Monday 3 June 2024 – Afternoon

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2406

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ____________________

Last name ____________________

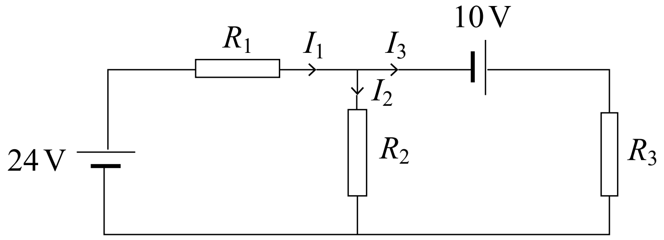
Date of birth D D M M Y Y Y Y

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The circuit diagram below contains three resistors with resistances $R_1\,\Omega$, $R_2\,\Omega$ and $R_3\,\Omega$. The circuit also contains two batteries with voltages 24 V and 10 V.



According to Kirchhoff's current law the currents I_1 A, I_2 A and I_3 A flowing through corresponding resistors with resistances R_1, R_2 and R_3, satisfy the following relationship.

$I_1 - I_2 - I_3 = 0$

According to Kirchhoff's voltage law the following relationships are also satisfied.

$24 - I_1R_1 - I_2R_2 = 0$

$10 + I_2R_2 - I_3R_3 = 0$

For this circuit $R_1 = 4$, $R_2 = 2$ and $R_3 = 5$.

(a) Derive a pair of linear simultaneous equations with unknowns I_2 and I_3.

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(b) Use your equations derived in part **(a)** to calculate the values of I_2 and I_3.

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(c) Calculate the total power consumed by the circuit.

You may use the formula $P = I^2 R$ given in the formula booklet.

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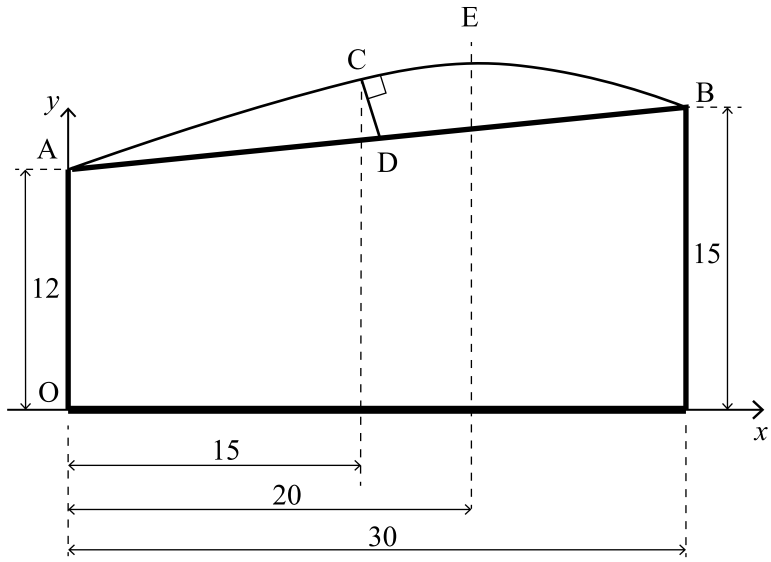
2 The diagram below shows the cross-section of a warehouse constructed on a horizontal base aligned within a Cartesian coordinate system, (x, y), with the origin at point O. The total width of the warehouse is 30 m. The warehouse has a curved roof which is supported on the top of vertical side walls. The cross-section shows a straight rigid joist which spans the entire width of the warehouse between points A and B. The wall on the left is 12 m high and the wall on the right is 15 m high. The cross-section of the roof has a shape modelled by the equation $y = ax^2 + bx + c$ and has a maximum height at the point where $x = 20$. The line CD represents a roof support.



Not to scale

(a) Calculate the values of a, b and c.

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The line CD in the diagram represents a straight support from the roof to the joist, where C is a point on the roof at which $x = 15$ and D is on the joist. The line CD is normal to the roof at point C.

(b) Determine the equation of the line CD.

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(c) Calculate the height of point D above the base.

