GCE

Further Mathematics B (MEI)

Y413/01: MEI Modelling with algorithms

AS Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance
1	(a)	(i)	3	B1 [1]	1.1	
		(ii)	4	B1 [1]	1.1	
	(b)		$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 & 0 \end{pmatrix} \end{array}$	B1 [1]	1.1	Do not accept '-' (oe) for a 0.
	(c)		e.g. the graph is not simple as there is a loop (at D) e.g. the graph is not simple as there is a repeated edge (between C and E)	B1 [1]	2.4	Either mention that the graph contains a loop (but not just a cycle) or a repeated edge (but not multiple edges) – technical language used must be correct. Condone additional but unnecessary reasons (e.g. the graph contains loops and cycles).
	(d)		5	B1 [1]	1.1	

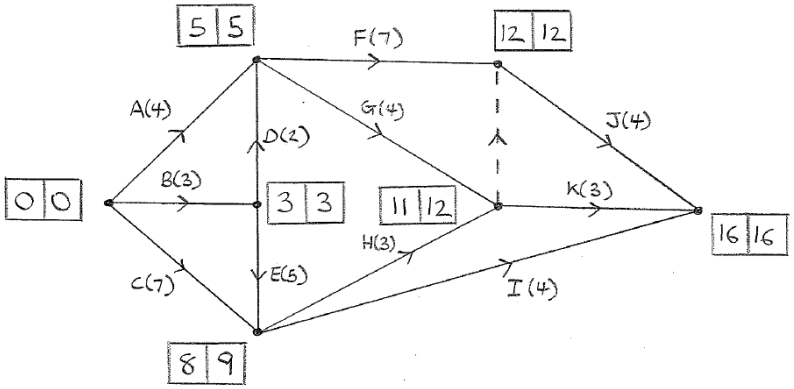
Question			Answer	Marks	AOs	Guidance
2	**(a)**		Bin 1: 31 13 Bin 2: 12 17 18 Bin 3: 25 11 8 Bin 4: 15	**B1** **[1]**	**1.1**	
	(b)		<u>31</u> 13 12 17 25 18 11 8 15 <u>13</u> 12 17 25 18 11 8 15 **31** (1st pass) <u>12</u> 11 8 **13** <u>17</u> 25 18 15 **31** (2nd pass) <u>11</u> 8 **12** **13** 15 **17** <u>25</u> 18 **31** (3rd pass) 8 **11** **12** **13** 15 **17** 18 **25** **31** (4th pass)	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	31 used as the first pivot and in the correct position after the first pass First two passes correct and for the remainder of the sort must be using quick (not slow) sort Correct sort Mark descending as a misread
	(c)		Bin 1: **31** **18** Bin 2: **25** **17** 8 Bin 3: **15** 13 12 Bin 4: 11	**M1** **A1** **[2]**	**1.1** **1.1**	First five numbers placed correctly (those in bold) cao (no additional/repeated values)
	(d)		e.g. Bin 1: 25 13 12 Bin 2: 31 11 8 Bin 3: 18 17 15 While first fit and first fit decreasing do find a solution efficiently to the problem of packing items with the given sizes into bins of capacity 50 there were not optimal solutions (as indicated by the fact that both methods required 4 bins) and hence first fit and first fit decreasing are examples of heuristic algorithms	**B1*** **B1dep*** **[2]**	**2.1** **1.2**	A correct full bin solution using exactly three bins Must give an understanding that FF and FFD do find a solution (e.g. ‘they are efficient’) but not necessarily optimal. Dependent on both the packing in part **(a)** and part **(c)** using exactly 4 bins.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>3</td><td>(a)</td><td></td><td>
<table>
<tr><th>Activity</th><th>Immediate Predecessor(s)</th></tr>
<tr><td>A</td><td>-</td></tr>
<tr><td>B</td><td>-</td></tr>
<tr><td>C</td><td>-</td></tr>
<tr><td>D</td><td>B</td></tr>
<tr><td>E</td><td>B</td></tr>
<tr><td>F</td><td>A, D</td></tr>
<tr><td>G</td><td>A, D</td></tr>
<tr><td>H</td><td>C, E</td></tr>
<tr><td>I</td><td>C, E</td></tr>
<tr><td>J</td><td>F, G, H</td></tr>
<tr><td>K</td><td>G, H</td></tr>
</table>
</td><td>B1

