OCR
Oxford Cambridge and RSA

Thursday 23 May 2024 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

1334920365

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \text{ or } \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

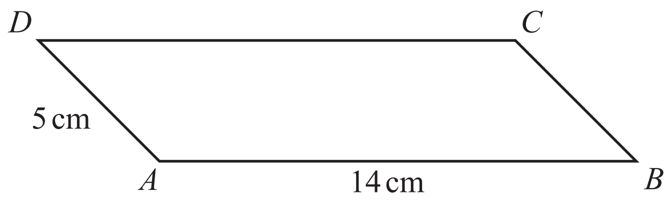
Section A
Pure Mathematics

1 **(a)** Write $3x^2 + 24x + 5$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be determined. **[3]**

The finite region R is enclosed by the curve $y = 3x^2 + 24x + 5$ and the x-axis.

(b) State the inequalities that define R, including its boundaries. **[2]**

2



The diagram shows a parallelogram $ABCD$ where $AB = 14\text{ cm}$, $AD = 5\text{ cm}$ and angle DAB is obtuse. The area of $ABCD$ is 50 cm^2.

(a) Determine, in degrees, the size of angle DAB. Give your answer correct to **1** decimal place. **[3]**

(b) Find the length, in centimetres, of the diagonal BD. Give your answer correct to **1** decimal place. **[2]**

3 The quadratic equation $kx^2 + 2kx + 2k = 3x - 1$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$4k^2 + 16k - 9 > 0.$$ **[4]**

(b) Hence find the set of possible values of k. Give your answer in set notation. **[2]**

4 **(a)** The curve $y = \sqrt{2x-1}$ is stretched by a scale factor $\frac{1}{4}$ parallel to the x-axis.

Find the equation of the curve after it has been stretched. **[2]**

(b) The curve $y = \sqrt{2x-1}$ is translated by the vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$, and as a result, the point on the curve where $x = 5$ is transformed to the point P.

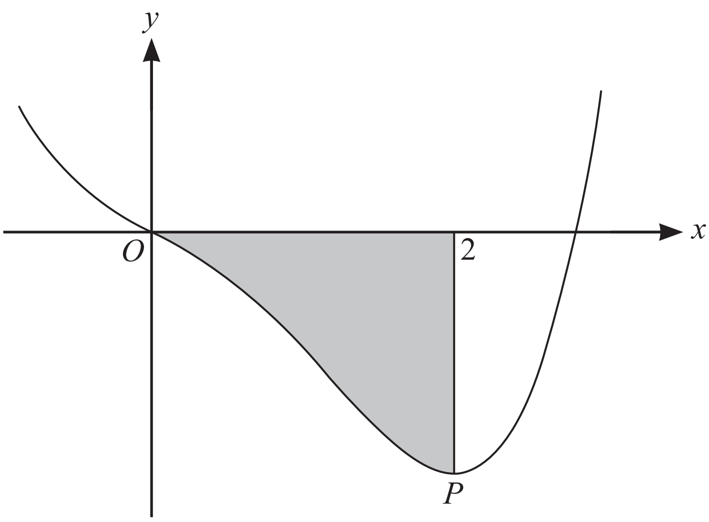
Find the coordinates of P. **[2]**

5 At the point P on the curve $y = e^{3x} - 21x - 8$ the gradient of the tangent is 3.

(a) Determine the x-coordinate of P. Give your answer in the form $\ln a$, where a is an integer to be determined. **[4]**

(b) Hence determine the equation of the tangent at P. Give your answer in the form $y = 3x + c$ where c is an exact constant to be determined. **[3]**

