Oxford Cambridge and RSA

Thursday 16 May 2024 – Afternoon

Level 3 Certificate Core Maths B (MEI)

H869/01 Introduction to Quantitative Reasoning

Insert

Time allowed: 2 hours

INSTRUCTIONS
- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION
- This Insert contains the pre-release material that you have already seen.
- This document has **8** pages.

A Spam emails

Spam emails, or junk emails, are uninvited emails which are sent out in very large quantities to recipients. Often, spam email is simply advertising.

Spam emails may also involve scams. A famous example is the plea for money to return a cosmonaut to Earth, who had been stranded on a secret Russian space station for 14 years. The scam claimed that the cosmonaut needed \$3 million to get back to Earth and that they would use their 14 years of backpay to return any money donated increased by a factor of 5. Several hundred people were taken in by this scam, including one person who gave \$30 000!

The person sending spam, the spammer, only needs an internet connection, software to allow thousands of identical emails to be sent out quickly and a list of recipients' email addresses. The overall cost per email sent is very low, typically about 0.01 p per email. There are companies that send spam emails for clients. They make use of specialised lists of email addresses to target individuals. Of course, they charge for their services.

All the main email providers have methods of blocking or filtering off spam email before it goes to the recipient's inbox. However, spammers regularly alter their methods. It is a cat-and-mouse situation; each new spam blocking or filtering method will be worked around by spammers.

The spammer's email must pass through three filters before the recipient accesses the advertiser's homepage. **Fig. A.1** shows these diagrammatically.

Fig. A.1

Does the email get through the spam filter?	→	If any spam email is let through, does the recipient bother to read it?	→	After reading the email does the recipient click to go to the advertiser's homepage?

It is not generally considered, but the recipients of spam emails also pay a cost. This comprises, for example: time wasted reading spam emails that get through "spam detectors", storing detected spam emails and buying the appropriate software to detect them. There is also an environmental cost. Data storage devices leave a carbon footprint because they need power to run and to keep cool.

> Most numbers involved with the internet are huge.
> You may find this useful:
> One billion is 1000 million.
> One trillion is 1000 billion.

B Volcanoes

Geologists need to give a number to describe the severity of a volcanic eruption.
The Volcanic Explosivity Index (VEI) is one way to do this.

It uses the volume of material ejected by the volcano and places this on a scale of 0 to 8.
The table in **Fig. B.1** shows the connection.

Fig. B.1

Volume of material ejected $E\,\text{km}^3$	**VEI (Volcanic Explosivity Index)**
$E < 10^{-5}$	0
$10^{-5} \leqslant E < 10^{-3}$	1
$10^{-3} \leqslant E < 10^{-2}$	2
$10^{-2} \leqslant E < 10^{-1}$	3
$10^{-1} \leqslant E < 1$	4
$1 \leqslant E < 10^{1}$	5
$10^{1} \leqslant E < 10^{2}$	6
$10^{2} \leqslant E < 10^{3}$	7
$10^{3} \leqslant E$	8

The VEI can be used with volcanoes many millions of years old. Measuring the depth and area of the ejected material allows its volume to be estimated.

About 2 billion years ago in Ecuador a volcanic eruption resulted in $456\,\text{km}^3$ of material being ejected. This had a VEI of 7.

Estimating the volume of material can be difficult and cannot be done while an eruption is in progress. A different approach involves measuring the height, H km, of the volcano's plume, as shown in **Fig. B.2**.

Fig. B.2

[redacted]

The VEI, V, is given by $V = \dfrac{H+25}{9}$; this is then rounded to the nearest whole number.

C Water-drops

Raindrops are studied because of the effect they can have on soil erosion. They dislodge earth particles which can be carried away by the flow of rainwater.

Raindrops, and water-drops generally, can be almost spherical or shaped like jelly beans.
Some typical water-drop shapes are shown in **Fig. C.1**.

Fig. C.1

Not to scale

As water-drops have such complicated shapes it is common to model them as spheres with the same volumes as the actual water-drops.

A water-drop in the shape of a sphere, diameter d mm, has a volume of $\frac{\pi d^3}{6}$ mm^3.

However, 1 mm^3 of water has a mass of 1 mg, so the water-drop has a mass of $\frac{\pi d^3}{6}$ mg.

So for a water-drop of mass M mg in the shape of a sphere of diameter d mm, $M = \frac{\pi d^3}{6}$,

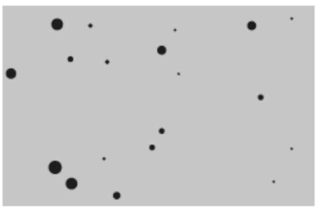
which can be rearranged to give $d^3 = \frac{6M}{\pi}$ or $d = \sqrt[3]{\frac{6M}{\pi}}$.

This means when a water-drop is modelled as a sphere, its diameter can be calculated from its mass.

The **stain method** is used to investigate the sizes and numbers of raindrops.

