OCR
Oxford Cambridge and RSA

Friday 17 May 2024 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

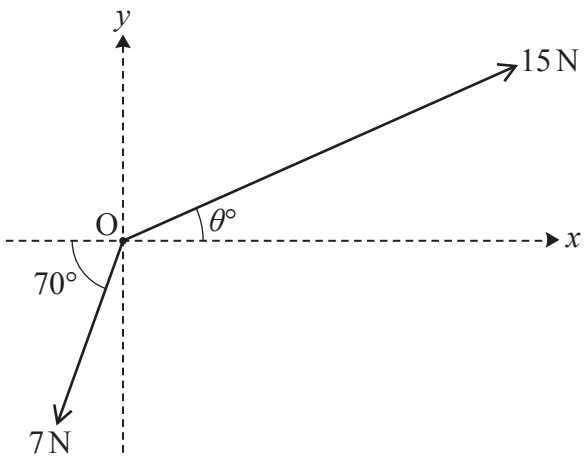
QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 Two horizontal forces of magnitudes 7 N and 15 N act at a point O.
The 15 N force acts an angle of $\theta°$ above the positive x-axis.
The 7 N force acts at an angle of 70° below the negative x-axis (see diagram).



The resultant of the two forces acts only in the positive x-direction.

(a) Calculate the value of θ. **[2]**

(b) Calculate the magnitude of the resultant of the two forces. **[2]**

2 **(a)** Find the dimensions of energy. **[1]**

The moment of inertia, I, of a rigid body rotating about a fixed axis is measured in kg m^2.

(b) State the dimensions of I. **[1]**

The kinetic energy, E, of a rigid body rotating about a fixed axis is given by the formula

$$E = \frac{1}{2}I\omega^2,$$

where ω is the angular velocity (angle per unit time) of the rigid body.

(c) Show that the formula for E is dimensionally consistent. **[2]**

When a rigid body is pivoted from one of its end points and allowed to swing freely, it forms a pendulum. The period, t, of the pendulum is the time taken for it to complete one oscillation. A student conjectures the formula

$$t = k(mg)^{\alpha} r^{\beta} I^{\gamma},$$

where

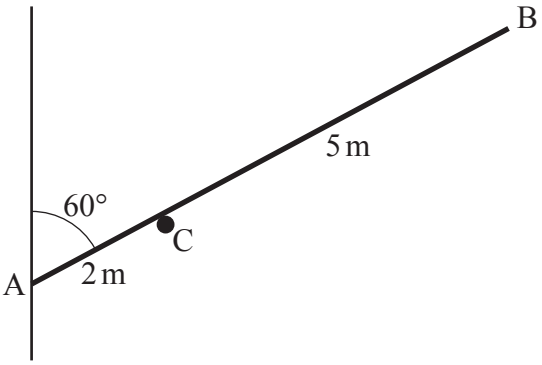
- k is a dimensionless constant,
- m is the mass of the rigid body,
- g is the acceleration due to gravity,
- r is the distance between the pivot point and the rigid body's centre of mass.

(d) Use dimensional analysis to find the values of α, β and γ. **[4]**

The moment of inertia of a thin uniform rigid rod of mass 1.5 kg and length 0.8 m, rotating about one of its endpoints, is $0.32\,\text{kg m}^2$. The student suspends such a rod from one of its endpoints and allows it to swing freely. The student measures the period of this pendulum and finds that it is 1.47 seconds.

(e) Using the formula conjectured by the student, determine the value of k. **[3]**

3 The diagram shows a uniform beam AB, of weight 80 N and length 7 m, resting in equilibrium in a vertical plane. The end A is in contact with a rough vertical wall, and the angle between the beam and the upward vertical is 60°. The beam is supported by a smooth peg at a point C, where AC = 2 m.



(a) Complete the diagram in the **Printed Answer Booklet** to show all the forces acting on the beam. **[2]**

(b) **(i)** Show that the magnitude of the frictional force exerted on the beam by the wall is 25 N. **[3]**

(ii) Hence determine the magnitude of the total contact force exerted on the beam by the wall. **[3]**

(c) Determine the direction of the total contact force exerted on the beam by the wall. **[2]**

The coefficient of friction between the beam and the wall is μ.

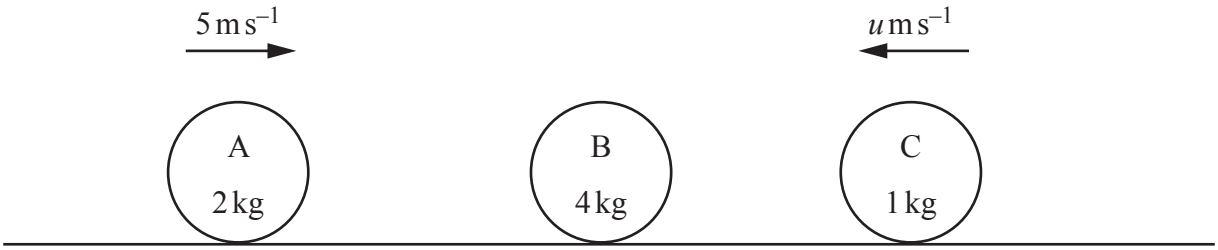
(d) Find the range of possible values for μ. **[2]**

(e) Explain how your answer to part **(b)(ii)** would change if the peg were situated closer to A but the angle between the beam and the upward vertical remained at 60°. **[1]**

4 Three spheres A, B, and C, of equal radius are in the same straight line on a smooth horizontal surface. The masses of A, B and C are 2 kg, 4 kg and 1 kg respectively.

