GCE

Further Mathematics B MEI

Y414/01: Numerical Methods

AS Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

   a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award NR (No Response)
   - if there is nothing written at all in the answer space and no attempt elsewhere in the script
   - OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
   - OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
   Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

   b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

   c. The following types of marks are available.

   M

5

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there

may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f.   Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   •   When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   •   When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
       NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g.   Rules for replaced work and multiple attempts:
   •   If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   •   If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   •   if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h.   For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
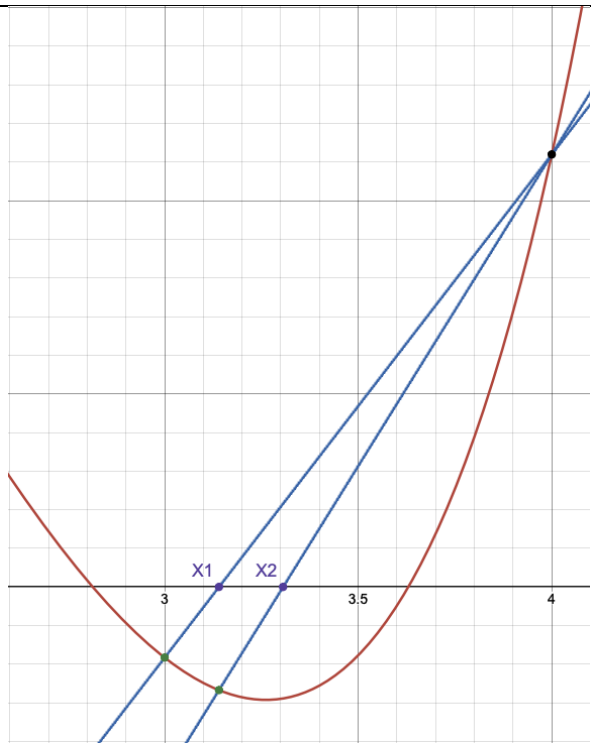
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i.   If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j.   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer					Marks	AO	Guidance
1	**(a)**							**M1**	**1.1**	At least three first differences correct

Table within answer:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0.8			
		–0.1		
1	0.7		–0.5	
		–0.6		12.3
2	0.1		11.8	
		11.2		36.2
3	11.3		48	
		59.2		72.9
4	70.5		120.9	
		180.1		
5	250.6			

Question		Answer	Marks	AO	Guidance
			A1	**1.1**	All correct
			[2]		
1	**(b)**	The third differences are **not** (approximately) **constant** So a cubic (polynomial) is not suitable (for modelling the data)	**B1** **[1]**	**2.2a**	Allow "equal" or "the same" for "constant" but not "similar" and not "equivalent" Allow "No" for this conclusion

Question			Answer	Marks	AO	Guidance
2	(a)		$\pi \approx 3.1415926\ldots$ rounds to 3.141593 $e \approx 2.7182818\ldots$ rounds to 2.718282	B1 [1]	1.1	Unrounded values seen for π and e, given to at least 7 dp, with either rounded values or equivalent explanation.
2	(b)		$\dfrac{3.141593-\pi}{\pi}$ soi $= 0.000\,000\,110\,26579$	M1 A1 [2]	1.1 1.1	Allow signs reversed in numerator for M1 only. Or 1.10266×10^{-7} oe. At least 2sf required, but if given to greater accuracy then must be correct rot
2	(c)		The spreadsheet stores the values (in cells B1 and B2) to a greater precision than it displays them And uses these (stored) values, not the displayed values, to compute the value (in cell B3)	B1 B1 [2]	1.2 2.4	Do not allow references to use of "true" or "exact" values Do not allow references to use of "true" or "exact" values Do not allow references to A1 and A2

Question			Answer	Marks	AO	Guidance
3	(a)		$[P =]\frac{(w-2)(w-4)}{(1-2)(1-4)} \times 52 + \frac{(w-1)(w-4)}{(2-1)(2-4)} \times 62.54 +$	M1	3.3	Allow one sign error and one substitution error, allow use of other variable(s)
			$\frac{(w-1)(w-2)}{(4-1)(4-2)} \times 79.9$	A1	1.1	All substitutions correct, allow use of other variable(s)
			$-0.62w^2 + 12.4w + 40.22$	A1	1.1	Two of three terms correct; allow use of other variable(s)
			$P = -0.62w^2 + 12.4w + 40.22$ **cao**	A1	1.1	Must be P in terms of w
				[4]		
3	(b)		$w = 6, P = 92.3$	B1FT	3.4	**FT** their three term quadratic
			$w = 7, P = 96.64$	B1FT	1.1	**FT** their three term quadratic
			so she can play at (the end of) week 7 but not at (the end of) week 6.	B1FT	3.2a	Dependent on B1B1, conclusion must reflect correct comparison of their values to 95%. Allow reference to $w = 6$ and $w = 7$ (or t etc.)
				[3]		
3	(c)		$w = 9, P = 101.6$ – a player can't be more than 100% fit so model not viable	B1FT	3.5a	**FT** their 3 term quadratic as long as their '101.6' > 100 Must calculate the value of P at $w = 9$ and explain in context. E.g - "cannot score more than 100% on a fitness test" B1 - "cannot have a percentage higher than 100" B0
				[1]		

