OCR
Oxford Cambridge and RSA

# Friday 14 June 2024 – Afternoon

## AS Level Further Mathematics B (MEI)

### Y414/01 Numerical Methods

**Time allowed: 1 hour 15 minutes**

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

**1** The difference table shows some values of $x$ and the associated values of $y$.

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 0 | 0.8 | | | |
| | | | | |
| 1 | 0.7 | | | |
| | | | | |
| 2 | 0.1 | | | |
| | | | | |
| 3 | 11.3 | | | |
| | | | | |
| 4 | 70.5 | | | |
| | | | | |
| 5 | 250.6 | | | |

**(a)** Complete the copy of the difference table in the **Printed Answer Booklet**. **[2]**

**(b)** Explain whether a cubic polynomial would be suitable to model the relationship between $y$ and $x$. **[1]**

**2** A spreadsheet is used to find the approximate value of $\pi + \text{e}$. The output is shown in the table.

| | A | B |
|---|---|---|
| 1 | $\pi$ | 3.141 593 |
| 2 | e | 2.718 282 |
| 3 | $\pi + \text{e}$ | 5.859 874 |

The formula in cell B1 is `= PI()`.

The formula in cell B2 is `= EXP(1)`

and the formula in cell B3 is `= BI+B2`.

**(a)** Show that the approximations to $\pi$ and e displayed in cells B1 and B2 respectively have both been rounded. **[1]**

**(b)** Find the relative error when 3.141 593 is used to approximate $\pi$. **[2]**

**(c)** Explain why the value displayed in cell B3 is not equal to $2.718\,282 + 3.141\,593$. **[2]**

**3** Following an injury a professional footballer has to undergo a rehabilitation programme before she can resume playing. As part of the programme she is required to take weekly fitness tests to assess her progress. When she achieves a score of 95% or over she is deemed fit enough to be part of the match day squad.

The footballer was ill at the end of week 3 and so did not take the fitness test. The percentage score she achieved, $P$, at the end of each week, $w$, is shown for $w = 1$, 2 and 4 in the table.

| $w$ | 1 | 2 | 4 |
|---|---|---|---|
| $P$ | 52.00 | 62.54 | 79.90 |

The medical staff at the footballer's club believe the data may be modelled by a polynomial.

**(a)** Use Lagrange's form of the interpolating polynomial to construct a polynomial of degree 2 to model the relationship between $P$ and $w$. **[4]**

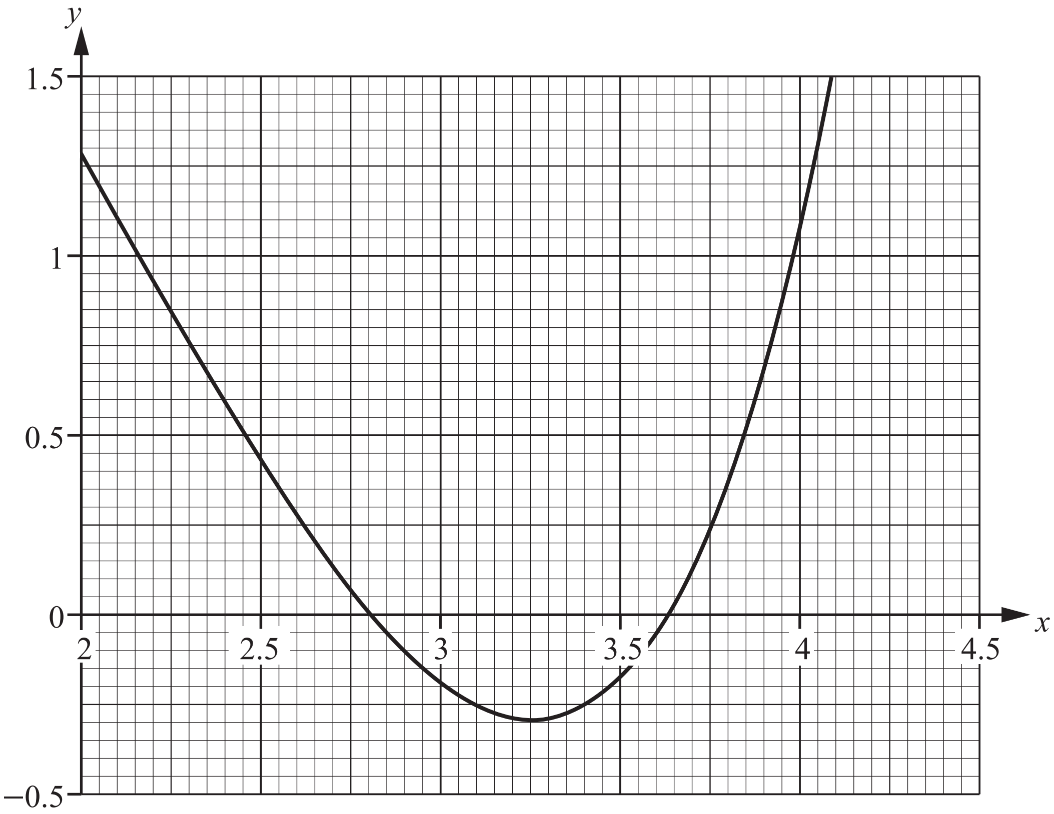
The club are due to play a match at the end of week 6 and another match at the end of week 7.

