OCR
Oxford Cambridge and RSA

Tuesday 18 June 2024 – Afternoon

Level 3 Free Standing Mathematics Qualification: Additional Mathematics

6993/01 Paper 1

Time allowed: 2 hours

You must have:
- the Formulae Sheet for Level 3 Free Standing Mathematics Qualification: Additional Mathematics (inside this document)

You can use:
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
FSMQ Additional Mathematics (6993)

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$, for positive integers, n,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$, $r \leqslant n$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Numerical methods

Trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Kinematics

Variable acceleration formulae

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

$s = \int v \, \mathrm{d}t$ and $v = \int a \, \mathrm{d}t$

Constant acceleration formulae

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 Solve the inequality $2-(x+1)>7$. **[2]**

1	

2 Write $\log_{10}2a+2\log_{10}b$ as a single logarithm. **[2]**

2	

3 You are given the quadratic polynomial $f(x)=x^2+6x-11$.

(a) Write $f(x)$ in the form $f(x)=(x+a)^2+b$ where a and b are integers to be found. **[3]**

(b) Using your answer to part **(a)**, solve the equation $f(x)=0$, giving exact answers. **[2]**

3(a)	
3(b)	

4 A group of 68 students was asked about their preference for pop music or classical music.
10 said they did not like either pop music or classical music.
20 said they liked classical music.
50 said they liked pop music.

(a) Draw a Venn diagram to illustrate this information. **[2]**

(b) Determine the number of students who said they liked both pop music and classical music. **[2]**

4(a)	
4(b)	

5 You are given the recurrence relationship $x_{n+2} = x_{n+1} + 2x_n$ for all positive integers n where $x_1 = 1$ and $x_2 = 1$.

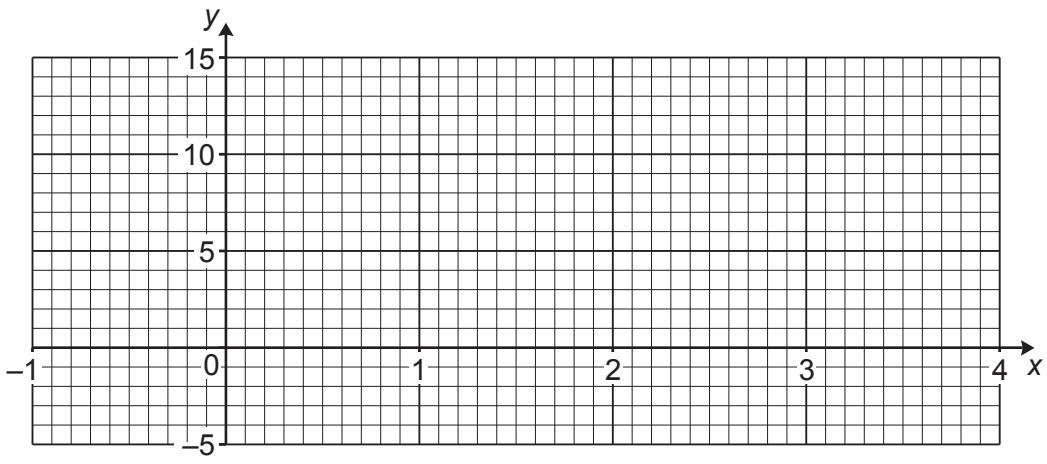
Determine the value of x_5. **[3]**

5	

6 **(a)** On the grid, plot the graphs of $y = x^2 - 4x + 1$ and $y = 2^x - 2$. **[4]**

(b) Hence write down the root of the equation $2^x = x^2 - 4x + 3$, giving your answer correct to **1** decimal place. **[1]**

6(a)



