OCR

Oxford Cambridge and RSA

Thursday 16 May 2024 – Afternoon

Level 3 Certificate Core Maths B (MEI)

H869/01 Introduction to Quantitative Reasoning

Time allowed: 2 hours

You must have:
- the Insert (inside this document)

You can use:
- a scientific or graphical calculator

* H 8 6 9 0 1 *

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working.
- Give your final answers to a degree of accuracy that is appropriate to the context.

INFORMATION
- The total mark for this paper is **72**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **28** pages.

ADVICE
- Read each question carefully before you start your answer.

1 This question refers to article A in the pre-release material, 'Spam emails'. You can find the article on the Insert accompanying this paper.

(a) In 2018 a total of 102.6 trillion emails were sent and received globally.
Of these, 53.9 trillion were spam emails.

(i) Use the definition of a trillion provided in article A to write 102.6 trillion in standard form.
Give your answer to **2** significant figures. **[2]**

1(a)(i)	

(ii) Some people find such large numbers hard to interpret.
It can be helpful to use index numbers.

Fig. 1.1 gives the spam emails in recent years as actual numbers and as index numbers with 2018 as the base year.
The index numbers are given to **1** decimal place.

Complete the table by filling in the empty cell. **[1]**

1(a)(ii) **Fig. 1.1**

Year	Spam emails (trillions)	Index number
2018	53.9	100.0
2019	60.6	....
2020	56.3	104.5
2021	53.2	98.7
2022	60.8	112.8

(b) *Useful Bookz* are online book sellers. They are planning to use a spamming company for an advertising campaign to sell their latest vegan recipe book.

The spamming company provides this information:

- Spam filters are able to **reject** spam emails 99.9% of the time.
- About 40% of recipients open any spam emails that have arrived in their inboxes.
- Of those who open the spam emails, about 2% will click on the link to go to the homepage. They **might** make a purchase there.

The spamming company use a specialised list and charge £0.0005 per spam email.

Useful Bookz plan to pay for 100 000 spam emails and they make a profit of £8 for each copy of their vegan recipe book they sell.

Is spamming a financially effective way for *Useful Bookz* to advertise the book?
Justify your decision using calculations.
State any assumptions you make. **[6]**

1(b)	

2 This question refers to article B in the pre-release material, 'Volcanoes'. You can find the article on the Insert accompanying this paper.

Table 2.1 shows the volume of material ejected by volcanoes and their corresponding VEIs.

Table 2.1

Volume of material ejected E km^3	VEI (Volcanic Explosivity Index)
$E < 10^{-5}$	0
$10^{-5} \leqslant E < 10^{-3}$	1
$10^{-3} \leqslant E < 10^{-2}$	2
$10^{-2} \leqslant E < 10^{-1}$	3
$10^{-1} \leqslant E < 1$	4
$1 \leqslant E < 10^{1}$	5
$10^{1} \leqslant E < 10^{2}$	6
$10^{2} \leqslant E < 10^{3}$	7
$10^{3} \leqslant E$	8

(a) In the year 79, Vesuvius erupted, ejecting $3.3\,\text{km}^3$ of material.

What was the VEI (Volcanic Explosivity Index) of this volcanic eruption? **[1]**

2(a)	

(b) An alternative method of calculating the VEI of a volcano makes use of the height, H kilometres, of its plume.

The VEI, V, is given by $V = \frac{H + 25}{9}$.

This is then rounded to the nearest whole number.

Vesuvius had another eruption in the year 512.
Written accounts at the time suggest that:

- The volume of material ejected was $0.008\,\text{km}^3$.
- The plume rose to a height of 9.0 km.

Use both methods to calculate values for the VEI of this eruption. **[3]**

2(b)	VEI of Vesuvius from **volume of material ejected**.
	VEI of Vesuvius from **height of plume**.

(c) **Table 2.2** shows the VEIs, determined using both methods, of a representative sample of volcano eruptions.