Initially the three spheres are at rest.

Spheres A and C are each given impulses so that A moves towards B with speed $5\,\text{m s}^{-1}$ and C moves towards B with speed $u\,\text{m s}^{-1}$ as shown in the diagram below.



The coefficient of restitution between A and B is $\frac{4}{5}$.

It is given that the first collision occurs between A and B.

(a) State how you can tell from the information given above that kinetic energy is lost when A collides with B. **[1]**

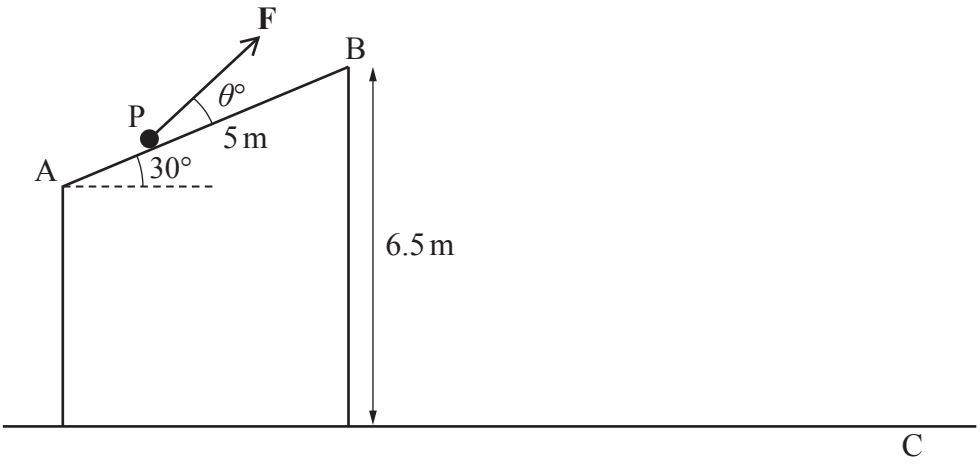
(b) Show that the combined kinetic energy of A and B decreases by 24% during their collision. **[5]**

Sphere B next collides with C. The coefficient of restitution between B and C is $\frac{2}{3}$.

(c) Given that a third collision occurs, determine the range of possible values for u. **[6]**

(d) State **one** limitation of the model used in this question. **[1]**

5 In the diagram below, points A, B and C lie in the same vertical plane. The slope AB is inclined at an angle of 30° to the horizontal and AB = 5 m. The point B is a vertical distance of 6.5 m above horizontal ground. The point C lies on the horizontal ground.



Starting at A, a particle P, of mass m kg, moves along the slope towards B, under the action of a constant force **F**. The force **F** has a magnitude of 50 N and acts at an angle of $\theta°$ to AB in the same vertical plane as A and B. When P reaches B, **F** is removed, and P moves under gravity landing at C.

It is given that

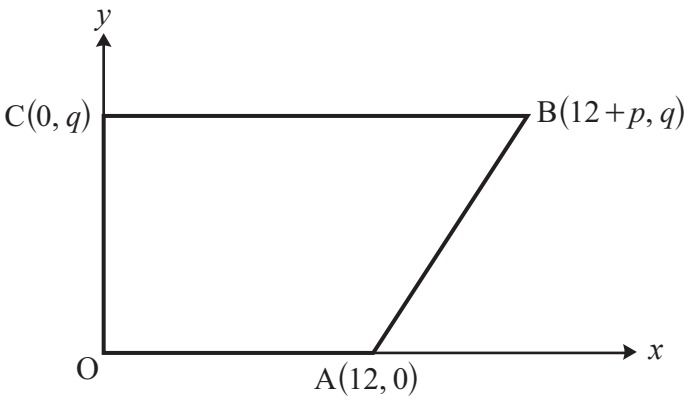
- the speed of P at A is $3\,\text{m}\,\text{s}^{-1}$,
- the speed of P at B is $6\,\text{m}\,\text{s}^{-1}$,
- the speed of P at C is $12\,\text{m}\,\text{s}^{-1}$,
- 58 J of work is done against non-gravitational resistances as P moves from A to B,
- 42 J of work is done against non-gravitational resistances as P moves from B to C.

(a) By considering the motion from B to C, show that $m = 4.33$ correct to **3** significant figures. **[3]**

(b) By considering the motion from A to B, determine the value of θ. **[4]**

(c) Calculate the power of **F** at the instant that P reaches B. **[2]**

6 A uniform lamina OABC is in the shape of a trapezium where O is the origin of the coordinate system in which the points A, B and C have coordinates (12, 0), $(12+p, q)$ and $(0, q)$ respectively.



(a) Determine, in terms of p and q, the coordinates of the centre of mass of OABC. **[4]**

The point D has coordinates $(7.6, q)$. When OABC is suspended from D, the lamina hangs in equilibrium with BC horizontal.

(b) Determine the value of p. **[3]**

When OABC is suspended from C, the lamina hangs in equilibrium with BC at an angle of 35° to the downward vertical.

(c) Determine the value of q, giving your answer correct to **3** significant figures. **[3]**

END OF QUESTION PAPER