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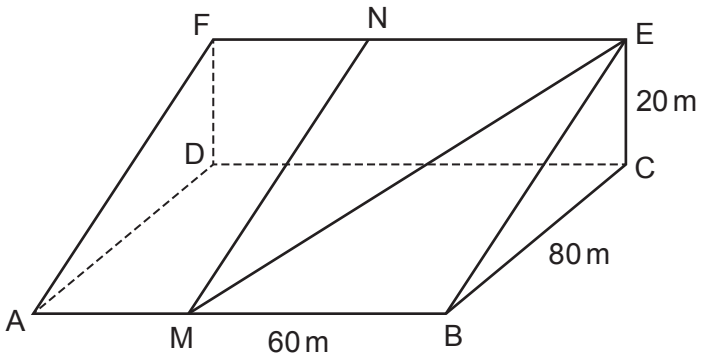
6(b)

7 A hillside can be modelled by a wedge as shown in the diagram.



The base ABCD is a horizontal rectangle with BC = AD = 80 metres.
The back of the wedge is a vertical rectangle CDFE with DF = CE = 20 metres.
M is a point on the line AB where BM = 60 metres.

A straight path goes up the hill from M to N where N is a point on FE such that MN is a line of greatest slope.

(a) Calculate the angle that the path MN makes with the horizontal, giving your answer correct to **1** decimal place. **[2]**

(b) A second straight path goes up the hill from M to E. Calculate the angle that this path makes with the horizontal, giving your answer correct to **1** decimal place. **[4]**

7(a)	
7(b)	

8 In this question you must show detailed reasoning.

You are given the cubic polynomial $f(x) = x^3 - 5x^2 + 2x + 8$.

(a) Using the factor theorem, factorise $f(x)$. **[4]**

(b) Hence solve the equation $f(x) = 0$. **[1]**

8(a)

8(b)

9 Two fair six-sided dice are rolled. If they both display the same number, then that number is recorded in the table below. When they display different numbers the higher of the two numbers is recorded in the table below.

(a) Complete the two way table showing the number that would be recorded. **[2]**

(b) Write down the probability that:

(i) a 1 is recorded **[1]**

(ii) a 6 is recorded. **[1]**

9(a)

2nd dice / 1st dice	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

9(b)(i)

9(b)(ii)

10 Determine the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x + 4$. **[5]**

10	

11 The gradient function of a curve is given by $\frac{dy}{dx} = 4 + 6x - 3x^2$ and the curve passes through the point (1, 5).

Determine the equation of the curve. **[4]**

11	

12 The table below gives the coordinates of six points.

x	0	1	2	3	4	5
y	1	1.13	1.43	2.11	3.69	7.30

The points are plotted on a grid and a smooth curve is drawn to pass through the points.

(a) Using the trapezium rule with 5 strips, find an estimate for the area between the x-axis, the y-axis, this curve and the line $x = 5$. Give your answer correct to **1** decimal place. **[3]**

(b) (i) Calculate an estimate for the rate of change of y with respect to x for this curve when $x = 3$. Give your answer correct to **1** decimal place. **[2]**

(ii) Explain how this estimate could be improved. **[1]**

12(a)	
12(b)(i)	
12(b)(ii)	

13 Tom has been asked to solve the equation $x^3 - x - 7 = 0$.

(a) Show by sign change that $x^3 - x - 7 = 0$ has a root, α, in the interval [2, 3]. **[2]**

(b) Tom decides to use the iterative formula $x_{r+1} = \sqrt[3]{x_r + 7}$ to find α.

(i) Explain how he obtained this iterative formula. **[2]**

(ii) Using this iterative formula, with $x_0 = 2$, find α correct to **3** significant figures. **[2]**

13(a)	
13(b)(i)	
13(b)(ii)	

14 Layla is given the following data.

n	1	3	5	7	9
x	0.5	1.1	2.5	5.7	12.8

She believes that the relationship between n and x can be modelled by the equation $x = ka^n$ where k and a are constants.

(a) Show by taking logs of both sides of this equation that a straight line will be obtained by plotting $\log_{10} x$ against n. **[3]**

(b) For the data given in the table, plot the points $(n, \log_{10} x)$ on the grid on page 13.