B1

[2]</td><td>1.1

1.1</td><td>Any 5 rows correct
(not including rows A, B, C)

cao (accept blanks for A, B, C)</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td> 
</td><td>M1

M1

A1

[3]</td><td>1.1

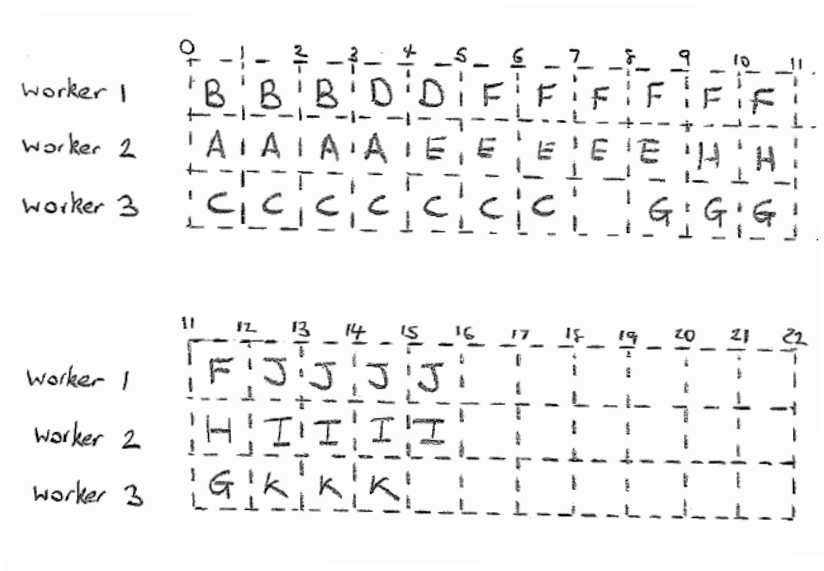
1.1

1.1</td><td>Forward pass – numbers increasing from source to sink (allow one slip) – all complete

Backward pass – numbers decreasing from sink to source (allow one slip) – all complete – condone missing 0 at the source

cao for backward and forward pass</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>Minimum project completion time is 16 (hours)</td><td>B1
[1]</td><td>1.1</td><td></td></tr>
<tr><td></td><td></td><td>(iii)</td><td>Critical activities are B, D, F and J</td><td>B1
[1]</td><td>1.1</td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td></td><td>(c)</td><td></td><td>Independent float for activity I is 16 – 9 – 4 = 3 (hour)</td><td>B1FT

[1]</td><td>1.1</td><td>FT their event times provided their independent float is > 0 – if working shown it must be correct for activity I.</td></tr>
<tr><td></td><td>(d)</td><td></td><td>e.g.
 
</td><td>M1

A1

[2]</td><td>3.1b

1.1</td><td>Three workers used at most, at least 9 activities placed, no activity finishing after the completion time for the entire project and at least 5 activities dealt with correctly
cao

<table>
<tr><th>Activity</th><th>IPA</th><th>Interval</th></tr>
<tr><td>A (4)</td><td>-</td><td>0 – 5</td></tr>
<tr><td>B (3)</td><td>-</td><td>0 – 3</td></tr>
<tr><td>C (7)</td><td>-</td><td>0 – 9</td></tr>
<tr><td>D (2)</td><td>B</td><td>3 – 5</td></tr>
<tr><td>E (5)</td><td>B</td><td>3 – 9</td></tr>
<tr><td>F (7)</td><td>A, D</td><td>5 – 12</td></tr>
<tr><td>G (4)</td><td>A, D</td><td>5 – 12</td></tr>
<tr><td>H (3)</td><td>C, E</td><td>8 – 12</td></tr>
<tr><td>I (4)</td><td>C, E</td><td>8 – 16</td></tr>
<tr><td>J (4)</td><td>F, G, H</td><td>12 – 16</td></tr>
<tr><td>K (3)</td><td>G, H</td><td>11 – 16</td></tr>
</table></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td></td><td>Maximise $(P=)2x-3y+z$
Subject to
$x+2y+3z\leqslant k$
$3x-y+z\leqslant 45$
$2x+4y-z\leqslant 50$
$x,y,z\geqslant 0$</td><td>B1

B1

B1

[3]</td><td>3.4

3.4

1.1</td><td>Correct objective function ($P-2x+3y-z=0$)