Make **two** comments comparing the values given by the two methods. **[2]**

Table 2.2

VEI (from volume of material ejected)	1	1	1	2	2	2	2	2
VEI (from plume height)	2	2	3	4	4	4	3	3

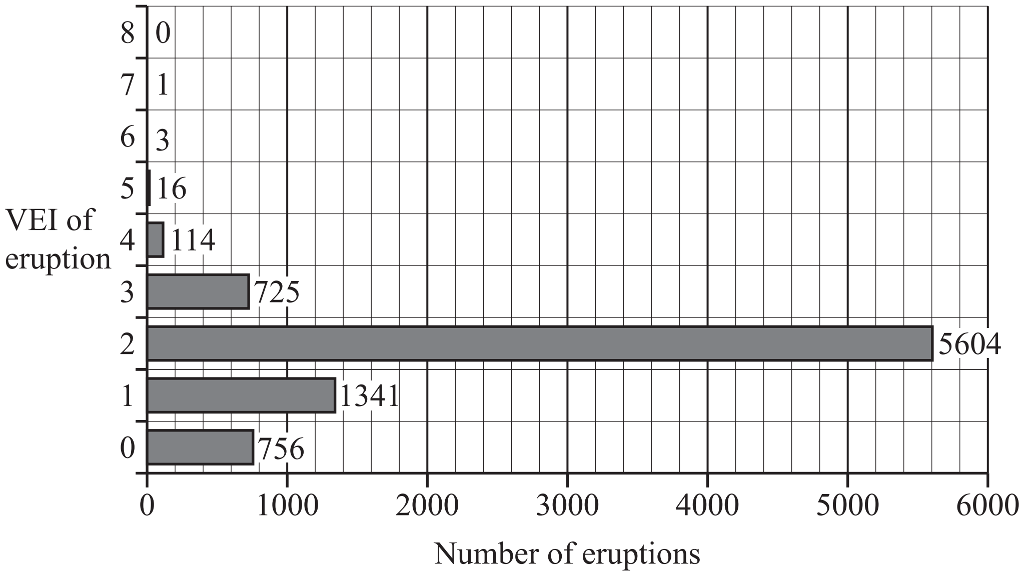
VEI (from volume of material ejected)	3	3	3	4	4	4	4	4
VEI (from plume height)	4	4	3	4	4	4	4	5

VEI (from volume of material ejected)	5	5	5	5	6	6	6	7
VEI (from plume height)	5	5	5	5	6	6	6	7

2(c)	Comment 1
	Comment 2

(d) The bar chart in **Fig. 2.3** shows the VEIs of known volcanoes over the last 500 years.

Fig. 2.3



(i) Eruptions of VEI 4 or more are considered devastating.
A TV programme states that,
"*A volcano eruption of Volcanic Explosivity Index 4 or more can be expected roughly once every ten years*".

Investigate whether this is supported by the data in the bar chart. **[3]**

2(d)(i)	

(ii) Give **one** reason, based on the bar chart, for assuming that the distribution of VEIs is **not** Normal? **[1]**

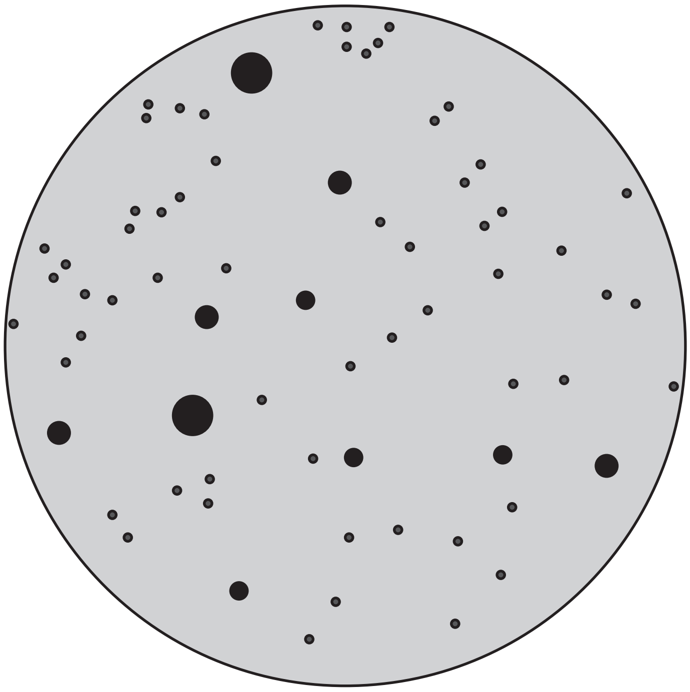
2(d)(ii)	

3 This question refers to article C in the pre-release material, 'Water-drops'.
You can find the article on the Insert accompanying this paper.