Use the spare row in the table above for any values you need to calculate. **[3]**

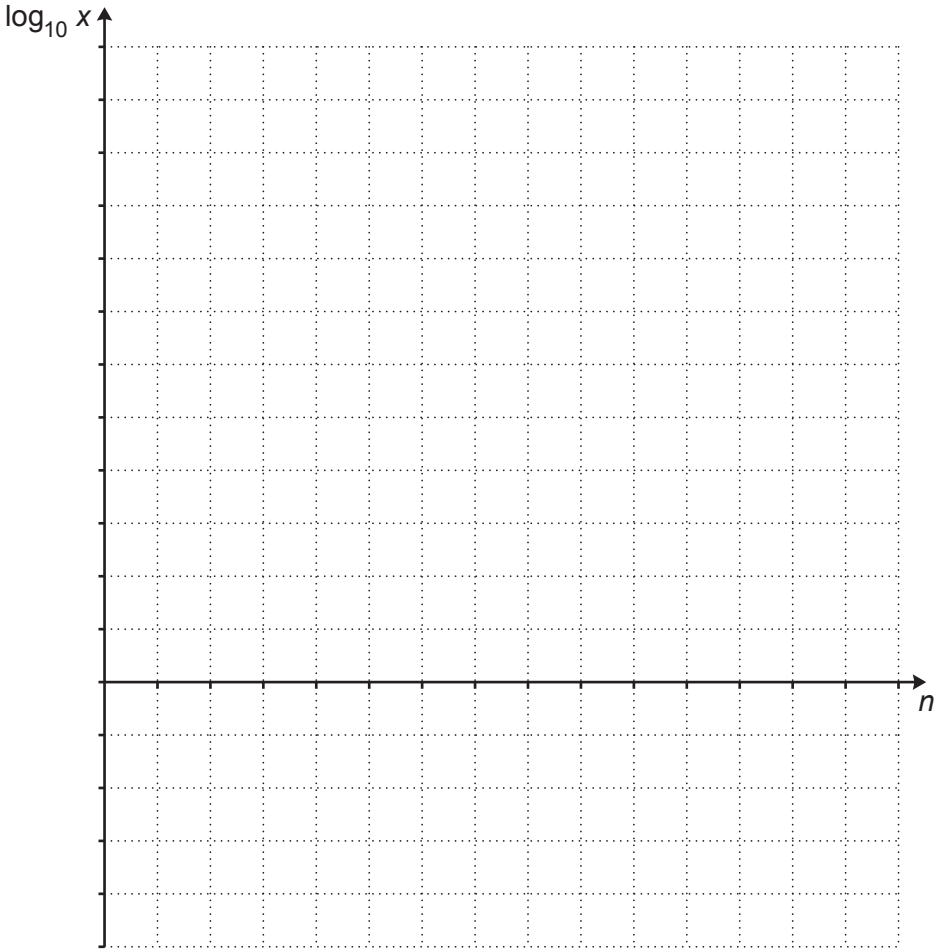
(c) **(i)** Draw the line of best fit for the points on the grid. **[1]**

(ii) Explain why the model is reasonable for these data. **[1]**

(d) Hence estimate values for k and a. **[3]**

14(a)	

14(b), (c)(i)



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14(c)(ii)

14(d)

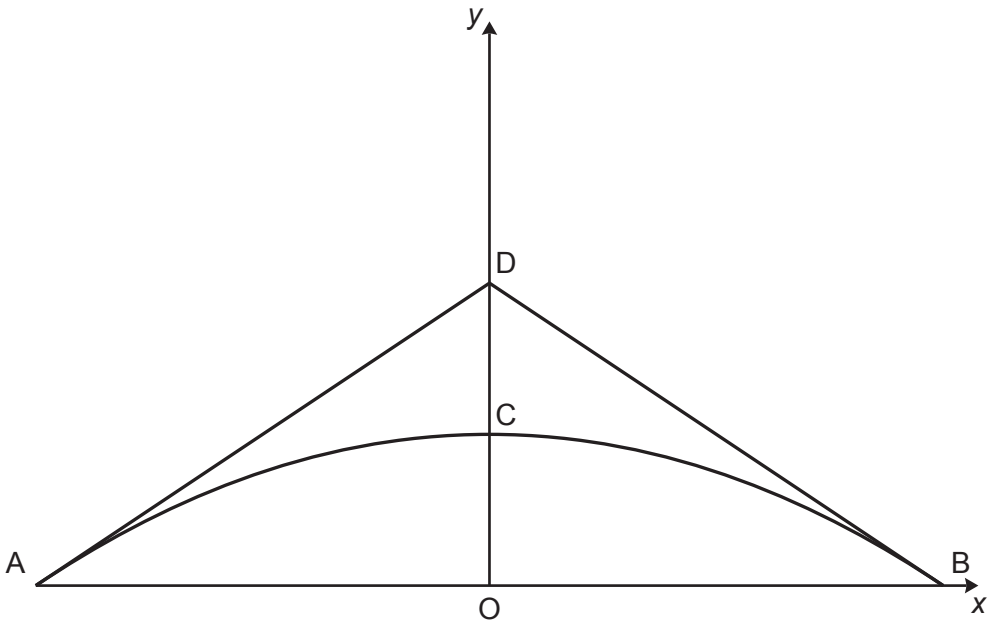
15 In this question you must show detailed reasoning.