Two correct inequalities or all three non-trivial constraints with slack variables
All three non-trivial constraints as inequalities and all three trivial constraints</td></tr>
<tr><td></td><td>(b)</td><td></td><td><table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>RHS</th></tr><tr><td>1</td><td>0</td><td>$\frac{7}{3}$</td><td>$-\frac{1}{3}$</td><td>0</td><td>$\frac{2}{3}$</td><td>0</td><td>30</td></tr><tr><td>0</td><td>0</td><td>$\frac{7}{3}$</td><td>$\frac{8}{3}$</td><td>1</td><td>$-\frac{1}{3}$</td><td>0</td><td>$k-15$</td></tr><tr><td>0</td><td>1</td><td>$-\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>0</td><td>$\frac{1}{3}$</td><td>0</td><td>15</td></tr><tr><td>0</td><td>0</td><td>$\frac{14}{3}$</td><td>$-\frac{5}{3}$</td><td>0</td><td>$-\frac{2}{3}$</td><td>1</td><td>20</td></tr></table></td><td>M1

A1

A1

[3]</td><td>3.4

1.1

1.1</td><td>Pivot (third) row and column (x) correct (condone one slip on one value in the pivot row but pivot column must be fully correct)

One of columns y,z,s_2 or RHS correct

cao</td></tr>
<tr><td></td><td>(c)</td><td></td><td>The solution after the second iteration is optimal as there are no negative values in the (new) objective row</td><td>B1

[1]</td><td>2.4</td><td>Correct reasoning – allow ‘top’ row or P row – do not allow ‘all values are positive’</td></tr>
<tr><td></td><td>(d)</td><td></td><td>$\frac{k+225}{8}=36\Rightarrow k=...$ or from $2\left(\frac{-k+135}{8}\right)+3\left(\frac{k-15}{8}\right)=36$
$x=\frac{135-'63'}{8}$ or $z=\frac{3}{8}('63'-15)$ or $s_3=\frac{5}{8}(17+'63')$
$x=9,z=18,s_3=50$</td><td>M1*

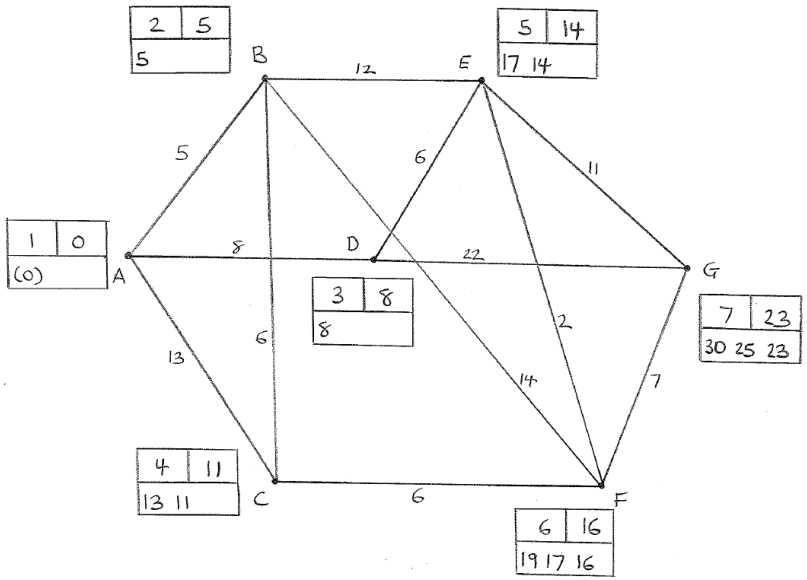
M1dep*

A1
[3]</td><td>3.1a

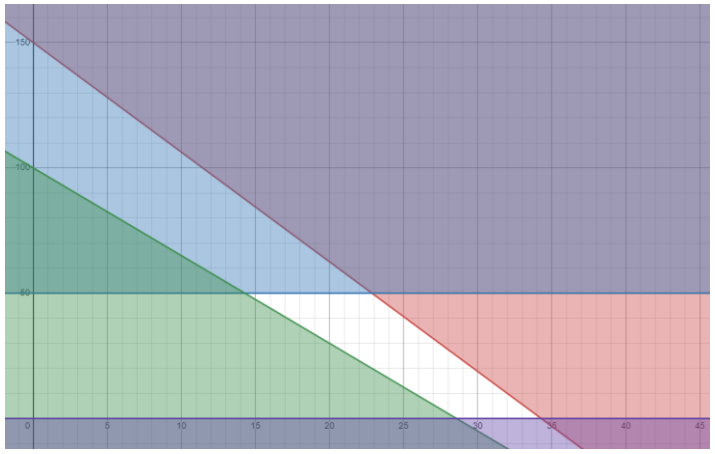
3.4

3.2a</td><td>Setting the value of the objective equal to 36 and solving for k (if correct k = 63)