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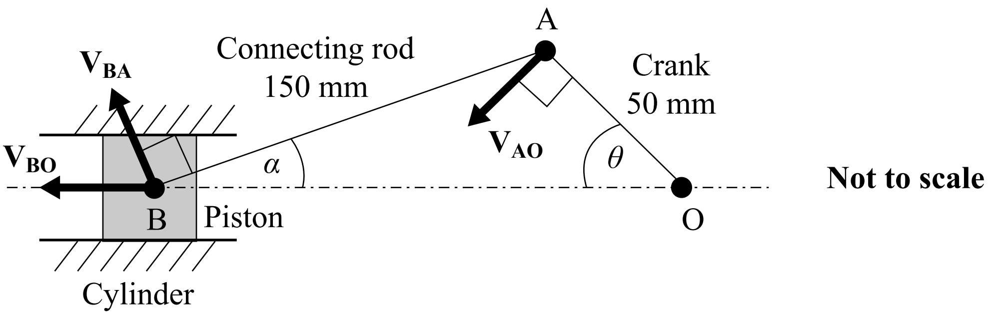
3 **Fig. 1** is a schematic diagram of a crank, connecting rod and piston mechanism. The crank, OA, rotates in an anticlockwise direction about a fixed point O. There is a pin at point A which freely connects the end of the crank to the end of the connecting rod, AB. There is also a pin at point B which freely connects the end of the connecting rod to the piston. Points O and B are on the same vertical level. The piston is constrained to move horizontally within a cylinder. As the crank rotates the end of the crank at point A forces the connecting rod to move which in turn forces the piston to move to the left.

Fig. 1



The heavy arrows in **Fig. 1**, labelled $\mathbf{V_{AO}}$, $\mathbf{V_{BA}}$ and $\mathbf{V_{BO}}$, represent relative velocity vectors. $\mathbf{V_{AO}}$ is the velocity vector of point A relative to point O, $\mathbf{V_{BA}}$ is the velocity vector of point B relative to point A and $\mathbf{V_{BO}}$ is the velocity vector of point B relative to point O. Note that $\mathbf{V_{AO}}$ is normal to AO, $\mathbf{V_{BA}}$ is normal to BA and $\mathbf{V_{BO}}$ remains horizontal. The length of the crank is 50 mm and the length of the connecting rod is 150 mm. The crank rotates in an anticlockwise direction at 1146 rpm (revolutions per minute).

(a) Show that the magnitude of $\mathbf{V_{AO}}$ is approximately $6\,\mathrm{m\,s^{-1}}$.

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The connecting rod makes an angle of α with the horizontal when the crank makes an angle of θ with the horizontal, as shown in **Fig. 1**.

Parts **(b)**, **(c)** and **(d)** refer to the case when $\theta = 30°$.

(b) Calculate the value of α.

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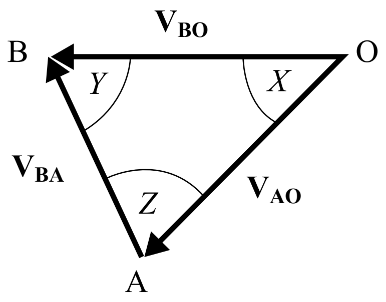
(c) **Fig. 2** is a diagram showing a closed triangle of the vectors $\mathbf{V_{AO}}$, $\mathbf{V_{BA}}$ and $\mathbf{V_{BO}}$ associated with **Fig. 1**.

Fig. 2



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Calculate

(i) angle X,

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(ii) angle Y,

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(iii) angle Z.

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(d) Calculate the magnitude of $\mathbf{V_{BO}}$.

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4 The voltage, v Volts, and the current, i Amps, in a particular AC circuit are given by the following equations.

$$v = 240\, sin\left(\omega t + \frac{\pi}{8}\right)$$

$$i = 4\, sin\left(\omega t + \frac{3\pi}{8}\right),$$

where $\omega = 100\pi$ and t seconds is time.

(a) Sketch a graph of v for $0 \leqslant t \leqslant 0.02$ on the grid below. Label the axes with appropriate values.

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(b) Use trigonometric identities from the formula booklet to show that

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right].$$

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(c) The instantaneous power, P VA (VoltAmps), at time t, consumed by the circuit is given by $P = v \times i$.