- A piece of absorbent or filter paper is covered with a colourless water-soluble dye.
- When the filter paper is exposed to rain, clear circular stains are produced by the raindrops, as illustrated in **Fig. C.2**.

Fig. C.2



- Stain diameters are related to the diameters of the raindrops producing them and to the type of filter paper used. Different types of filter paper vary in how well they absorb water. Whatman No. 1 filter paper is commonly used.
- To investigate raindrops the filter paper needs to be calibrated. This involves dropping water-drops of known masses and measuring the diameters of the stains they produce on the type of filter paper used.

D CubeSats

CubeSats are a type of small satellite with a low mass; one is shown in **Fig. D.1**.

As they are both light and small it is possible for CubeSats to be launched into orbit by hitching a ride on rockets with spare capacity that are being sent on other missions. They are ejected from the rocket using a simple spring system.

Fig. D.1

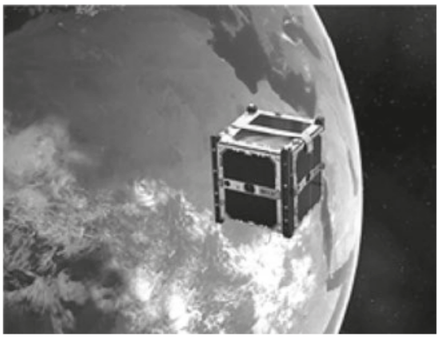
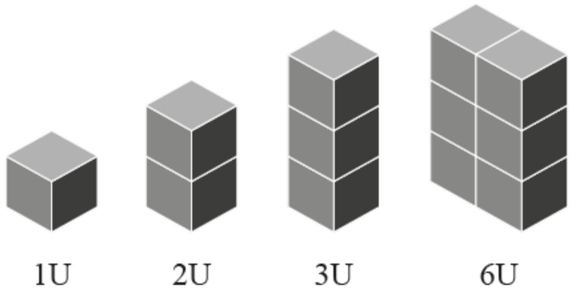


Fig. D.2



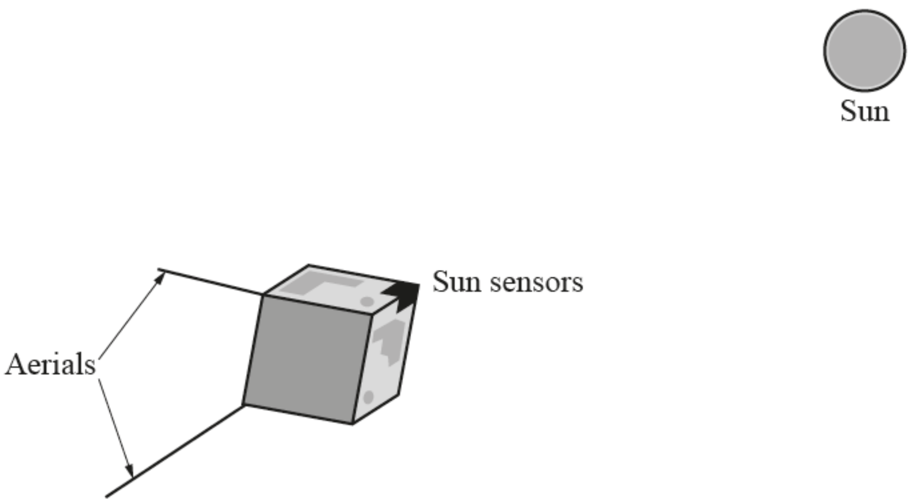
An important feature is that they are built up from cubes measuring $10\,\text{cm} \times 10\,\text{cm} \times 10\,\text{cm}$, as shown in **Fig. D.2**.
Two 1U cubes, as they are called, make up a 2U CubeSat ($10\,\text{cm} \times 10\,\text{cm} \times 20\,\text{cm}$).
3U CubeSats measure $10\,\text{cm} \times 10\,\text{cm} \times 30\,\text{cm}$.
Using standard sizes means that cheap off-the-shelf components can be used.

These factors make it inexpensive to launch CubeSats, typically about $6000 per kilogram of satellite. This makes it possible for universities and even groups of schools to have a CubeSat launched into orbit. At the time of writing over 2000 CubeSats have been put into orbit.

CubeSats have a limited lifetime. They do not usually have thrusters so cannot avoid eventually falling to Earth. They orbit between 200 km and 1000 km above the Earth. At this height particles from the sun slow satellites down, forcing them into lower and lower orbits, so eventually they burn up in the Earth's atmosphere. A 1U CubeSat of mass 0.8 kg launched into orbit at 250 km could burn up after about a week in orbit.

Most CubeSats use sunlight for their power. Photocells are fixed on their faces. **Fig. D.3** illustrates the arrangement that gives the maximum exposure to the Sun. The photocells are on three faces which share a corner that points directly to the Sun.

Fig. D.3



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