OCR
Oxford Cambridge and RSA

Thursday 16 May 2024 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 The triangle ABC has an obtuse angle at A. The angle at B is 15°. The length of AC is 10 cm and the length of BC is 13 cm.

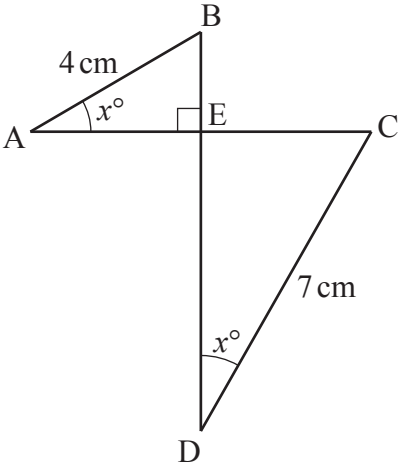
Calculate the size of the angle at A. **[2]**

2 Two forces $\mathbf{F}_1$ N and $\mathbf{F}_2$ N are given by $\mathbf{F}_1 = -6\mathbf{i} + 2\mathbf{j}$ and $\mathbf{F}_2 = -8\mathbf{i} + \mathbf{j}$.

Show that the magnitude of the resultant of these two forces is $\sqrt{205}$ N. **[2]**

3 Prove that, when n is an even number, $n^3 + 4$ is a multiple of 4 but not a multiple of 8. **[3]**

4 The perpendicular lines AC and BD intersect at E as shown in the diagram. The point E is the midpoint of AC. The angles BAC and BDC are each equal to $x°$. The lengths of AB and CD are 4 cm and 7 cm respectively.



Determine the value of x. **[4]**

5 **In this question you must show detailed reasoning.**

(a) Show that the gradient of the curve $y = \sqrt{x}\left(\frac{1}{x^2} - 2x\right)$ at the point $\left(\frac{1}{4}, \frac{31}{4}\right)$ is $-\frac{99}{2}$. **[4]**

(b) Find the equation of the tangent to the curve at $\left(\frac{1}{4}, \frac{31}{4}\right)$ giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[2]**

6 The polynomial $x^3 - 4x^2 + 10x - 21$ is denoted by $\mathrm{f}(x)$.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $\mathrm{f}(x)$. **[2]**

(b) The polynomial $\mathrm{f}(x)$ can be written as $(x - 3)(x^2 + bx + c)$ where b and c are constants.

Find the values of b and c. **[2]**

(c) Show that $x = 3$ is the only real root of the equation $\mathrm{f}(x) = 0$. **[2]**

7 The velocity of a particle moving in a straight line is modelled by $v = 0.6t^2 - 2.1t + 1.5$ where v is the velocity in metres per second and t is the time in seconds.

(a) Determine the times at which the particle is stationary. **[2]**

(b) Find the acceleration of the particle at the first of the times at which it is stationary. **[2]**

(c) Find the distance travelled by the particle between the times at which it is stationary. **[2]**

8 A circle with centre C has equation $x^2 + y^2 - 6x - 16y + 48 = 0$.

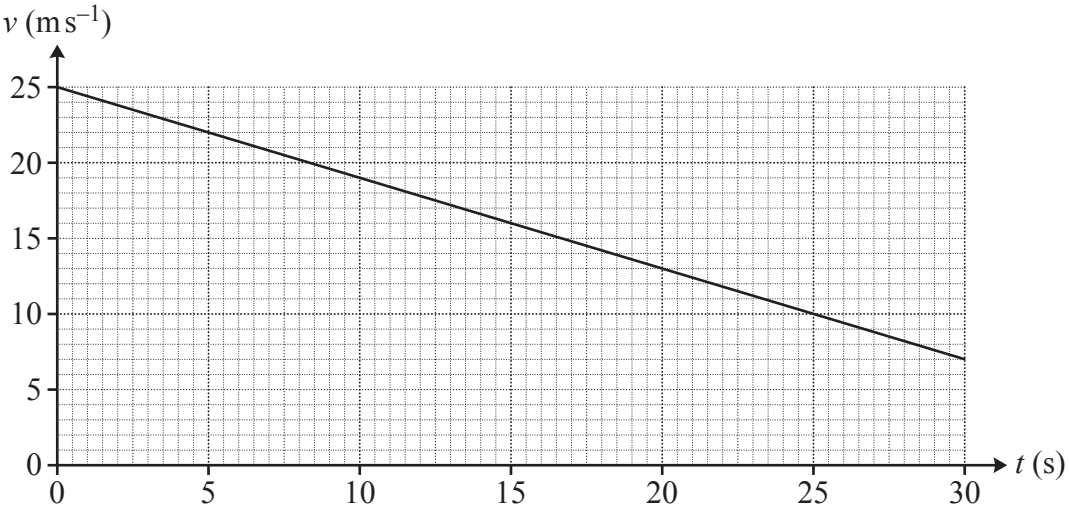
(a) Find the coordinates of C. **[2]**

A line has equation $y = x - 2$ and intersects the circle at the points A and B. The midpoints of AC and BC are A′ and B′ respectively.

