the wrong formula for the acceleration, such as mv or $\frac{1}{2}mv^2$.

Question 2 (c)

After the string has broken P starts to move directly up a smooth slope which is fixed to the plane and inclined at an angle $\theta°$ above the horizontal. Particle P moves a distance of 20 m up the slope before coming to instantaneous rest.

(c) Use an energy method to determine the value of θ. **[3]**

This is the first question in the paper to combine different quantities to solve a problem, in this case the use of conservation of energy to equate loss of KE with gain in PE. This was generally done well, but a considerable number of candidates made one or more avoidable errors. Common errors included equating GPE with the (negative) change in KE, not resolving the distance travelled to find the GPE, using the speed from part (a) instead of part (b), giving the answer in radians, or other calculation errors.

Question 3 (a)

3 A small object P of mass m is suspended from a fixed point by a light inextensible string of length l. When P is displaced and released in a certain way, it oscillates in a vertical plane. The time taken for one complete oscillation is called the period and is denoted by τ.

A student is carrying out experiments with P and suggests the following formula to model the value of τ.

$$\tau = cg^{\alpha}l^{\beta}m^{\gamma}$$

in which

- g is the acceleration due to gravity,
- c is a dimensionless constant.

(a) Use dimensional analysis to determine the values of the constants α, β and γ. **[5]**

Virtually all candidates were able to make good progress with this now well-established topic, with the majority gaining full marks. It is pleasing to see that almost all candidates are now using the correct dimensional symbols M, L and T (which should be capitalised), rather than specific units such as kg, metres, seconds, and using an explicit overall dimensional equation with logarithmic equations for the individual dimensions.

Almost all candidates solved their derived equations correctly, with a relatively small number of numerical errors seen. In very rare cases candidates were not able to produce a meaningful response. Common errors included using the wrong units for g, usually MT^{-2} or MLT^{-2} instead of LT^{-2}, and less commonly using T^{-1}, LT^{-1}, or even LT^{-2} for τ (which is defined in the question as being the time taken).

Assessment for learning

Present students with a number of right and wrong dimensions (such as, but not limited to, those mentioned above) and see if they can sort them into correct and incorrect statements, discussing the reasons for their answers.

Get them also to consider the problems in context, e.g. can they explain why, from a physical point of view, mass is unlikely to affect the period of oscillation, rather than just relying on mathematical reasoning to get the answer?

Question 3 (b) (i) and (ii)

(b) **(i)** Determine the effect on the period, according to the model, if the length of the string is then multiplied by 4, all other conditions being unchanged. **[2]**

(ii) Determine the effect on the period, according to the model, if **instead** the mass of the object is multiplied by 4, all other conditions being unchanged. **[1]**

This question was again generally answered well. In part (i) most candidates, having obtained the correct value of $\beta = \frac{1}{2}$, were able to apply this to a scale factor of 4 for the length to give a scale factor of 2 for the time taken. A small number of candidates did not realise that they needed to justify their answer as they are instructed to 'determine' the effect. At the very least, we require the statement or clear implication that $4^{\frac{1}{2}} = 2$, but the most diligent candidates used the entire formula with 4L substituted for L, or at least stated that $(4L)^{\frac{1}{2}} = 2L^{\frac{1}{2}}$ or equivalent. Candidates should also check their use of language as a scale factor of 2 is twice or two times as big, not two times *bigger* (which is actually three times as large).

However, a significant minority did not realise the significance of the value of β as a power, and just assumed that the scaling was linear, that the time would decrease, or that the scale factor of 4 related to the dimensionless constant. Some even thought that the values of α and β would cancel each other out since they add up to 0, or that time would increase by 4 seconds.

Part (ii) contrasts the effect of a zero index for M with the non-zero index for L in part (i). A relatively large number of candidates did not gain the mark for this, mostly because they did not explain the answer adequately or at all. At the very least, acknowledging that the period has no dependency on *m* (as $\gamma = 0$) is required. Again, a small number of candidates thought that it was because 4 is a dimensionless constant, or assumed a linear scaling, or that the time would increase by 4 (seconds).