**(b)** Determine whether, according to the model, the footballer will be available to play in either of these matches. **[3]**

After returning to the match day squad, the medical staff continue to give the footballer a fitness test at the end of each week.

**(c)** Explain why the model is no longer viable from the end of week 9. **[1]**

**4** The graph shows part of the graph of $y = \mathrm{f}(x)$.




The equation $\mathrm{f}(x) = 0$ has two positive roots, $\alpha$ and $\beta$, where $\alpha > \beta$.

The method of False Position is used to find $\alpha$. A spreadsheet is used to find a sequence of approximations to $\alpha$. The output is shown in **Table 4.1**.

**Table 4.1**

| *r* | V | W | X | Y | Z | AA |
|---|---|---|---|---|---|---|
| 3 | *a* | f(*a*) | *b* | f(*b*) | | |
| 4 | 3 | −0.18289 | 4 | 1.11963 | 3.14041 | −0.26707 |
| 5 | 3.14041 | −0.26707 | 4 | 1.11963 | 3.30597 | −0.28855 |
| 6 | 3.30597 | −0.28855 | 4 | 1.11963 | 3.44818 | −0.22333 |
| 7 | 3.44818 | −0.22333 | 4 | 1.11963 | 3.53995 | −0.13196 |
| 8 | 3.53995 | −0.13196 | 4 | 1.11963 | 3.58845 | −0.06598 |
| 9 | 3.58845 | −0.06598 | 4 | 1.11963 | 3.61136 | −0.03027 |
| 10 | 3.61136 | −0.03027 | 4 | 1.11963 | 3.62159 | −0.01334 |
| 11 | 3.62159 | −0.01334 | 4 | 1.11963 | 3.62604 | −0.00577 |
| 12 | 3.62604 | −0.00577 | 4 | 1.11963 | 3.62796 | −0.00248 |

**(a)** On the copy of the graph in the **Printed Answer Booklet**, show how the method of False Position works to find the next **two** approximations to $\alpha$ using starting values of $x = a = 3$ and $x = b = 4$. **[2]**

The formula in cell W4 is $= 0.2 * \text{EXP}(\text{V4}) - 0.8 * \text{V4}^\wedge 2 + 3$ .

**(b)** Write down the equation which is being solved. **[2]**

The formula in cell V5 is $= \text{IF}(\text{AA4} < 0, \text{Z4}, \text{V4})$ .

**(c)** Write down a similar formula for cell X5. **[1]**

**(d)** By considering the output in **Table 4.1**, state the value of $\alpha$ as accurately as possible. You must justify the precision quoted. **[1]**

The Newton-Raphson method is used with $x_0 = 2$ to find a sequence of approximations to $\beta$. The associated spreadsheet output is shown in **Table 4.2**.

**Table 4.2**

| $r$ | $x_r$ | difference | ratio of differences |
|---|---|---|---|
| 0 | 2 | | |
| 1 | 2.741969 | 0.741969 | |
| 2 | 2.811136 | 0.069167 | 0.09322 |
| 3 | 2.814353 | 0.003217 | 0.04651 |
| 4 | 2.814361 | 7.67E-06 | 0.00238 |
| 5 | 2.814361 | 4.38E-11 | 5.7E-06 |

**(e)** Use the information in **Table 4.2** to state the value of $\beta$ as accurately as possible. You must justify the precision quoted. **[1]**

**(f)** Explain what the ratios of differences tell you about the order of convergence of the sequence of approximations to $\beta$. **[2]**

**(g)** State **two** advantages of using the Newton-Raphson method instead of the method of False Position for finding the roots of an equation. **[2]**

**5** **Table 5.1** shows some values of $x$ and the associated values of $f(x)$.

**Table 5.1**

| $x$ | 5 | 5.0001 | 5.001 | 5.01 | 5.1 |
|---|---|---|---|---|---|
| $f(x)$ | 12.182494 | 12.183103 | 12.188587 | 12.243559 | 12.807104 |