(a) Some students use the stain method to investigate the number and sizes of raindrops. They expose a piece of circular filter paper, covered with a colourless water-soluble dye, to a shower of rain for **10 seconds**.

Fig. 3.1 shows the resulting stains on the filter paper.
The diameter of the filter paper is 9 cm.

Fig. 3.1

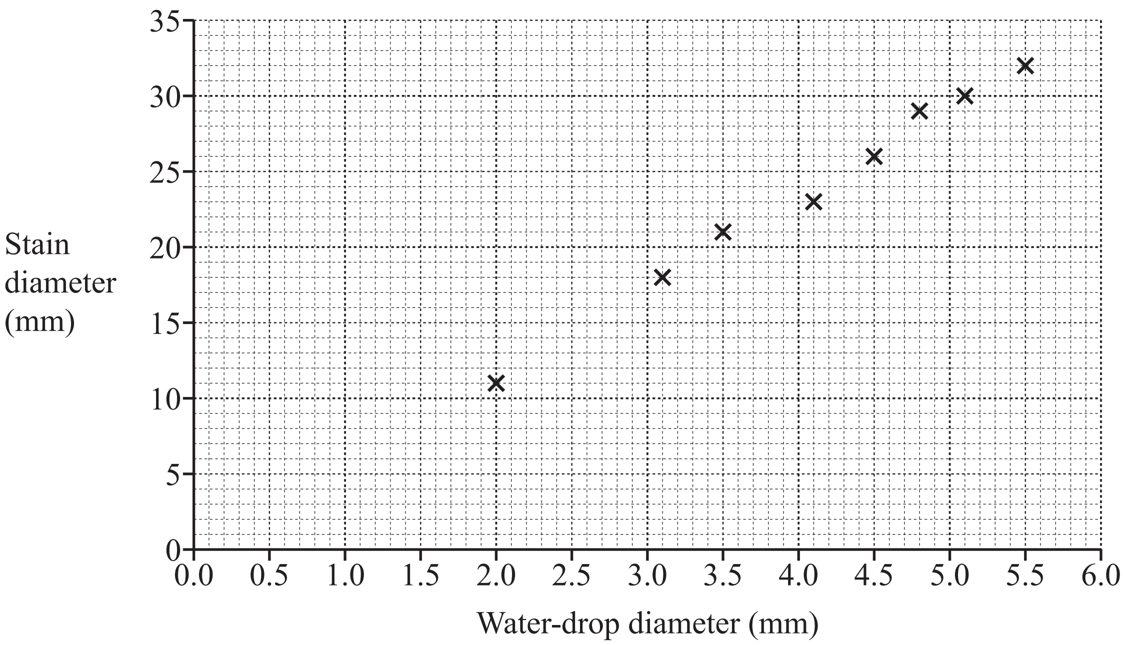


Calculate an estimate of the number of raindrops **per minute** that fall on each **square metre** of ground the shower falls on.
Give your answer to an appropriate degree of accuracy.
Show your calculations. **[5]**

3(a)	

(b) **Fig. 3.2** shows the scatter diagram for water-drop diameters and the resulting stain diameters when they fall onto Whatman No.1 filter paper.

(i) Draw a line of best fit on the scatter diagram. **[1]**

3(b)(i)	**Fig. 3.2** 

The straight line model represented by your line of best fit can be used to estimate the stain diameter for a given water-drop diameter, and vice versa.

(ii) Use your line of best fit to estimate the diameter of a water-drop which leaves a stain of diameter 15 mm. **[1]**

3(b)(ii)	

(iii) A formula is proposed that gives stain diameter, s mm on Whatman No. 1 filter paper, as a function of water-drop diameter, d mm. The formula is $s = 6d - 1$.

Does the formula fit your answer to part **(b)(ii)**? **[2]**

3(b)(iii)	

(c) The formula $d = \sqrt[3]{\frac{6M}{\pi}}$ gives the diameter d mm of a spherical water-drop mass M mg.

Without using a calculator, estimate the value of d in $d = \sqrt[3]{\frac{6 \times 3.8}{\pi}}$.

Show all the approximations that you make. **[2]**

3(c)	

(d) **Table 3.3** shows the distribution of raindrop diameters in a heavy shower.