Assessment for learning

Get students to consider and explain their answers to variety of problems that involve different types of proportional reasoning. As this topic is also covered in GCSE maths, it can be included as a starter/refresher activity at some point during the teaching of dimensional analysis, and then applied later in the lesson in a dimensional analysis context.

Question 4

4 A particle B of mass 5 kg is at rest at the bottom of a slope which is angled at $\sin^{-1} 0.2$ above the horizontal. A constant force D initially acts directly up the slope on B.

The total resistance to the motion of B is modelled as being a constant 12 N.

At the instant that D stops acting, the speed of B is 18 m s^{-1} and B has moved 90 m up the slope.

Determine the average power of D over the time that D has been acting on B. **[6]**

This question is the first substantial question that proved to be a serious challenge for a significant number of candidates. This is likely due to the complexity of the problem, which required candidates to combine three separate energy terms, i.e. KE, GPE and work done against resistance, and equate the sum of these to the energy output from the driving force. Candidates then had to use suvat to find the time taken, to find the average power generated by the force D.

Many candidates chose to use force and N2L to find D, and multiply by the (average) speed instead. This is perhaps slightly more complicated but a valid method nonetheless, especially for many candidates that still prefer the familiarity of force equations over the less familiar technique of considering energy. A few even found the total energy output and then converted it into a value for D by dividing by the distance travelled (since D is constant), before proceeding to multiply by the (average) speed.

Only a minority gained full marks in the question, with a slightly larger number gaining 4 marks and most of the rest gaining 2 or 3 marks. The most common issue was many candidates' inability to handle more than two force or energy terms, with most candidates omitting at least one of the terms, sometimes two. Some candidates used the wrong sign for at least one term, suggesting that they were perhaps thinking of conservation of energy rather than work done. One or two candidates replicated terms (usually GPE) on both sides of their energy equation, which then cancelled out.

A significant number of candidates attempted an energy equation but then restarted by using a force equation, or vice versa, showing the challenge that some candidates find in this type of problem. Other issues including using the wrong values, e.g. multiplying the friction by the height gained instead of the distance travelled, multiplying the distance by $\cos\theta$ or not multiplying by $\sin\theta$ when finding the GPE.

Among those using F = ma to find the driving force, some then went on to multiply by the final, rather than the average, speed, and therefore lost the last 2 marks unless they then divided the result by 2. Many candidates did not handle suvat equations correctly, for example using the height gained rather than the distance travelled when attempting to find the acceleration, or confused distance, time or acceleration.

Assessment for learning

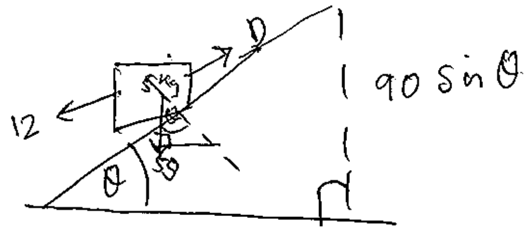
Present students with a variety of problems involving energy and get them to discuss and explain what energy terms would be present in each problem, without solving any of the problems at first. Answers could perhaps be provided to cut out and match to each problem.

Once they have worked out how to analyse the questions consistently and correctly and learned what terms to look for, they can then attempt to solve the resulting equations together and review what they have learned from the exercise.

Exemplar 2

4

12 7 8



when D = 0
V = 18
S = 90

sin⁻¹ 90
θ = S⁻¹(0.2)
θ = 11.53

P = FV
W.D = P/T

when D = 0

F = ma

−(12 + 5g sin θ) = 5 × a

−21/8 = 5a

~~a = 0.44 ms⁻²~~ a = −4.36

~~S = 90~~ S = 90 ~~S = vt − ½ at²~~ S = vt − ½ at²

~~u = 0~~ u
~~v = 18~~ v = 18 ~~90 = 18t − ½ × 0.44 × t²~~ 90 = 18t + 2.18t²
~~a = 0.44~~ a = −4.36
~~t = ?~~ t = ? 2.18t² + 18t − 90 = 0