The diagram shows a symmetric, triangular cross-section of a roof, ADB, where AB is horizontal. Under this roof is the ceiling, ACB, of a room.

In a coordinate system the mid-point of AB is the origin, O, with the x-axis passing through A and B and the y-axis passing through C and D.

With respect to this coordinate system the equation of the curve ACB is $y = 2 - \frac{1}{18}x^2$.

The lines AD and BD are tangents to the curve at A and B respectively. Units are metres.



(a) Find the coordinates of the points A, C and B. **[3]**

(b) Find the equation of the line AD. **[4]**

(c) Find the coordinates of the point D. **[1]**

(d) Find the cross-sectional area of the space between the roof and the ceiling. **[5]**

15(a)	

15(b)	
15(c)	
15(d)	

16 Two drugs, X and Y, are used in combination to treat dogs with a particular dietary condition. Both drugs can have a positive effect and this is measured by "curative" units per mg of the drug. However both drugs can also have a negative effect and this is measured by "toxic" units per mg of the drug.

The table summarises the number of curative and toxic units per mg present in the two drugs.

Drug	Curative units per mg	Toxic units per mg
X	2000	40
Y	1000	50

A dog is given x mg of drug X and y mg of drug Y.

(a) (i) The dog needs at least 5000 curative units.

Write this condition as an inequality in x and y. **[1]**

(ii) The dog must not receive more than 190 toxic units.

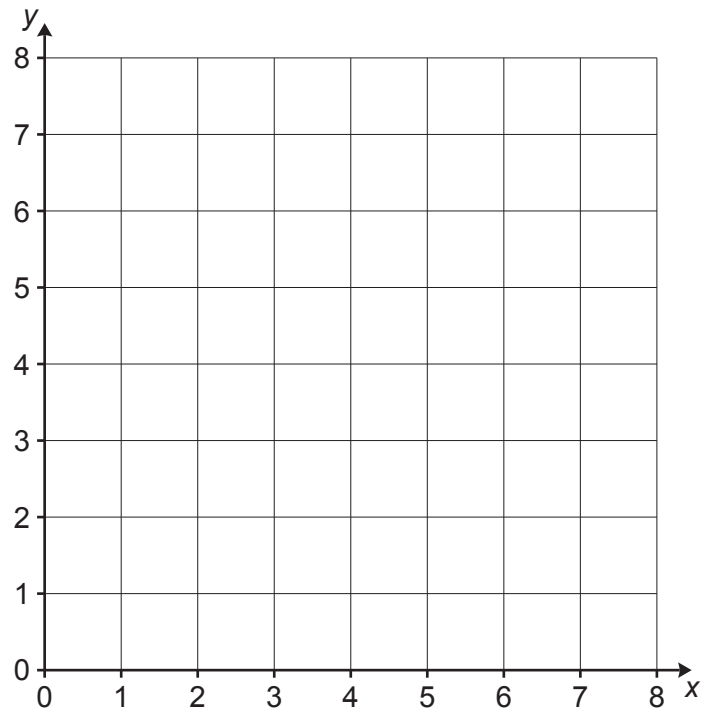
Use this information to write down a second inequality in x and y. **[1]**

(iii) Write each of the inequalities you found in **(a)(i)** and **(a)(ii)** in their simplest form. **[1]**

16(a)(i)	
16(a)(ii)	
16(a)(iii)	

(b) On the grid below illustrate the two inequalities. You should shade the region that does **not** satisfy the inequalities. **[4]**

16(b)



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(c) (i) Find the combination of X and Y that will minimise the total intake of the two drugs in mg given the restrictions above. **[2]**

(ii) The vet prescribing the drugs decides that the dog should receive equal quantities of X and Y.