	Question		Answer	Marks	AO	Guidance
4	(a)			B1	1.1	Chord from $(3, f(3))$ to $(4, f(4))$ shown
				B1	1.1	Both chords shown with X_1 and X_2 clearly marked
						These two points must be identified on the *x*-axis (e.g. with a dot or cross) and at least the second (X_2) must be labelled (e.g. with a letter such as *d* or "second approximation" – need not indicate '2').
						Condone labelling the point directly below on the curve but only if there is a line (or arrow etc.) linking this to the *x*-axis.
						isw further markings (e.g. a third chord)
						NB for reference: $X_1 = 3.14041, X_2 = 3.30597$
				[2]		
4	(b)		$0.2e^x - 0.8x^2 + 3 = 0$	B1	1.1	Allow omission of =0 and/or one coefficient/sign error only
				B1	1.1	All correct
				[2]		
4	(c)		=IF(AA4>0,Z4,X4)	B1	1.1	or =IF(AA4<0,X4,Z4)
				[1]		
4	(d)		3.62796 and 3.62604 agree to 2 dp so 3.63 seems secure	B1	2.2b	Allow "best two" or "last two" approximations or reference to Z11 (or V12) and Z12
				[1]		

	Question		Answer	Marks	AO	Guidance
4	**(e)**		2.814361 since the last two estimates agree to this precision	**B1** [1]	**2.2b**	Allow "best two" or x_4 and x_5
4	**(f)**		(ratio of differences) **decreasing**	**B1**	**1.1**	Allow "reduces"
			so (convergence) **faster than first order** (in this case)	**B1** [2]	**2.2a**	Allow "quicker" Condone "greater" or "larger"
4	**(g)**		Newton-Raphson **converges faster** (because it is generally 2nd order and False Position is 1st order)	**B1**	**1.2**	For a correct advantage relating to the speed of convergence
			Newton-Raphson is easier (to apply on a spreadsheet) because it doesn't require a sign change (as the first step)	**B1** [2]	**2.4**	For a correct advantage relating to the ease of application e.g. • "Does not require a pair of starting values that straddle the root" • "The method requires fewer steps to set up (so is easier to apply" (may be supported by examples of these steps) • "Does not require a change of sign" Do not accept: • "Gradient needs to be between -1 and 1" • "Is more accurate" • "Needs to be close to the root at first" • "Easier to apply" (without giving a reason)

	Question		Answer		Marks	AO	Guidance
5	(a)		h	approximation to f'(5)	M1	1.1a	Use of forward difference formula soi
			0.1	6.2461	A1	1.1	6.2461
			0.01	6.1065	A1	1.1	6.1065
			0.001	6.093	A1	1.1	6.093
			0.0001	6.09	A1	1.1	6.09
					[5]		
5	(b)		6.09 since the last two estimates agree to this precision		B1	2.2b	Allow "best two"
					[1]		
5	(c)		Ali's approximation is (probably) less accurate		B1	2.2b	
			because Ali's calculation involves the subtraction of nearly equal numbers (in the numerator) **and** the denominator is very small oe, (so accuracy is lost)		B1	2.3	
					[2]		
5	(d)		$0.02 \times$ their 6.09		M1	1.1a	Allow $0.02 \times 6.0396\ldots$ Must see this step or equivalent working, may see [f(5.02) $\approx$]12.182494 + 0.02 × ′6.09′[= 12.304294]
			–0.1218		A1	1.1	awrt ±0.12, allow 0.1218
					[2]		

	Question		Answer	Marks	AO	Guidance
6	(a)		$[x_0 =]$ 2	M1	1.1	1.5518… and 1.4063… seen to 2 or more dp rot
			$[x_1 =]$ 1.551846			
			$[x_2 =]$ 1.406319			
			$[x_3 =]$ 1.362062			
			$[x_4 =]$ 1.348881			
			$[x_5 =]$ 1.344981			
			$[x_6 =]$ 1.343828			
			$[x_7 =]$ 1.343488			
			$[x_8 =]$ 1.343388			
			$[x_9 =]$ 1.343358			
			$[x_{10} =]$ 1.343349			
			$[x_{11} =]$ 1.343347			
			$[x_{12} =]$ 1.343346			
			$[x_{13} =]$ 1.343346			
			1.343346	A1	1.1	1.343346 seen
				A1	2.2a	Accuracy justified by repeat of 1.343346 (two consecutive iterates at or beyond x_{12} to at least 6dp) **or** by change of sign (as a minimum stating f(1.3433455) < 0 and f(1.3433465) > 0)
				[3]		
6	(b)	(i)	central difference (generally) 2nd order method whereas forward difference (generally) 1st order method **oe** **or** central difference (approximation) straddles the root but forward difference doesn't	B1	1.2	Must make a comparison and refer to both methods (but may say 'it' instead of 'central difference' etc.) Condone "central difference **converges faster** than forward difference"
				[1]		