~~0.22t² − 18t + 90 = 0~~

t = 3.508 or t = −11.76

t > 0
∴ t = 3.508

W.D = mgh + Δ KE

mgh = 5 × 9.8 × 90 sin θ
= 882

initial KE = 0
final KE = ½ × 5 × 18²
= 810

W.D = 810 + 882
= 1692

W.D = P/t

1692 = P/3.508

P = 5935.54 W

In this example, the candidate has a reasonable diagram with driving force, friction, expression for height gained, angle of elevation (which is not actually needed) and values for *v* and *s*, and an incorrect value of *D* = 0. The component of weight down the slope is not shown, but this does not stop the candidate from including it later. Crucially they have missed the fact that *u* = 0, making it impossible to use suvat to find *t* or *a*. There are also two formulae involving power, the second of which is incorrect and used later.

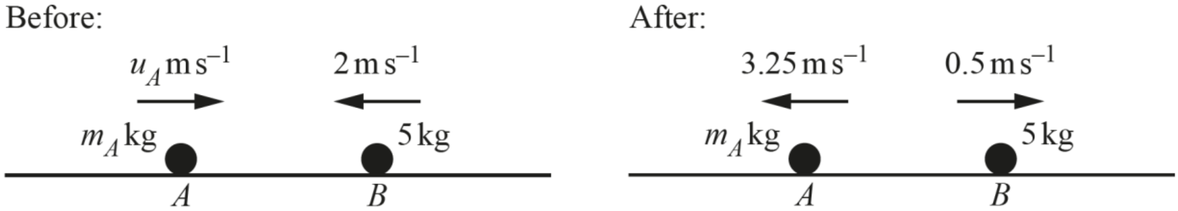
They then appear to start off by considering the forces on the object using N2L but have used this incorrectly to find *a* by assuming that *D* = 0. They then use the wrong suvat equation (which should be using *u* = 0) to find an incorrect value of *t*, so they lose the first M mark.

They then proceed by attempting to find the work done by *D*, although they previously stated that *D* = 0. Despite including the friction in the force equation, they have only included the KE and GPE in the energy mix, so they lose the first B mark but gain the second and third mark.

They have then used an incorrect formula for the power, multiplying the energy by *t* instead of dividing, so this loses the second M mark and the final A mark, giving 2 marks overall.

Question 5 (a)

5 Two particles, A of mass m_A kg and B of mass 5 kg, are moving directly towards each other on a smooth horizontal floor. Before they collide they have speeds $u_A\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively. Immediately after they collide the direction of motion of each particle has been reversed and A and B have speeds $3.25\,\mathrm{m\,s^{-1}}$ and $0.5\,\mathrm{m\,s^{-1}}$ respectively (see diagram). The coefficient of restitution between A and B is 0.75.



(a) Determine the value of m_A and the value of u_A. **[5]**

This question uses the well-established duo of conservation of momentum (COLM) and Newton's elasticity law (NEL). A large majority of candidates were able to handle the equations confidently and correctly despite the unknowns being the first mass and its initial velocity, rather than the velocities after the collision, as is often the case in standard problems.

Sign errors, usually occurring due to not assigning opposite directions with opposite signs, and numerical errors were the most common reason for losing marks. A few candidates attempted to use kinetic energy instead of one of the two required equations. Candidates should be aware that answers in this type of questions are likely to be simple whole or rational numbers, and to carefully check that their answers work with the details given in the original question.

Question 5 (b)

(b) Show that approximately 41% of the kinetic energy of the system is lost in this collision. **[3]**

This was handled well by nearly all candidates, who clearly knew how to find the total kinetic energy before and after, and then find the percentage change. A few used incorrect values, e.g. 3.5 instead of 3.25 for the final speed of particle A, or confused the mass and speed elements, or only calculated the values for one of the particles. One or two tried to work backwards by calculating 59% of their initial KE, but the result did not work because they had got the wrong answer in part (a).

Question 5 (c)

After the collision between A and B, B goes on to collide directly with a third particle C of mass 3 kg which is travelling towards B with a speed of $5.5\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between B and C is denoted by e.

