GCE

Further Mathematics A

Y534/01: Discrete Mathematics

AS Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

   YOU MUST MARK THE REQUIRED NUMBER OF PRACTICE AND STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. The RM Assessor **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
   If you have any questions or comments for your team leader, use the phone, the RM Assessor messaging system, or e-mail.

6. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

7. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

8. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award No Response (NR) if:

- there is nothing written in the answer space

Award Zero '0' if:

- anything is written in the answer space and is not worthy of credit (this includes text and symbols).

Team Leaders must confirm the correct use of the NR button with their markers before live marking commences and should check this when reviewing scripts.

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

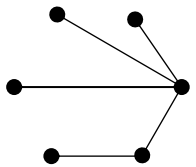
   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
   If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

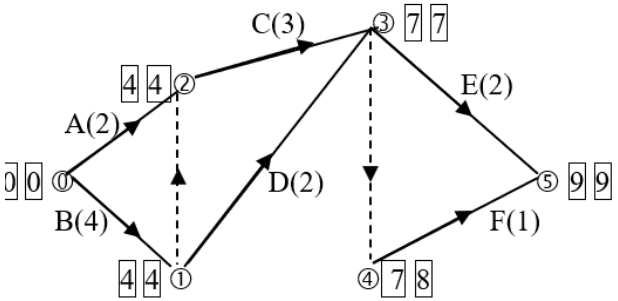
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance
1	(a)	C and D	B1 [1]	1.2	cao
1	(b)		B1 [1]	1.1	A tree that is isomorphic to this with vertex degrees 1, 1, 1, 1, 2, 4
1	(c)	(1+1+1+3+3+3)/2 = 6	M1 A1 [2]	3.1a 1.1	NOT from a sketch diagram (unless working is also seen) Seen or implied from working 6 **SC B1** answer 6 without appropriate method
1	(d)	A tree with 6 vertices would have 6 – 1 = 5 edges, but graph G has 6 edges (from part c)	B1 [1]	2.4	Using result from part c to show why G is not a tree In G the number of edges = the number of vertices, so not a tree
1	(e)	Pigeons are the actual degrees of the 6 vertices and pigeonholes (options) are the possible vertex degrees, but cannot have both 0 and 5 because if any vertex has degree 0 then it is not joined to the others so max degree is 4 OR if any vertex has degree 5 it is connected to all the others so min degree is 1 so no vertex with degree 0	B1 M1 A1 [3]	2.1 1.1 2.2a	Identified in any way e.g. 6 pigeons/vertices <u>and</u> 5 degrees Vertex degrees cannot be all of {0, 1, 2, 3, 4, 5} Allow for {1, 2, 3, 4, 5} or {0, 1, 2, 3, 4} seen Explaining why {0, 1, 2, 3, 4, 5} is not possible <u>and</u> using pigeonhole principle to conclude result If graph is assumed to be connected (i.e. assuming 1 to 5 only) ⇒ B1 M1 max

Question		Answer	Marks	AO	Guidance
2	(a)	<table><tr><th></th><th>C</th><th>D</th><th>E</th><th>H</th><th>I</th><th>row min</th></tr><tr><td>A</td><td>4</td><td>1</td><td>3</td><td>2</td><td>2</td><td>1</td></tr><tr><td>B</td><td>0</td><td>2</td><td>1</td><td>2</td><td>1</td><td>0</td></tr><tr><td>F</td><td>0</td><td>1</td><td>1</td><td>2</td><td>3</td><td>0</td></tr><tr><td>G</td><td>2</td><td>0</td><td>3</td><td>3</td><td>3</td><td>0</td></tr><tr><td>J</td><td>1</td><td>2</td><td>3</td><td>0</td><td>2</td><td>0</td></tr></table> Play-safe strategy for player 1 is A	**M1** **A1** **[2]**	**1.1** **1.1**	Finding row minima for table of pay-offs for player 1 Accept 1, 0, 0, 0, 0 or an equivalent written description A (from correct row minima)
2	(b)	2 > 0 and 2 > 1 3 > 2 3 > 2 3 > 2 3 > 0 1 > 0 1 > 0 2 > 0 2 > 1 <table><tr><th></th><th>C</th><th>D</th><th>E</th><th>H</th><th>I</th></tr><tr><td>A</td><td>2</td><td>2</td><td>0</td><td>1</td><td>1</td></tr><tr><td>B</td><td>3</td><td>1</td><td>2</td><td>1</td><td>2</td></tr><tr><td>F</td><td>3</td><td>2</td><td>2</td><td>1</td><td>0</td></tr><tr><td>G</td><td>1</td><td>3</td><td>0</td><td>0</td><td>0</td></tr><tr><td>J</td><td>2</td><td>1</td><td>0</td><td>3</td><td>1</td></tr></table>	**B1** **B1** **[2]**	**1.1** **1.1**	These comparisons seen (may omit repeated 3 > 2 and 2 > 0 for E and repeated 2 > 1 for I, i.e. 2 > 0, 3 > 2, 1 > 0 and 2 > 1, 3 > 2, 3 > 0, 1 > 0 or similarly) Ignore extras Deleting columns E and I (both) Or saying that both E and I are dominated Or indicating both E and I
2	(c)	When they play A and C the total pay-off is 6 but for (all the) other cells total pay-off is 3	**B1** **[1]**	**2.2a**	Identifying cell (A, C) in any form and an appropriate comparison (of total or mean) with at least one other cell