With this extra restriction, what combination of X and Y will minimise the total intake of the two drugs in mg? **[2]**

16(c)(i)	
16(c)(ii)	

17 In this question you must show detailed reasoning.

You are given that $\tan\theta = \dfrac{n+1}{n-1}$ where n is a number greater than 1.

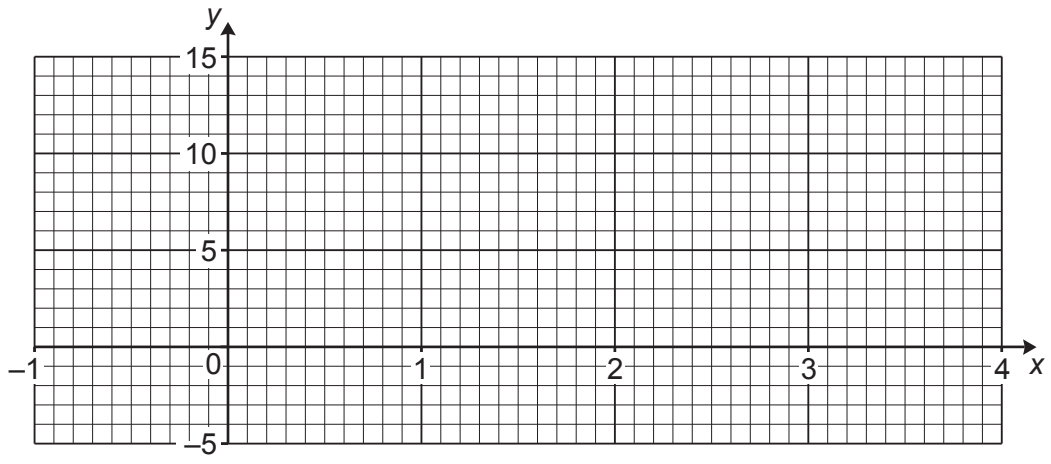
(a) Determine the ranges of possible values of θ, given that θ can only take values between 0° and 360°. **[4]**

(b) Given that θ is acute, find an expression for $\sin\theta$ in the form $\dfrac{an+b}{\sqrt{cn^2+d}}$ where a, b, c and d are integers. **[4]**