(c) Given that, after B and C collide, there are no further collisions between A, B and C determine the range of possible values of e. **[6]**

This type of question, which typically requires candidates to find an unknown value or range of values of *e*, is designed to be more challenging than the standard type of problem encountered in part (a). Therefore, there was considerable variability in responses, with only a few candidates getting full marks on this. Most candidates started out correctly by once again using the standard COLM/NEL equations with the unknown value of *e*, and made a good attempt at the first 3 marks.

Unfortunately, there were considerable issues with some candidates who attempted to anticipate the direction of travel of the two particles after the collision. This led to many cases of sign error, with it being hard to tell at times if left or right was being considered as positive, or what to do with unknowns pointing in different directions. For example, if there is a left arrow on B and a right arrow on C, it is hard to know whether to count V_B (the velocity of B after the second collision) as positive or negative, depending on whether it is meant to be a speed or a velocity or if it has no real significance.

This confusion then led some candidates to lose marks for inconsistent signing of values. It is far better for candidates to adopt a consistent approach, such as always counting right as positive. They can then assume that unknowns are in the positive direction, so that the correct direction can then be determined by whether the answer is positive or negative.

Quite a few candidates realised that V_B was related to the velocity of A after the first collision. Some candidates caused confusion by using different orientations between the parts of the question (e.g. switching from left to right in part (a) to right to left in part (b), or often a mixture of both). This led to them wrongly stating the relationship, for example, saying that $V_B \le$ (or $<$) 3.25 when seemingly counting right as the positive direction, or similar incorrect statements. Again, a consistent approach as above would make it considerably less likely for candidates to make such errors.

A fair number of candidates made a good attempt at eliminating V_C to give V_B in terms of *e* only (or sometimes vice versa), which was the crucial step required to find the critical value of *e*. Using $V_B \le -3.25$ (if done correctly) then gives $e \le 2/3$. Many had unfortunately made errors by this point and got the wrong value or the wrong inequality. Some also worked out *e* when $V_B = V_C$, giving $e = 0$, which is unnecessary.

A certain number of candidates realised that they could greatly simplify the problem by assuming from the start that $V_B = -3.25$ to get the critical value of *e* instead of leaving V_B as an unknown, and a few managed to get all the way to the correct answer with the correct inequalities in the correct direction.

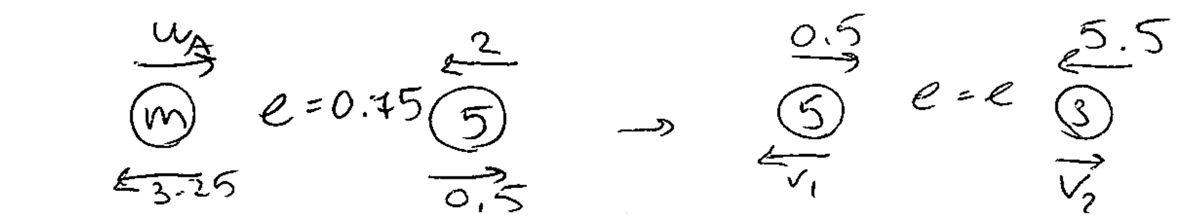
Assessment for learning

Ask students to work with examples of momentum questions where directions have not been applied consistently, e.g. with unknowns going in different directions and directions oriented right to left (i.e. left is positive) instead of left to right. Then get them to try examples which show consistent use of left to right directions.

Ask them to discuss which method worked best and why and encourage them to follow consistently good practice in the future.

Exemplar 3

5(a)



5(c)

$e = \frac{V_2 + V_1}{0.5 + 5.5} = \frac{V_2 + V_1}{6}$, $6e = V_2 + V_1 \rightarrow V_2 = 6e - V_1$,

$(0.5 \times 5) + (-5.5 \times 3) = -5V_1 + 3V_2$,

$-14 = -5V_1 + 3(6e - V_1)$,

$-14 = -5V_1 + 18e - 3V_1$,

$8V_1 = 18e - 14$, $V_1 = \frac{18e - 14}{8} = \frac{9e - 7}{4}$

5(c) (continued) no further collisions $\rightarrow V_1 < -3.25$,

$\frac{9e - 7}{4} < -3.25$ $| \times 4$, $9e - 7 < -13$,

$9e <$ ~~20~~ -6, $e > -\frac{2}{3}$,

$-\frac{2}{3} < e < 1$

This response is a good illustration of the pitfalls of not using consistent directions in collision questions, and of checking work, especially when the results are not as expected.