Question		Answer	Marks	AO	Guidance
3	(a)	$9 \times 8 \times 7 \times 6$ or ${}^{9}P_{4}$	M1	1.1	Appropriate working seen or explained
		= 3024	A1	1.1	cao **SC B1** 3024 seen with no relevant working
			[2]		
3	(b)	$(9 \times 8 \times 7) \times 6$	M1	1.1	Sight of <u>calculation</u> $504 \times N$ or $252 \times N$ o.e., for positive integer N OR a <u>calculation</u> leading to final answer 3024, 6048, 1512 or 504
		= 3024	A1	1.1	cao **SC B1** 3024 seen with no relevant working
			[2]		
3	(c)	9^4	M1	2.1	soi
		= 6561	A1	1.1	cao
		Alternative solution			
		aabb $\Rightarrow {}^{9}C_{2} \times 6 = 216$ aaab $\Rightarrow {}^{9}P_{2} \times 4 = 288$ aaaa $\Rightarrow 9$	M1		At least two of 216, 288, 9
		3024 + 3024 + 216 + 288 + 9 = 6561	A1		6561
			[2]		

<table>
<tr><th colspan="2">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td>So that E and F
do not start at the same vertex (event ③)
and also end at the same vertex (event ⑤)</td><td>B1

[1]</td><td>2.4</td><td>Uniqueness (E and F)
To make graph simple
Ignore references to durations or start/finish times</td></tr>
<tr><td>4</td><td>(b)</td><td> 

Minimum project completion time
= 9 (minutes)</td><td>M1

A1

[2]</td><td>1.1

1.1</td><td>Forward pass seen with at most one independent error

<table><tr><td>⓪</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td></tr><tr><td>0</td><td>4</td><td>4</td><td>7</td><td>7</td><td>9</td></tr></table>

9 from correct forward pass seen
SC B1 for 9 without forward pass seen</td></tr>
<tr><td>4</td><td>(c)</td><td>Backward pass seen with at most one independent error

$Float_{ij} = LFT_j – EST_i - duration_{ij}$
<table><tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr><tr><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td></tr></table></td><td>M1

A1

B1
[3]</td><td>1.1

1.1

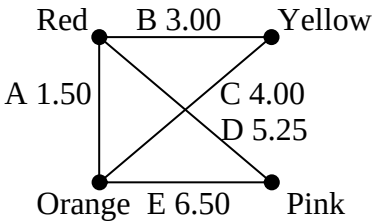
1.1</td><td>May be implied from A mark achieved

<table><tr><td>⓪</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td></tr><tr><td>0</td><td>4</td><td>4</td><td>7</td><td>8</td><td>9</td></tr></table>