6

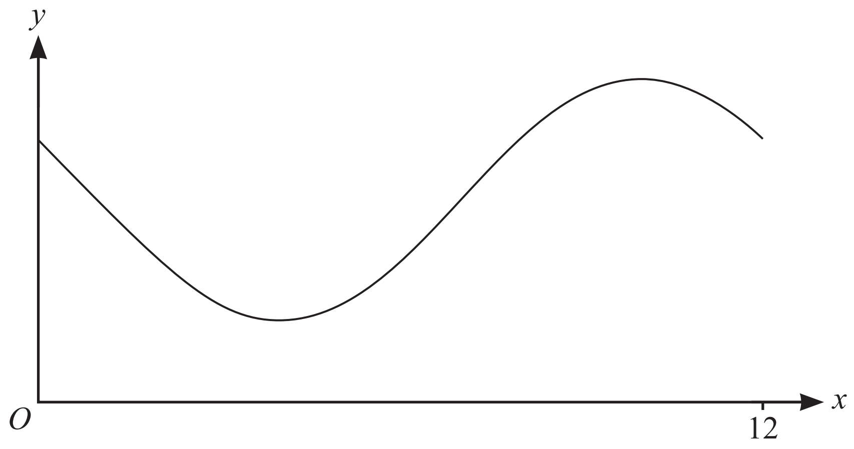


The diagram shows the curve with equation $y = 5x^4 + ax^3 + bx$, where a and b are integers. The curve has a minimum at the point P where $x = 2$.

The shaded region is enclosed by the curve, the x-axis and the line $x = 2$.

Given that the area of the shaded region is 48 units^2, determine the y-coordinate of P. **[7]**

7



The diagram shows the curve with equation $y = 5 + 3\cos(30x + 60)°$, for $0 \leqslant x \leqslant 12$.

(a) **(i)** State the greatest value of y for points on the curve. **[1]**

(ii) Determine the value of x for which this greatest value of y occurs. **[3]**

There are two points on the curve for which the value of y is 7.

(b) Determine the values of x at these two points. **[5]**

8 The points A and B have coordinates (4, 8) and (6, p) respectively, where p is a positive constant and $p \neq 8$.

(a) Find, in terms of p, the coordinates of the midpoint of AB. **[1]**

(b) Find, in terms of p, the gradient of the line AB. **[1]**

The perpendicular bisector of AB intersects the positive x-axis at the point where $x = \frac{7}{5}p$.

(c) Determine the x-coordinate of this point of intersection. **[5]**

Section B
Mechanics

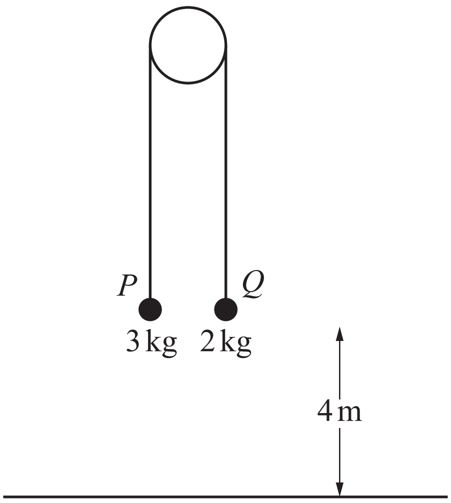
9 A particle P of mass 2.5 kg is in equilibrium under the action of three horizontal forces $\mathbf{F}_1 = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$N, $\mathbf{F}_2 = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$N and $\mathbf{F}_3$.

(a) Find the force $\mathbf{F}_3$. **[2]**

The force $\mathbf{F}_3$ is changed to $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$N.

(b) Find the acceleration of P, giving your answer in column vector form. **[2]**

10



Two small balls P and Q, of masses 3 kg and 2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The balls are held at a height of 4 m above a horizontal floor, with the string taut. The parts of the string which are not in contact with the pulley are vertical (see diagram).

The system is set in motion in such a way that P moves vertically downwards.

(a) Determine the tension in the string immediately after the system is set in motion. **[4]**

The initial downward speed of P is $4.2\,\mathrm{m\,s^{-1}}$ and when P reaches the floor it is immediately brought to rest. It is given that Q does not reach the pulley in the subsequent motion.

(b) Find the speed of P as it hits the floor. **[2]**

(c) Determine the time, after the start of the motion, at which Q reaches its greatest height above the floor. **[4]**

(d) State what it would mean about the accelerations of the balls if the string could not be assumed to be inextensible. **[1]**

11 A man P runs in a straight line from O to A, leaving O at time $t = 0$. At time t seconds his velocity $v\,\mathrm{m\,s^{-1}}$ is given by $v = 5 + 0.003t^2$.

Another man Q runs from O to A at a constant speed of $7.08\,\mathrm{m\,s^{-1}}$, starting 5 seconds after P.

(a) Show that the times T s when P and Q are the same distance from O satisfy the equation

$T^3 - 2080T + 35400 = 0$. **[5]**

(b) **In this question you must show detailed reasoning.**

As they run from O to A there are two times at which P and Q are the same distance from O. The second of these is when $T = 30$.

Find the acceleration of P at the first of these times. **[5]**

END OF QUESTION PAPER