Show that

$$P = 480\left(\frac{1}{\sqrt{2}} + sin\ 2\ \omega t\right).$$

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5 For this question use the following values.

Density of water: $\rho_w = 1000\ \text{kg}\,\text{m}^{-3}$

Density of steel: $\rho_s = 8000\ \text{kg}\,\text{m}^{-3}$

A perfectly spherical, hollow, steel ball has an outside diameter of 2 m and a uniform wall thickness of t m.

(a) Calculate the mass of the ball when $t = 0.05$.

The mass of air in the ball can be neglected.

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When a body is fully immersed in water it displaces a volume of water equal to the volume of the body. A body placed in water will sink if the mass of the body is greater than the mass of the water it displaces; otherwise it will float.

(b) Determine whether the ball will sink or float when $t = 0.05$.

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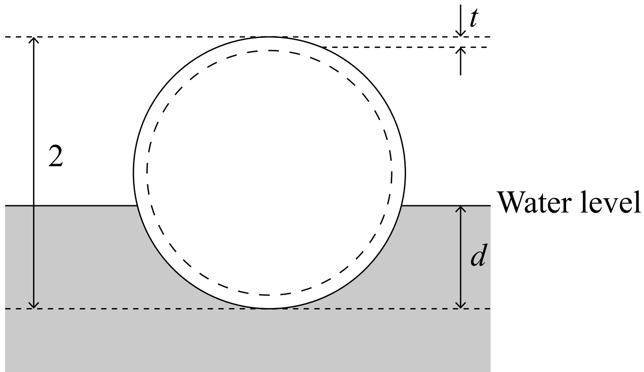
According to Archimedes' principle, when a body floats in water the mass of water displaced is equal to the **total** mass of the body. The diagram below shows a similar ball, but with a different wall thickness, floating in water so that its lowest point is d m below the surface of the water.



The mass of water displaced by this ball, M_w kg, is given by the following formula.

$$M_w = \pi\rho_w \int_0^d (2x - x^2)\, dx \text{ for } 0 \leqslant d \leqslant 2.$$

(c) Calculate M_w when $d = 0.5$.

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(d) Calculate the value of t when the ball is floating and $d = 0.5$.

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6 The heating system for an office is turned on at 7 am. The power supplied to the heating system is reduced once the office temperature has reached a comfortable level. On a particular day the office temperature is recorded for a period of ten hours from 7 am to 5 pm. This temperature is modelled by the following formula.

$$T = 9te^{-\frac{t}{3}} + 15,$$

where t is the number of hours after 7 am.

(a) Calculate the time of day when the maximum office temperature is reached, showing that the temperature reached at this time is a maximum.

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For the next part of the question you should use the following information.

The average value of a function, f(x), in the interval $x_0 \leqslant x \leqslant x_1$ is given by

$$\frac{1}{x_1 - x_0} \int_{x_0}^{x_1} \mathrm{f}(x)\,\mathrm{d}x\,.$$

(b) Calculate the average office temperature during the ten-hour period.

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7 The acceleration, $a\,\mathrm{m\,s^{-2}}$, and the velocity, $v\,\mathrm{m\,s^{-1}}$, of a small light aircraft while accelerating along a runway before taking off are modelled by

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{15t + 66}{t^2 + 7t + 10},$$

where t seconds is the time from when the aircraft started to move.

(a) Express $\frac{15t + 66}{t^2 + 7t + 10}$ as the sum of two partial fractions.

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For the next part of this question you are given the following information.

$$\int \frac{a}{x+b}\,\mathrm{d}x = a\ln(x+b) + C$$

(b) Determine a formula for v in terms of t, given that $v = 0$ when $t = 0$.

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The aircraft takes off when $t = 12$.

(c) Calculate the acceleration of the aircraft at this time.

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(d) Calculate the speed of the aircraft at this time.

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(e) Immediately after take-off the acceleration of the aircraft remains constant at the value calculated in part **(c)** until it reaches a speed of $40\,\mathrm{m\,s^{-1}}$.

Calculate the time after take-off for the aircraft to reach this speed.

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END OF QUESTION PAPER

EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.