Table 3.3

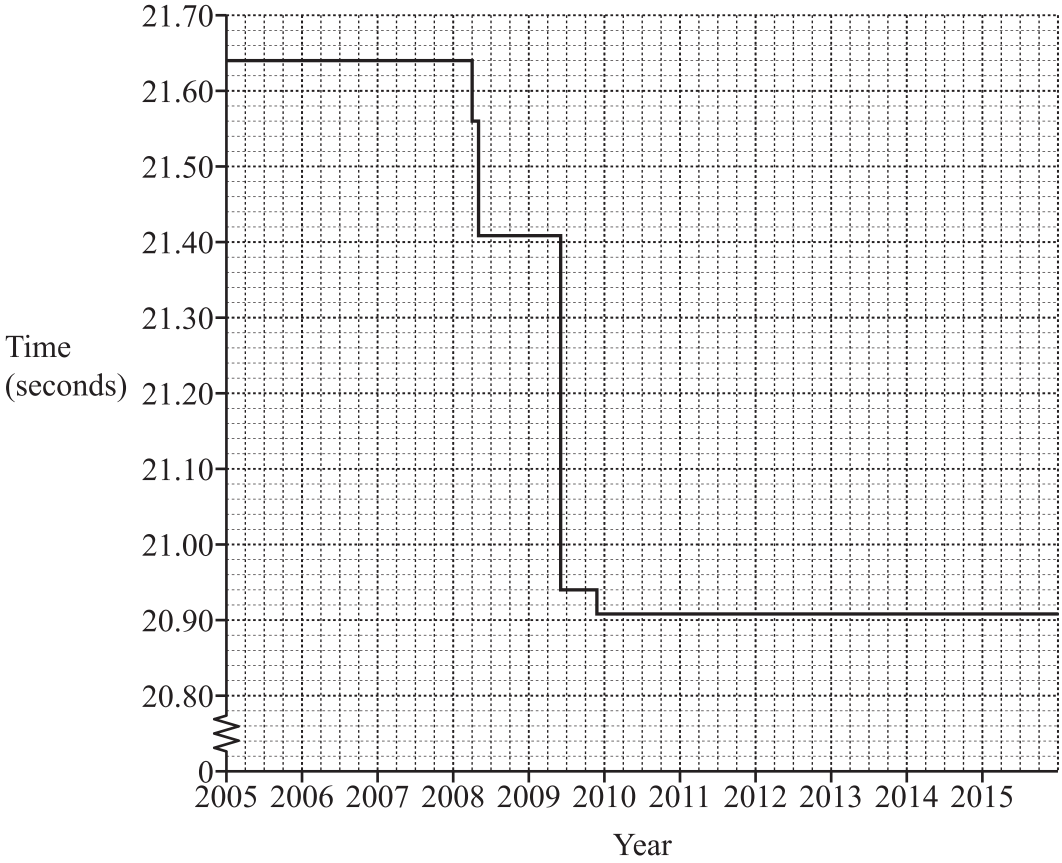
Diameter *d* mm	**Frequency**	**Mid-interval**	**Frequency × Mid-interval**
$0.5 \leqslant d < 1.0$	61	0.75	45.75
$1.0 \leqslant d < 1.5$	109	1.25	136.25
$1.5 \leqslant d < 2.0$	116	1.75	203.00
$2.0 \leqslant d < 2.5$	30	2.25	67.50
$2.5 \leqslant d < 3.0$	3	2.75	8.25

Calculate an estimate of the mean raindrop diameter.
Give your answer correct to the nearest 0.1 mm. **[3]**

3(d)	

4 The international 50 m swimming race is one length of a 50 m long pool.

(a) The step graph shows the World record times for the 50 m men's race from 2005 to 2015.



(i) What was the World record time in 2005? **[1]**

4(a)(i)	

(ii) In which year did the World record time fall below 21.45 seconds? **[1]**

4(a)(ii)	

(iii) Explain why the lines on the graph are either horizontal or vertical. **[2]**

4(a)(iii)	

(iv) Make **one** detailed observation about how the World record times changed from 2005 to 2015. **[1]**

4(a)(iv)	

(b) Below is a screenshot from a spreadsheet. Columns B and C are in number format, set to **2** decimal places.

Column B shows the ten best women's times in the 50 m swimming race at the 2020 Olympics.
These times are used to calculate the average speed of each swimmer, shown in column C.