**(a)** Use the forward difference method to obtain four approximations to $f'(5)$. Write your answers in the copy of **Table 5.2** in the **Printed Answer Booklet**. **[5]**

**Table 5.2**

| $h$ | approximation to $f'(5)$ |
|---|---|
| 0.1 | |
| 0.01 | |
| 0.001 | |
| 0.0001 | |

**(b)** State the value of $f'(5)$ as accurately as you can. You must justify the precision quoted. **[1]**

Ali uses a smaller value of $h$ to find an approximation to $f'(5)$. Ali's spreadsheet output is shown in **Table 5.3**.

**Table 5.3**

| $h$ | approximation to $f'(5)$ |
|---|---|
| 1.00E-14 | 6.039613 |

**(c)** Explain whether Ali's approximation to $f'(5)$ is likely to be more accurate or less accurate than your answer to part **(b)**. **[2]**

**(d)** Determine an approximation of the error in using $f(5)$ to approximate $f(5.02)$. **[2]**

**6** The equation $5\log_3 x - x = 0$ has two roots, $\alpha$ and $\beta$.

You are given that $1 < \alpha < 2$.

**(a)** Use the iterative formula $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x_n) = 3^{\frac{x_n}{5}}$ and $x_0 = 2$, to determine the value of $\alpha$ correct to **6** decimal places. **[3]**

You are given that $\beta \approx 10.9$.

The table shows some approximations to $\mathrm{g}'(10.9)$ found using the central difference method.

| $h$ | approximation to $\mathrm{g}'(10.9)$ |
|---|---|
| 0.1 | 2.410 093 |
| 0.01 | 2.409 901 |
| 0.001 | 2.409 899 |

**(b)** **(i)** Explain why it is preferable to use the central difference method instead of the forward difference method where possible. **[1]**

**(ii)** Show how the approximation with $h = 0.1$ is calculated. **[2]**

**(iii)** Write down the value of $\mathrm{g}'(10.9)$ as accurately as you can. You must justify the precision quoted. **[1]**

**(c)** Explain why your answer to part **b(iii)** suggests that the iterative formula

$$x_{n+1} = \mathrm{g}(x_n), \text{ where } \mathrm{g}(x_n) = 3^{\frac{x_n}{5}},$$

may not be used to successfully find $\beta$, however close the starting value is to $\beta$. **[1]**

**(d)** Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n) \text{ with } \lambda = -0.269 \text{ and } x_0 = 10.9$$

to determine the value of $\beta$ correct to **6** decimal places. **[3]**

**7** A student uses the midpoint rule, the trapezium rule and Simpson's Rule to find a series of approximations to $\int_0^1 f(x)\,dx$. The associated spreadsheet output, together with some further analysis, is shown in the table.

| $r$ | F | G | H | I | J | K |
|---|---|---|---|---|---|---|
| 1 | $n$ | $M_n$ | $T_n$ | $S_{2n}$ | difference | ratio of differences |
| 2 | 1 | 0.8660254 | 0.5 | 0.74401694 | | |
| 3 | 2 | 0.81484183 | 0.6830127 | 0.77089879 | 0.0268819 | |
| 4 | 4 | 0.79598231 | 0.74892727 | 0.78029729 | 0.0093985 | 0.35 |
| 5 | 8 | 0.78917173 | 0.77245479 | 0.78359942 | 0.0033021 | 0.351 |
| 6 | 16 | 0.78673795 | 0.78081326 | 0.78476305 | 0.0011636 | 0.352 |
| 7 | 32 | 0.78587285 | 0.78377561 | 0.78517377 | 0.0004107 | 0.353 |
| 8 | 64 | 0.78556617 | 0.78482423 | 0.78531885 | 0.0001451 | 0.353 |

**(a)** Write down a suitable spreadsheet formula for cell I2. **[2]**

The formula in cell J3 is $= \text{I3} - \text{I2}$ .

**(b)** Write down a suitable spreadsheet formula for cell K4. **[1]**

**(c)** By considering the values in column H, explain what may be inferred about the shape of the curve $y = f(x)$. **[1]**

**(d)** State the order of convergence of the sequence of values in column K. **[1]**

**(e)** Explain what the values of the ratios of differences tell you about the order of Simpson's Rule in this case. **[2]**

**In this question you must show detailed reasoning.**

**(f)** Determine the value of $\int_0^1 f(x)\,dx$ as accurately as you can. You must justify the precision quoted. **[5]**

**END OF QUESTION PAPER**