Substituting their value of k into at least one of the correct expressions for x, z or s_3
These three and no others (condone P = 36)</td></tr>
</table>

Question			Answer	Marks	AOs	Guidance
5	(a)		The objective function contains both BC and CB to allow for the possibility that the shortest path from A to G could go from B (directly) to C or from C (directly) to B	**B1** **[1]**	**3.3**	Allow edge (BC) is undirected.
	(b)		The purpose of the constraint AD + ED – DE – DG = 0 is to allow for the fact that if the shortest path passes through D then the path can enter D via A or E and leave via E or G (which accounts for all the arcs incident to D).	**B1** **[1]**	**3.3**	Explanation that mentions D and the ability for the path to pass through this node. So must mention D and the (possible) path through that node (but mention of the path through any other node is **B0**).
	(c) (d)	(i)	 Length of shortest path from A to G is 23	**M1** **A1** **[2]** **M1** **A1** **A1** **A1** **A1FT** **[5]**	**1.1** **1.1** **1.2** **1.1a** **1.1a** **1.1** **1.1**	All arcs present (allow one absent/one extra/one error) cao (including weights) – allow arrows on arcs from source node and allow arrows on arcs to sink node only Correct working values at C Working values Labels Order of labelling Follow through their label at G only

Question			Answer	Marks	AOs	Guidance
	(d)	**(ii)**	Shortest path from A to G is ADEFG	**B1** **[1]**	**2.2a**	cao
	(e)		$t \approx 0.014\left(\frac{700}{7}\right)^2$ 140 (seconds)	**M1** **A1** **[2]**	**3.1b** **2.2b**	Stating or using the fact that Dijkstra's algorithm has quadratic complexity
	(f)	**(i)**	Prim's: AB, BC, CF, EF, ED, FG	**M1** **A1** **[2]**	**1.1** **1.1**	First three arcs (or four nodes) chosen correctly in the correct order cao (must be stated as arcs in the correct order) – **M1** only if starting at a different node (than A) – explicit rejection of arcs scores **M1** only
	(f)	**(ii)**	Weight of MST is 32	**B1** **[1]**	**2.2a**	cao

Question			Answer	Marks	AOs	Guidance
6	(a)		$5x = 2z$ (ratio of small to large constraint)	**B1**	**2.1**	Implied by a correct inequality or objective function in terms of x and y only
			$25x + 40y + 60z \leqslant 6000 \Rightarrow 5x + 8y + 12z \leqslant 1200$ OR $35x + 8y \leqslant 1200$ (time of production constraint)	**B1**	**3.1b**	Correct constraint for time of production either in terms of x, y and z or just in terms of x and y
			$y \leqslant 50$	**B1**	**1.1**	Constraint for 'at most 50 medium'
			$x + y + z \geqslant 100$ OR $7x + 2y \geqslant 200$	**B1**	**3.3**	Constraint for 'at least 100' either in terms of x, y and z or just in terms of x and y
			(Maximise) $5x + 6y + 8z$ OR $25x + 6y$	**B1**	**3.3**	Correct objective (stated or implied by later working) either in terms of x, y and z or just in terms of x and y
				**B1**	**1.1**	Any one correct line
				B1	**1.1**	Any two correct lines
				B1	**1.1**	All line correct + correct feasible region (either implied by shading or labelled) **Final 2 marks are dependent on the correct feasible region**
			Find correct optimal vertex or point testing	**M1**	**1.1a**	Find correct optimal vertex $\left(\frac{160}{7}, 50\right)$ **or** (22.9, 50) or better, or testing two points $(n, 50)$ where $14 \leqslant n \leqslant 23$ in a correct objective fn.
			24 small T-shirts, 45 medium T-shirts and 60 large T-shirts	**A1**	**3.2a**	Must be in context

Question			Answer	Marks	AOs	Guidance
				[10]		
6	**(b)**		(£) 870	**B1** **[1]**	**3.4**	
6	**(c)**		e.g. the total income (of £870) assumes that all the T-shirts will be sold	**B1** **[1]**	**3.5b**	

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