	A	B	C
1	**Rank order**	**Time (s)**	**Speed ($m\,s^{-1}$)**
2	1	24.02	2.08
3	2	24.12	2.07
4	3	24.15	2.07
5	4	24.26	2.06
6	5	24.31	2.06
7	6	24.36	2.05
8	7	24.37	2.05
9	8	24.41	2.05
10	9	24.52	2.04
11	10	24.55	2.04

Write down the formula which should be in cell C7. **[2]**

4(b)	

(c) Time measurements and distances in international races must be accurate within certain limits.

- Times must be accurate to ± 0.005 seconds.
- The length L, in metres, of a 50 m length pool must be such that $50 \leqslant L \leqslant 50.010$.

(i) Would a pool judged to be 5 mm longer than 50 m be acceptable?
Show how you decided. **[2]**

4(c)(i)	

(ii) Two women in an international 50 m race are given the same time but video evidence shows that one of them finished 0.8 cm ahead of the other.

Explain how this happened.
State clearly any estimations you make in your calculations. **[5]**

4(c)(ii)	

5 In this question, compound interest is charged, and the rate quoted is applied daily.

Amit and Kai are discussing short term loans.
Although daily interest rates look small, the total amount owed increases rapidly.

(a) To illustrate this, Kai builds a spreadsheet showing the total amount owed on £100 borrowed for various times in days and daily rates of compound interest. This is shown in **Fig. 5.1**.

For example, borrowing £100 for 21 days at a daily compound interest rate of 6% results in a total amount owed of £339.96.

Fig. 5.1

	Daily compound interest rate (%)						
Days	**0.1**	**0.4**	**0.8**	**1.2**	**3.0**	**5.0**	**6.0**
1	100.10	100.40	100.80	101.20	103.00	105.00	106.00
2	100.20	100.80	101.61	102.41	106.09	110.25	112.36
3	100.30	101.20	102.42	103.64	109.27	115.76	119.10
4	100.40	101.61	103.24	104.89	112.55	121.55	126.25
5	100.50	102.02	104.06	106.15	115.93	127.63	133.82
6	100.60	102.42	104.90	107.42	119.41	134.01	141.85
7	100.70	102.83	105.74	108.71	122.99	140.71	150.36
8	100.80	103.25	106.58	110.01	126.68	147.75	159.38
9	100.90	103.66	107.43	111.33	130.48	155.13	168.95
10	101.00	104.07	108.29	112.67	134.39	162.89	179.08
11	101.11	104.49	109.16	114.02	138.42	171.03	189.83
12	101.21	104.91	110.03	115.39	142.58	179.59	201.22
13	101.31	105.33	110.91	116.77	146.85	188.56	213.29
14	101.41	105.75	111.80	118.18	151.26	197.99	226.09
15	101.51	106.17	112.70	119.59	155.80	207.89	239.66
16	101.61	106.60	113.60	121.03	160.47	218.29	254.04
17	101.71	107.02	114.51	122.48	165.28	229.20	269.28
18	101.82	107.45	115.42	123.95	170.24	240.66	285.43
19	101.92	107.88	116.35	125.44	175.35	252.70	302.56
20	102.02	108.31	117.28	126.94	180.61	265.33	320.71
21	102.12	108.74	118.21	128.47	186.03	278.60	339.96

(i) A loan of £100 is borrowed at a daily rate of 3%.

After how many days will the total amount owed on it exceed £150? **[1]**

5(a)(i)	

(ii) The total amount owed is proportional to the amount borrowed.

Find the total amount owed on **£300** borrowed for 14 days at a daily interest rate of 0.8%. **[2]**

5(a)(ii)	

(b) Kai finds an approximate rule to show how quickly the total amount owed increases. It gives the number of days needed for the total amount owed to be double the original loan.

Divide 72 by the daily interest rate, % per day.
This figure, if not a whole number, is rounded up to the next whole number.

(i) Use the rule to calculate how many days the total amount owed would take to first become at least double the total amount initially borrowed at a daily interest rate of 6%. **[1]**

(ii) How does your answer to part **(b)(i)** compare to the one given by Kai's spreadsheet (**Fig. 5.1**)? **[1]**

5(b)(i)	
5(b)(ii)	

(c) Amit is not sure of Kai's figures and asks to see the spreadsheet coding.
Part of this is shown in **Fig. 5.2**.