The diagram at the top of part (a) shows that for their velocity V_1 (object B after the second collision, V_B in the mark scheme) a positive value would be to the left, meaning that their equations for NEL and COLM are correct by their diagram, so the first 3 marks are gained.

The candidate has then correctly eliminated V_2 between the equations but made a slip in the form of a sign error when moving -14 from the LHS to the RHS. This still gains the third M mark but loses any further A marks.

They have then stated that $V_1 < -3.25$. Unfortunately, by this point they have forgotten that V_1 is pointing to the left in their diagram, so the limit should be a positive, not a negative value, and this therefore loses the fifth mark. It should also be an inclusive inequality, but this is not penalised until the final mark.

Their equation then gives $e < -^2/_3$, which is impossible, but the candidate has changed this to $e > -^2/_3$ without justification rather than going back and checking their work for prior errors.

The final answer is wrong, so loses the final mark. It also includes negative values for *e*¸ which the candidate has not attempted to address.

Question 6

6 A motorbike and its rider, together denoted by M, have a combined mass of 360 kg. The resistive force experienced by M when it is in motion is modelled as being proportional to the speed it is moving at. All motion of M is on a straight horizontal road.

It is found that with the engine of the motorbike working at a rate of 12 kW, the maximum constant speed that M can move at is $10\,\mathrm{m\,s^{-1}}$.

Determine the speed of M such that with the engine working at a rate of 12 kW the acceleration of M is $1.5\,\mathrm{m\,s^{-2}}$. **[7]**

This question was tackled well by most candidates, who were usually given 6 or 7 marks. However, a substantial minority did not realise that there are two distinct parts to the problem or did not know how to connect them properly. A small number did not know how to use the given information to find the coefficient of resistance, and therefore made little or no progress.

The first 3 marks are for using the first scenario to find the coefficient of resistance (e.g. *k*), which the great majority of candidates attempted with some degree of success. The remaining marks are then for using this result to solve the second scenario where the motorbike is now moving at a different speed and acceleration. Candidates who lost marks here typically assumed that the driving force and/or friction were the same in both scenarios despite the speeds being different, or tried to do the second part without first finding *k*, or used the acceleration from the second part to find *k*. Other common errors included not dividing the power by the unknown *v,* or dividing the power by the resultant force, thus tacitly assuming that there was no friction.

Those who obtained and solved the correct quadratic in *v* then had to deduce which of the two solutions was correct. Many candidates did not realise that the answer is non-negative because *v* is the speed of the motorbike rather than the velocity, and not for any other reason. It is, for example, quite possible for the velocity and acceleration to be in opposite directions, so the contrary assertion is not a correct reason for the answer to be positive.

Question 7 (a)

7 A particle P of mass 3.5 kg is attached to one end of a rod of length 5.4 m. The other end of the rod is hinged at a fixed point O and P hangs in equilibrium directly below O.

A horizontal impulse of magnitude 44.1 N s is applied to P.

In an initial model of the subsequent motion of P the rod is modelled as being light and inextensible and all resistance to the motion of P is ignored. You are given that P moves in a circular path in a vertical plane containing O. The angle that the rod makes with the downward vertical through O is θ radians.

(a) Determine the largest value of θ in the subsequent motion of P. **[5]**

This question is a standard vertical circle problem, with a given impulse at the start rather than the usual initial speed. Some candidates were given full marks or most of the marks for this question and some were given few marks, with not many in the middle, suggesting that candidates generally either did or did not know how to solve such problems.

Most candidates were able to find the initial speed from the given impulse and then correctly used it to find the initial KE required to solve the problem. One or two thought they were calculating the acceleration and in some cases equated it to $\frac{v^2}{r}$ to find v, or mistook the impulse for a force. As seen in previous years, a very small number thought they were doing a horizontal circle problem, so did not get more than 1 or 2 marks in this question.

