



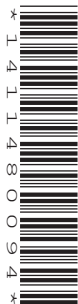
Oxford Cambridge and RSA

**Monday 13 May 2024 – Afternoon**

**AS Level Further Mathematics B (MEI)**

**Y410/01 Core Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**QP**

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

- 1 The quadratic equation  $x^2 + ax + b = 0$ , where  $a$  and  $b$  are real constants, has a root  $2 - 3i$ .
- (a) Write down the other root. [1]
- (b) Hence or otherwise determine the values of  $a$  and  $b$ . [3]
- 2 The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are given by  $\mathbf{A} = \begin{pmatrix} 1 & a \\ -1 & 2 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$ , where  $a$  is a constant.
- (a) By multiplying out the matrices on both sides of the equation, verify that  $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$ . [4]
- (b) State the property of matrix multiplication illustrated by this result. [1]
- 3 (a) Using standard summation formulae, write down an expression in terms of  $n$  for  $\sum_{r=1}^{2n} r^3$ . [1]
- (b) Hence show that  $\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2(an + b)(cn + d)$ , where  $a$ ,  $b$ ,  $c$  and  $d$  are integers to be determined. [5]
- 4 **In this question you must show detailed reasoning.**
- The roots of the cubic equation  $x^3 - 3x^2 + 19x - 17 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ .
- (a) Find a cubic equation with integer coefficients whose roots are  $\frac{1}{2}(\alpha - 1)$ ,  $\frac{1}{2}(\beta - 1)$  and  $\frac{1}{2}(\gamma - 1)$ . [4]
- (b) Hence or otherwise solve the equation  $x^3 - 3x^2 + 19x - 17 = 0$ . [3]

- 5 (a) Find the volume scale factor of the transformation with associated matrix  $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix}$ . [2]

(b) The transformations S and T of the plane have associated  $2 \times 2$  matrices **P** and **Q** respectively.

- (i) Write down an expression for the associated matrix of the combined transformation S followed by T. [1]

The determinant of **P** is 3 and  $\mathbf{Q} = \begin{pmatrix} k & 3 \\ -1 & 2 \end{pmatrix}$ , where  $k$  is a constant.

- (ii) Given that this combined transformation preserves both orientation and area, determine the value of  $k$ . [3]

- 6 You are given that  $\mathbf{M} = \begin{pmatrix} 4 & -9 \\ 1 & -2 \end{pmatrix}$ .

- (a) Prove that  $\mathbf{M}^n = \begin{pmatrix} 1+3n & -9n \\ n & 1-3n \end{pmatrix}$  for all positive integers  $n$ . [6]

- (b) A student thinks that this formula, when  $n = 0$  and  $n = -1$ , gives the identity matrix and the inverse matrix  $\mathbf{M}^{-1}$  respectively.

Determine whether the student is correct. [3]

- 7 Three planes have equations

$$x + 2y - 3z = 0,$$

$$-x + 3y - 2z = 0,$$

$$x - 2y + kz = k,$$

where  $k$  is a constant.

- (a) For the case  $k = 0$ , the origin lies on all three planes.

Use a determinant to explain whether there are any other points that lie on all three planes in this case. [2]

- (b) You are now given that  $k = 1$ .

- (i) Show that there are no points that lie on all three planes. [3]

- (ii) Describe the geometrical arrangement of the three planes. [1]

- 8 In an Argand diagram, the point P representing the complex number  $w$  lies on the locus defined by  $\{z: \arg(z-7) = \frac{3}{4}\pi\}$ . You are given that  $\operatorname{Re}(w) = 1$ .

(a) Find  $w$ . [2]

The point P also lies on the locus defined by  $\{z: |z+3-9i| = k\}$ , where  $k$  is a constant.

- (b) Find the complex number represented by the other point of intersection of the loci defined by  $\{z: |z+3-9i| = k\}$  and  $\{z: \arg(z-7) = \frac{3}{4}\pi\}$ . [7]

9 In this question you must show detailed reasoning.

Find a vector  $\mathbf{v}$  which has the following properties.

- It is a unit vector.
- It is parallel to the plane  $2x + 2y + z = 10$ .
- It makes an angle of  $45^\circ$  with the normal to the plane  $x + z = 5$ . [8]

**END OF QUESTION PAPER**

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