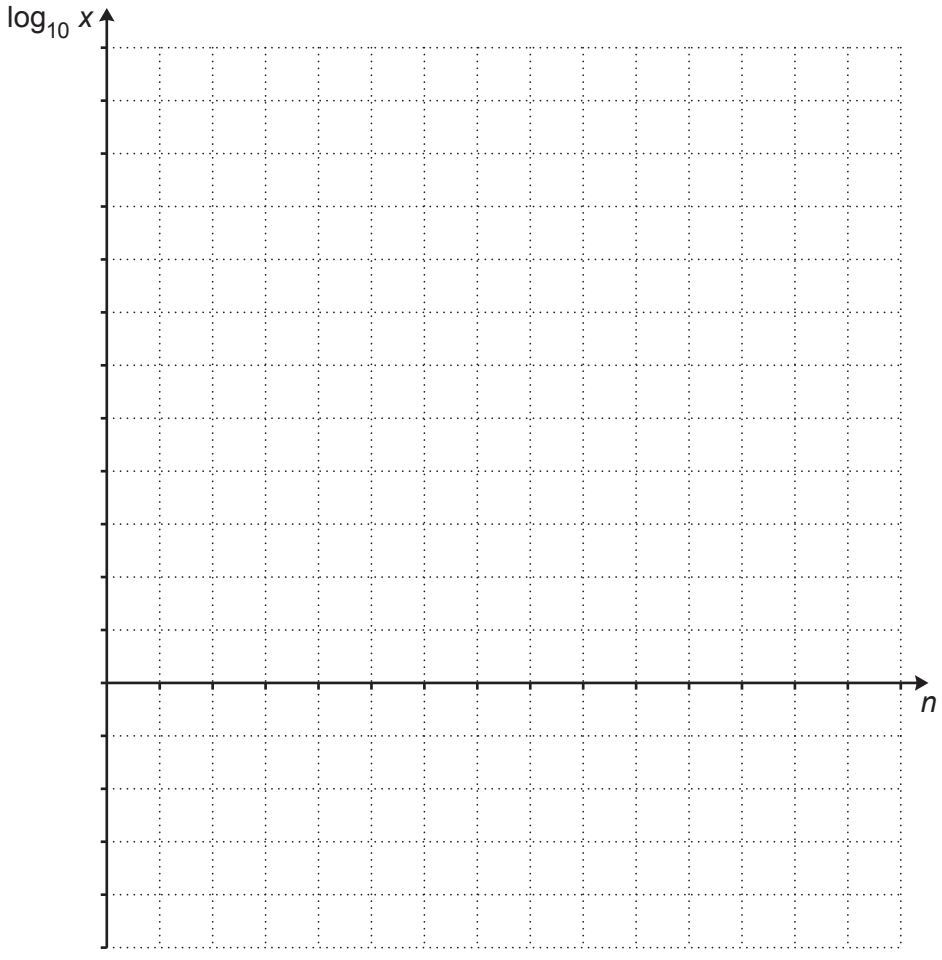
17(a)	
17(b)	

END OF QUESTION PAPER

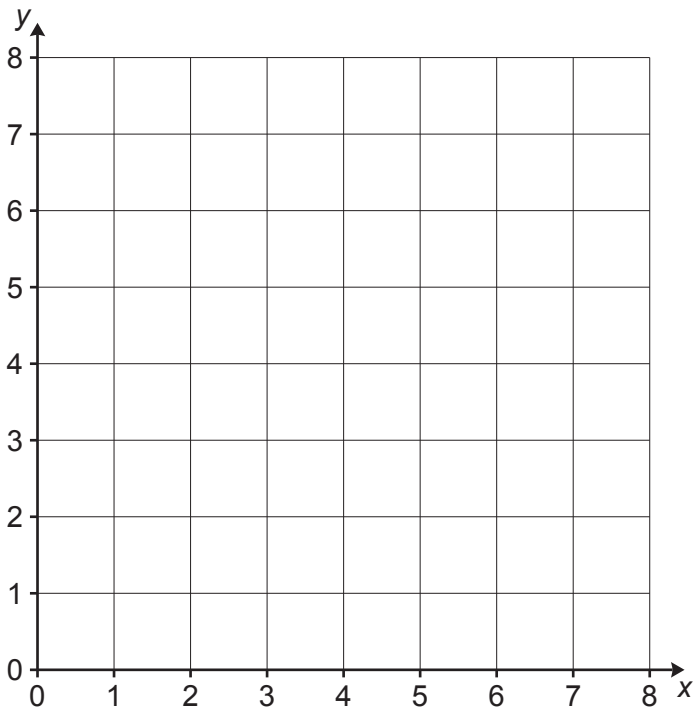
6(a) ***This is a spare copy of the grid for question 6(a). Only write on this page if you want to offer a second attempt at the graph.***
If you do so, then you must cross through the first attempt on page 5.



14(b), (c)(i) ***This is a spare copy of the grid for question 14(b) and 14(c)(i). Only write on this page if you want to offer a second attempt at the graph.***
If you do so, then you must cross through the first attempt on page 13.



16(b) ***This is a spare copy of the grid for question 16(b). Only write on this page if you want to offer a second attempt at the graph.***
If you do so, then you must cross through the first attempt on page 17.



EXTRA ANSWER SPACE

If you need extra space use these lined pages. You must write the question numbers clearly in the margin.

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