Fig. 5.2

	A	B	C
1			
2	**Days**	**0.1**	**0.4**
3	**1**	=100*(1+(B$2/100))^$A3	=100*(1+(C$2/100))^$A3
4	**=A3+1**	=100*(1+(B$2/100))^$A4	=100*(1+(C$2/100))^$A4
5	**=A4+1**	=100*(1+(B$2/100))^$A5	=100*(1+(C$2/100))^$A5

Write down the algebraic formula for the total amount owed, £T

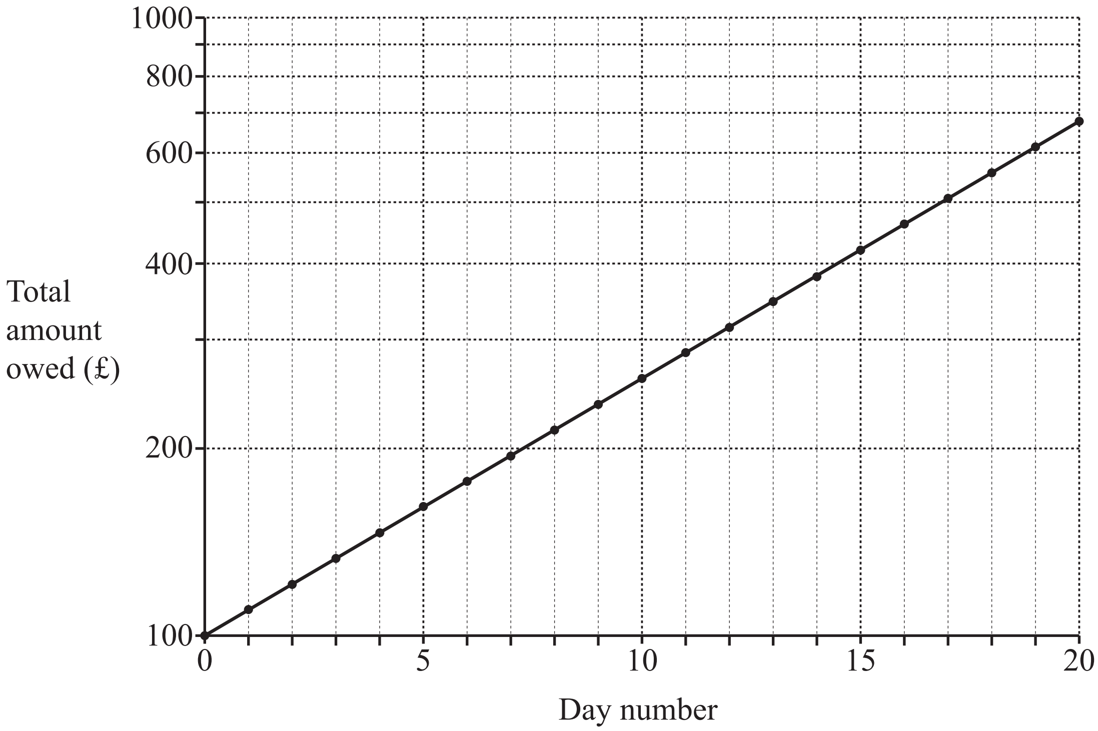
- when borrowing £100
- for n days
- at a daily interest rate of r%.

You may find the spreadsheet formulae in **Fig. 5.2** useful. **[2]**

5(c)	

(d) **Fig. 5.3** shows the graph Kai correctly obtained when plotting total amount owed against day number for a 10% per day interest rate.
The same general shape was found for all the daily percentage interest rates.

Fig. 5.3



Amit concludes that:
"The straight line shows that the increase in the total amount owed is the same each week."

Explain the error Amit has made. **[1]**

5(d)	

(e) Amit agrees with all Kai's calculations.
Daily compound interest charges are set at a **maximum** of 0.8% by law.
Fig. 5.4 shows the total amount owed for various daily compound interest rates for £100 borrowed, calculated by Kai.

Fig. 5.4

	Daily compound interest rate (%)					
Days	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**
1	100.20	100.40	100.60	100.80	101.00	101.20
2	100.40	100.80	101.20	101.61	102.01	102.41
3	100.60	101.20	101.81	102.42	103.03	103.64
4	100.80	101.61	102.42	103.24	104.06	104.89
5	101.00	102.02	103.04	104.06	105.10	106.15
6	101.21	102.42	103.65	104.90	106.15	107.42
7	101.41	102.83	104.28	105.74	107.21	108.71
8	101.61	103.25	104.90	106.58	108.29	110.01
9	101.81	103.66	105.53	107.43	109.37	111.33
10	102.02	104.07	106.16	108.29	110.46	112.67

Brisk Bread offers Amit a loan of £250 **for a week** with a total of £15 interest charged.