Float correct for non-critical activities: A = 2, D = 1, F = 1

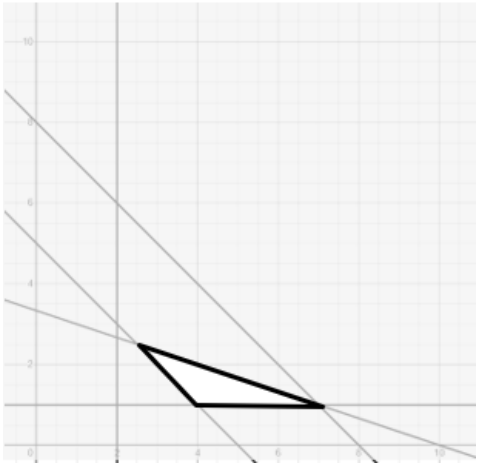
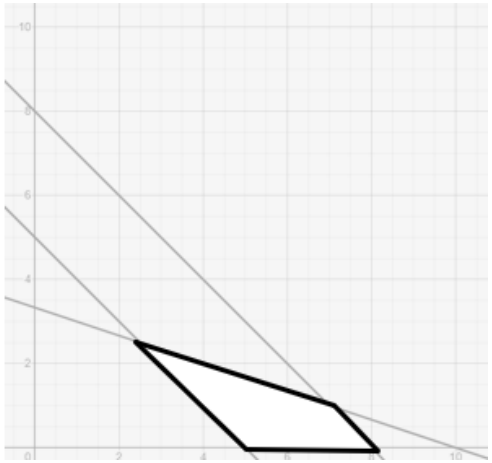
Critical activities B, C, E with 0 float</td></tr>
</table>

Question		Answer	Marks	AO	Guidance
4	(d)	No change (A is still not critical)	B1 [1]	1.1	No effect, none
4	(e)	If $m < 2$ then D becomes critical instead of C and the minimum project completion time is 8 (minutes) If $2 \leq m < 3$ then C is still critical and minimum project completion time is $6+m$ (minutes)	B1 B1 [2]	1.1 1.1	$m < 2 \Rightarrow 8$ or $m \leq 2 \Rightarrow 8$ Or from Max{8, $6+m$} $6+m$ Note: m need not be an integer, so listing time for e.g. $m = 1$, $m = 2$ scores B0, B0

Question		Answer	Marks	AO	Guidance
5	(a)	Vertices are the 4 colours (red, yellow, orange, pink), arcs are the 5 pack types and arc weights are costs	M1	3.3	A network (<u>weighted</u> graph) with 4 vertices and 5 arcs
			A1	1.1	cao Vertices need to be labelled appropriately (R,Y,O,P) Arcs need not be labelled with pack types but do need to be weighted with appropriate costs Ignore working for part (b) if seen
			[2]		
5	(b)	Use of Prim or Kruskal	M1	3.4	Any spanning tree for their network, written or drawn
		Arcs A, B, D or RO, RY, RP or 1.50, 3.00, 5.25	A1	3.4	cao, written or drawn (ignore total weight if given here)
			[2]		
5	(c)	No, cost of MST is £9.75 but C+D includes all four colours and only costs £9.25	B1	3.2b	Comparing <u>cost</u> of a valid pair (with 9.75) C and D costs less May imply ‘not’
		Alternative solution No, cost of MST is £9.75 but B+E includes all four colours and only costs £9.50	B1		B and E costs less
			[1]		

Question		Answer	Marks	AO	Guidance
5	(d)				5 variables (number of packs of each type) used consistently
		Minimise 1.5A + 3B + 4C + 5.25D + 6.5E	B1	2.5	'Minimise' an appropriate objective function
		Subject to A+B+C+D+E ≤ 4	B1	3.3	At most 4 packs
		A+B+D ≥ 1, B+C ≥ 1, A+C+E ≥ 1, D+E ≥ 1	M1 A1	3.1b 1.1	At least one of each colour (any one of these), allow > 0 All four correct (ignore extras)
		0 ≤ A, B, C, D, E	B1	1.1	Non-negativity (or 0-1 variables)
			[5]		

Question		Answer	Marks	AO	Guidance
6	(a)	Maximise $1.5x + 4y$	B1	1.1	$P = 1.5x + 4y$ (allow + constant)
		Subject to $5 \leq x + y \leq 8$	B1	1.1	Or as two separate inequalities
		$2(x – 2) + 6(y – 1) \leq 10$	B1	3.1b	Inequality for amount spent, $2x + 6y \leq 20$ o.e.
		$x – 2$ and $y – 1$ are non-negative (integers)	B1	1.1	Allow $x, y \geq 0$ Ignore extras BOD if < used for ≤ and/or > used for ≥
			[4]		