	Question		Answer	Marks	AO	Guidance
6	(b)	(ii)	$\dfrac{3^{\frac{11.0}{5}}-3^{\frac{10.8}{5}}}{0.2}$ **oe**	**M1**	**1.1**	Central difference formula used with one pair of points from the table, allow $\dfrac{g(11.0)-g(10.8)}{0.2}$. Condone 0.1 and f for M1 only.
				A1	**2.1**	Fully correct with numerical values or $\dfrac{11.211...-10.72...}{0.2}$ (calculation not required)
				[2]		
6	(b)	(iii)	2.40990 or 2.4099 seem justified as difference between successive estimates rapidly reducing	**B1**	**2.2b**	Allow "because the last/best two estimates agree to this precision"
				[1]		
6	(c)		(almost certainly) $g'(\beta) > 1$ (so the criterion for convergence is not met)	**B1**	**2.4**	Allow "the derivative of g at β is not between -1 and 1" Must include: - Gradient or derivative - g - At (or near) β or 10.9 - >1 (or not between -1 and 1)
				[1]		

	Question		Answer		Marks	AO	Guidance
6	**(d)**		$[x_0 =]$	10.9	**M1**	**1.1**	10.8817… and 10.87036… seen to 2 or more dp rot
			$[x_1 =]$	10.8817291			
			$[x_2 =]$	10.8703639			
			$[x_3 =]$	10.8632705			
			$[x_4 =]$	10.8588339			
			$[x_5 =]$	10.8560554			
			$\vdots$	$\vdots$			
			$[x_{20} =]$	10.851387			
			$[x_{21} =]$	10.851386			
			$[x_{22} =]$	10.851385			
			$[x_{23} =]$	10.851384			
			$[x_{24} =]$	10.851384			
			$[x_{25} =]$	10.851383			
			$[x_{26} =]$	10.851383			
			$[x_{27} =]$	10.851383			
		10.851383			**A1**	**1.1**	10.851383 seen
							If M0 then **SCB1** for 10.851383 (max. 1/3)
					A1	**2.2a**	Accuracy justified by repeated iterates (two consecutive iterates at or beyond x_{25} to at least 6dp) or by change of sign (as a minimum must see g(10.8513825) > 0 and g(10.8513835) < 0)
					[3]		

	Question		Answer	Marks	AO	Guidance
7	(a)		= (2*G2 + H2)/3 or = (4* H3 – H2)/3 **oe**	M1	1.1	Evidence of either correct integration formula (allow this mark if not in 'spreadsheet form' or if missing '=' or given as 'I2=', but must have correct form and correct cell references)
				A1 [2]	1.1	all correct
7	(b)		= J4/J3	B1 [1]	1.1	
7	(c)		trapezium rule estimates are **increasing** (as h decreases), which suggests curve is **concave down**	B1 [1]	2.2b	Allow "convex up" but not just "downwards" Do not accept a sketch alone
7	(d)		first order	B1 [1]	1.2	
7	(e)		(the ratios of differences are converging to $r \approx$ 0.353 and) $0.25 < r < 0.5$	B1	2.1	Accept $0.25 < 0.353 < 0.5$ **oe**
			hence the order (of convergence) is between 1st and 2nd order	B1 [2]	2.2b	Accept "higher than 1st order but lower than 2nd order" but not "faster… and slower…"
7	(f)		$S + D \times \dfrac{r}{1-r}$	M1 M1	3.1a 3.1a	S is value of S_{2n} from the table, D is the associated difference and r = 0.35 or 0.353 Condone $\pm$ for M marks only and deduct M1 as follows: eg $0.78517377 \pm 0.0004107 \times 0.35$ **M1M0** (S_{64} used and only partial extrapolation) eg $0.78531885 \pm 0.0001451 \times 0.35$ **M1M0** (S_{128} correctly used but still only partial extrapolation) eg $0.78517377 \pm 0.0004107 \times \frac{0.35}{1-0.35}$ **M1M1A0** (S_{64} used) **M1M1** may be implied by sight of correct value in range with evidence of use of correct formula.

Question	Answer	Marks	AO	Guidance
	$0.78531885 + 0.0001451 \times \dfrac{0.353}{1-0.353}$	**A1**	**1.1**	All values correct including S_{128}, allow $r = 0.35$ or 0.353 Ignore wrongly attributed values if calculation correct e.g. writing $S_{64} = 0.78531885$
	0.78539692 to 0.78539802	**A1**	**1.1**	
	0.785398 or 0.78540 or 0.7854 is probable since extrapolation greatly improves accuracy	**A1**	**3.2a**	Allow 0.785 is certain since best **Simpson's** estimate and extrapolated value agree to this precision. Condone reference to S_{64} instead of S_{128} if meaning is clear. If M1M0 allow SC**B1** for 0.785 because best **Simpson's** estimate and extrapolated value agree to this precision. (max. 2/5) If **M0M0** allow SC**B2** for $\dfrac{16 \times 0.78531885 - 0.78517377}{15} = 0.785328(522\ldots)$ **oe** (SC**B1** for the method and **B1** for the answer, max. 2/5)
		[5]		

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