Use the table to decide if this is a legal loan.
Show your working. **[3]**

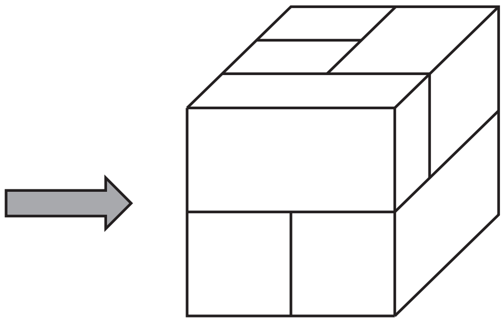
5(e)

6 This question refers to article D in the pre-release material, 'CubeSats'.
You can find the article on the Insert accompanying this paper.

(a) **Fig. 6.1** shows a drawing of a CubeSat package consisting of two 1U, two 2U and two 3U CubeSats, six CubeSats in total.
Like those shown in **Fig. D.2** on the Insert, all six CubeSats are cuboids.

Draw to scale the side view of the CubeSat package in the direction of the arrow. **[2]**

Fig. 6.1

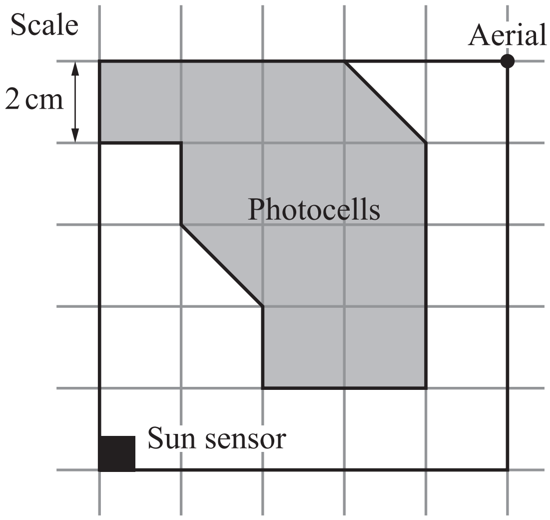


6(a)

(b) A university is making a 1U CubeSat to investigate how bacteria behave in space.
Fig. 6.2 shows a scale drawing, on a square grid, of one of the three identical faces which have photocells on them. The photocells are shown in grey.

When in orbit the CubeSat will be positioned so as to catch as much sunlight as possible.

Fig. 6.2



- To run the experiments and transmit data back to Earth the CubeSat will need 2.5 watts of electrical power in total.
- When the CubeSat is in position, each square centimetre of photocell will generate 0.019 watts of electrical power.

Does the planned CubeSat have sufficient electrical power in total?
Show your working. **[3]**

6(b)	

(c) (i) The number of CubeSats launched between 2003 and 2014 was 178.
Of these, only 108 completed their mission.

Use this information to estimate the probability that a randomly chosen CubeSat launched between 2003 and 2014 successfully completed its mission. **[1]**

6(c)(i)	