Question		Answer	Marks	AO	Guidance
	(b)				Mark grids first, tolerance $\pm$ 0.5 grid square
			B1 **B1** **B1**	**1.1** **1.1** **3.4**	$2x + 6y \leq 20$ drawn correctly $5 \leq x + y$ and $x + y \leq 8$ both correct, BOD scales if not labelled Shading to give a FR (between parallels and below other line) Allow 
		Check integer-valued points near their vertices (3, 2) (4,2) (5, 0) (7, 1) (8, 0) 12.5 14 7.5 14.5 12 Max is £14.50 when $x = 7$, $y = 1$ $7 - 2 = 5$ and $1 - 1 = 0$	**M1**	**3.4**	At least one (of theirs) correct Or profit line $y = 0.25P - 0.375x$, gradient $-\frac{3}{8}$ written or drawn May be implied from correct answer or from $x = 7$, $y = 1$ or (7, 1) or from 14.50
		Buy 5 silver tokens 0 gold tokens	**A1** **[5]**	**3.2a**	Solution in context, 5 silver May imply 0 gold tokens if not mentioned

Question		Answer	Marks	AO	Guidance
7	(a)	e.g. Bin 1 8 5 4 Bin 2 7 3 3	B1 [1]	1.1	Any valid packing of 8 5 4 7 3 3 into two bins of size $k = 18$ e.g. Bin 1 8 7 3 e.g. Bin 1 8 7 Bin 2 5 4 3 Bin 2 5 4 3 3
7	(b)	Total weight = 30 Need three bins, two are not enough so $k < 15$ (but 14 is possible) Three bins are sufficient so $k \geq 10$ $k = 10$ would mean no waste but bin with 8 would not be full, hence $k \geq 11$ $k = 11$ is achievable, e.g. 8+3, 7+4, 5+3 So $11 \leq k \leq 14$	B1 M1 A1 [3]	2.1 3.1b 2.2a	UB < 15 or UB = 14 Stated or by showing packing into two bins each with exactly 15 Sensible attempt to find LB as 10 or 11 $k \in \{11, 12, 13, 14\}$ Given that k is an integer so $11 \leq k < 15$, $10 < k \leq 14$ or $10 < k < 15$ are also correct Or given as separate statements
7	(c)	8 5 4 7 3 3 Bin 1 **8 4** Bin 2 **5** 7 Bin 3 3 3	M1 A1 [2]	1.1 1.1	First-fit with $k = 12$, evidenced by 8, 5, 4 correctly placed All correct
7	(d)	(Solution is not necessarily optimal because) there may be a packing that uses fewer bins	B1 [1]	2.2b	First-fit will find a good packing but may not find the packing that uses the fewest bins

Question		Answer	Marks	AO	Guidance
7	(e)	$8 \Rightarrow 0$ (first item), $5 \Rightarrow 1$ (check bin 1) $4 \Rightarrow 0$ (fits in bin 1), bin 1 full $7 \Rightarrow 0$ (fits in first bin that is not full), bin 2 full $3 \Rightarrow 0$, $3 \Rightarrow 0$ (fits in first bin that is not full)	**M1**	**1.1**	Evidence that $5 \Rightarrow 1$ and $4 \Rightarrow 0$ e.g. $0 + 1 + 0 + \ldots$
		$0 + 1 + 0 + 0 + 0 + 0 = 1$	**A1** **[2]**	**1.1**	1 from valid working seen
7	(f)	When $k = 13$, 8 and 5 will go in bin 1 and it will then be full and no longer considered 4 and 7 then go in bin 2 which is not full	**M1**	**2.4**	Bin 2 will not be full This may be done by showing packing Bin 1: 8 5 Bin 2: 4 7 Bin 3: 3 3 May be implied from correct A mark
		But bin 2 does not have enough room for a 3 So need to check bin 2 before putting 3 (or 3's) into bin 3	**A1** **[2]**	**2.3**	Explain why argument is wrong First 3 adds 1 to the count Each 3 adds 1 to the count Total count will be 0+0+0+0+1+1 = 2
7	(g)	Need full bins until last 8+5+4	**M1**	**2.2b**	May show packing with 8, 5, 4 in Bin 1 and 7, 3, 3 in Bin 2 (packed in this order)
		= 17	**A1** **[2]**	**1.1**	17 **SC B1** 17 with no appropriate reasoning

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Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.