(b) Determine the exact distance A′B′. **[8]**

9 Two trains are travelling in the same direction on parallel straight tracks and train A overtakes train B. At time t seconds after the front of train A overtakes the front of train B the velocities of trains A and B are $v_A \text{ m s}^{-1}$ and $v_B \text{ m s}^{-1}$ respectively.

The velocity of train A is modelled by $v_A = 25 - 0.6t$. The velocity-time graph of train A is shown below.



(a) A student argues that the speed of train A changes by 18 m s^{-1} in 30 seconds so its acceleration is 0.6 m s^{-2}.

Comment on the validity of the student's argument. **[1]**

(b) When the front of train A overtakes the front of train B, train B has a velocity of 10 m s^{-1}. The acceleration of train B is constant and is modelled as 0.15 m s^{-2}.

Write down the equation for v_B in terms of t that models the velocity of train B. **[1]**

(c) Draw the velocity-time graph of train B on the copy of the diagram in the Printed Answer Booklet. **[1]**

(d) Determine the distance between the fronts of the trains at the time when the trains are travelling at the same velocity. **[3]**

(e) Explain why the model for train A would not be valid for large values of t. **[1]**

10 A boat pulls a water skier of mass 65 kg with a light inextensible horizontal towrope. The mass of the boat is 985 kg. There is a driving force of 2400 N acting on the boat. There are horizontal resistances to motion of 400 N and 1200 N acting on the skier and the boat respectively.

(a) Draw a diagram showing all the horizontal forces acting on the skier and the boat. **[2]**

(b) (i) Write down the equation of motion of the skier. **[1]**

(ii) Find the equation of motion of the boat. **[2]**

(c) Find the acceleration of the skier and the boat. **[1]**

The driving force of the boat is increased. The skier can only hold on to the towrope when the tension is no greater than her weight.

(d) Determine her greatest acceleration, assuming that the resistances to motion stay the same. **[2]**

11 A student records the time a pendulum takes to swing for different lengths of pendulum.

The student decides to plot a graph of $\log_{10} T$ against $\log_{10} l$ where T is the time in seconds that the pendulum takes to return to its start position and l is the length in metres of the pendulum. They use a model for $\log_{10} T$ in terms of $\log_{10} l$ of the form $\log_{10} T = \log_{10} k + n \log_{10} l$.

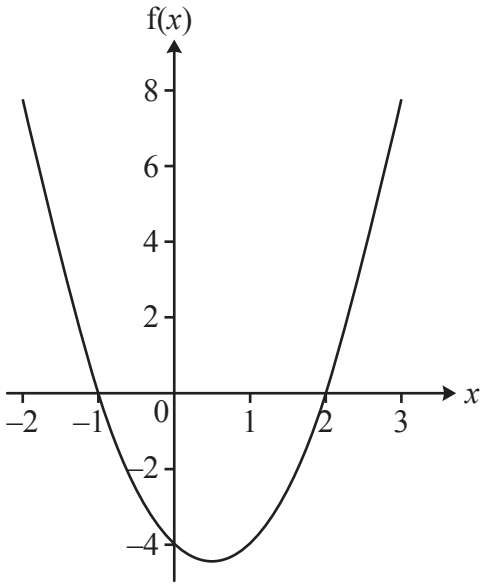
The student records the following data points.

$\log_{10} l$	–0.097	0.146
$\log_{10} T$	0.254	0.376

(a) Determine the values of k and n that best model the data. Give your values correct to **2** significant figures. **[4]**

(b) Using these values of k and n, write the student's model as an equation expressing T in terms of l. **[2]**

12 The diagram shows the graph of $f(x) = k(x-p)(x-q)$ where k, p and q are constants. The graph passes through the points (–1, 0), (0, –4) and (2, 0).



(a) Find $f(x)$ in the form $ax^2 + bx + c$. **[3]**

A cubic curve has gradient function $f(x)$. This cubic curve passes through the point (0, 8).

(b) Find the equation of the cubic curve. **[4]**

(c) Determine the coordinates of the stationary points of the cubic curve. **[3]**

END OF QUESTION PAPER