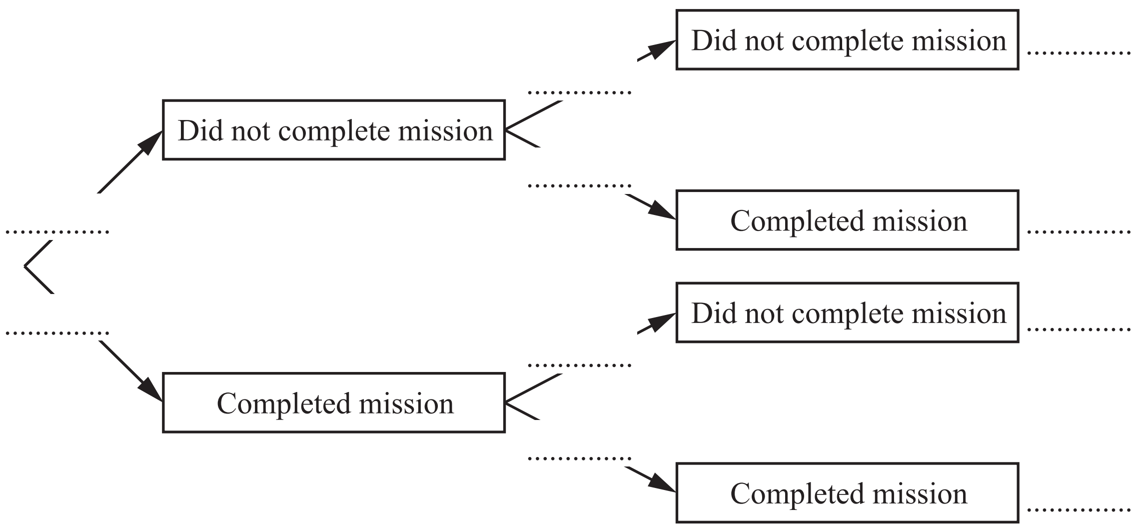
(ii) Since 2015 the reliability of CubeSats has increased.
The probability of one completing a mission once in orbit is now 0.80.

It is common to have several CubeSats working together.
Two CubeSats are working together. The mission needs both to work.

Complete the tree diagram in **Fig. 6.3**. **[2]**

6(c)(ii)

Fig. 6.3

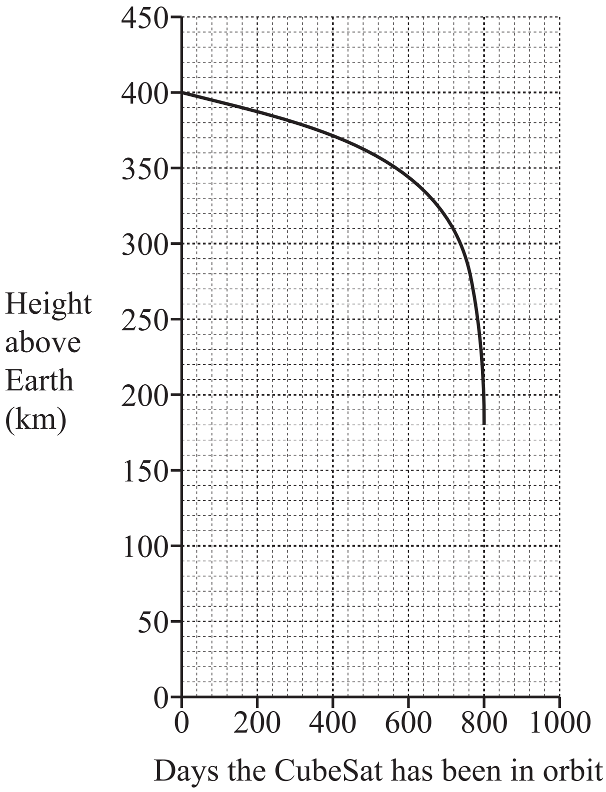


(iii) Work out the probability of **both** CubeSats completing the mission. **[1]**

6(c)(iii)	

(d) Unless they have some form of propulsion, all CubeSats will eventually fall to Earth and burn up in the Earth's atmosphere.

The graph shows how the height above the Earth of a particular 1U CubeSat changes with time.



(i) What was the CubeSat's height 400 days after being put into orbit? **[1]**

(ii) Estimate the rate that the CubeSat is falling to Earth when it has been in orbit for 720 days.
Give the units of your answer. **[3]**

6(d)(i)	
6(d)(ii)	

(iii) Calculations are carried out to estimate the expected lifetimes of 3U CubeSats of various masses and initial orbital heights.

The table below shows the lifetimes (in days) of 3U CubeSats of various masses and initial orbit heights to **2** significant figures.

	Height launched into orbit (km)			
Mass of 3U CubeSat (kg)	**200**	**250**	**400**	**500**
5	1.9	16	1000	11 000
4	1.6	13	820	8 700
3	1.2	10	610	6 500
2	0.8	6.6	410	4 400
1	0.4	3.4	210	2 200

A student stated that the lifetime for a particular mass of CubeSat is directly proportional to its initial orbit height.

Use the data in the table to investigate whether this is **true** or **false**.
Support your answer with a reason. **[1]**

6(d)(iii)	The statement that the lifetime for a particular mass of CubeSat is directly proportional to its initial orbit height is .... because

END OF QUESTION PAPER

EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.