Some candidates did not know how to express the height correctly to get the GPE, sometimes using $r\cos\theta$ or $(1-\cos\theta)$ instead of $r(1-\cos\theta)$, for example, or missed out other terms. Some tried to simplify the problem by working with the angle to the upward vertical or to the horizontal, but then did not always correctly derive the angle to the downward vertical. We would recommend that candidates learn the correct expression for the height, as it is almost always required in this type of problem.

Finally, some gave the answer in degrees instead of radians as specified by the question and unfortunately lost the final mark. We would always advise candidates to double-check that they have answered each question exactly as it has been asked.

Question 7 (b)

In a **revised** model the rod is still modelled as being light and inextensible but the resistance to the motion of P is not ignored. Instead, it is modelled as causing a loss of energy of 20 J for every metre that P travels.

(b) Show that according to the **revised** model, the maximum value of θ in the subsequent motion of P satisfies the following equation.

$343(1+2\cos\theta)=400\theta$ **[2]**

This question proved extremely challenging for even the most successful candidates, with many not attempting the question or being given 0 marks, and remainder fairly equally split between 1 and 2 marks. Few candidates recognised that the key to this problem was recognising that the motion of P is an arc, the length of which then needs to be multiplied by 20 and subtracted from the final GPE as work done against resistance.

Those candidates that did attempt to solve this problem often did not know that the correct formula for the length of an arc is $r\theta$ (in radians), or worked in degrees instead and scaled incorrectly, e.g. by multiplying by $\frac{\theta}{360}$ instead of $\frac{\theta}{180}$, worked out the circumference of the circle or did not multiply it by the force. Others attempted to work backwards or in some cases tried to solve the equation instead of deriving it, or even assumed a given distance and divided by r to find θ.

Those that found the correct resistance term sometimes added it to the initial KE by mistake or did not always succeed in completing satisfactorily to the given answer, but a small proportion of candidates did succeed in answering fully and correctly.

Question 7 (c)

You are given that $\theta = 1.306$ is the solution to the above equation, correct to **4** significant figures.

(c) Determine the difference in the predicted maximum vertical heights attained by P using the two models. Give your answer correct to **3** significant figures. **[3]**

The simplest approach here was to equate the loss of energy due to resistance from part (b) to the loss of GPE and then divide by mg to give the height difference. Candidates generally either knew how to solve this question or did not; most either omitted the question, were given full marks or were given 0 marks.

Most opted to work out the actual height achieved in each scenario from the angle found in part (a) and the angle given in part (c). This avoided the tricky issue of finding the work done against resistance, which most had not done in part (b). Candidates were successful in most cases where they had already solved part (a) using a correct expression for the height in terms of θ. Those that used other methods to solve part (a) were not always as successful, often gaining only 1 or 2 marks.

Question 7 (d)

(d) Suggest **one** further improvement that could be made to the model of the motion of P. **[1]**

To succeed at this modelling question, candidates needed to suggest something that had not previously been covered in the text of the question, or which contradicted an existing assumption. The majority of candidates were not given the mark for this. The most popular correct response was to consider the mass or weight of the rod, as it could materially affect the motion. Suggesting that the rod was extensible was also a valid comment, as was the idea that resistance might vary with the speed of the pendulum, that there might be some spin, or that the size and/or shape of the particle may affect the motion.

Most candidates either omitted the question or simply repeated learned answers without considering the information in the question. Answers such as including air resistance or friction, both of which are already covered by part (b), were given no marks. Some mentioned the tension in the rod without relating it to the motion, while a few even thought that the rod was in fact a string.

Assessment for learning

Present students with a variety of questions and contexts and justify what modelling assumptions have been made to create a manageable but reasonably realistic problem to solve.

Then get students to discuss which assumptions are missing (and should therefore be included), and which ones could be changed or replaced to improve the model, justifying their answers.

Then get them to critique a variety of right and wrong responses to these or other questions.

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