

Advanced General Certificate of Education  
Advanced Subsidiary General Certificate of Education

**MEI STRUCTURED MATHEMATICS**

**EXAMINATION FORMULAE AND TABLES**



# **MEI STRUCTURED MATHEMATICS**

## **EXAMINATION FORMULAE AND TABLES**

**Arithmetic series**

General ( $k$ th) term,  $u_k = a + (k - 1)d$   
 last ( $n$ th) term,  $l = u_n = a + (n - 1)d$   
 Sum to  $n$  terms,  $S_n = \frac{1}{2} n(a + l) = \frac{1}{2} n[2a + (n - 1)d]$

**Geometric series**

General ( $k$ th) term,  $u_k = a r^{k-1}$   
 Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}$   
 Sum to infinity  $S_\infty = \frac{a}{1 - r}, -1 < r < 1$

**Binomial expansions**

When  $n$  is a positive integer  
 $(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots b^n, n \in \mathbb{N}$

where

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!} \quad \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

General case

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1) \dots (n-r+1)}{1.2 \dots r} x^r + \dots, |x| < 1, n \in \mathbb{R}$$

**Logarithms and exponentials**

$$e^{x \ln a} = a^x \quad \log_a x = \frac{\log_b x}{\log_b a}$$

**Numerical solution of equations**

Newton-Raphson iterative formula for solving  $f(x) = 0, x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Complex Numbers**

$$\{r(\cos \theta + j \sin \theta)\}^n = r^n(\cos n\theta + j \sin n\theta)$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

The roots of  $z^n = 1$  are given by  $z = \exp(\frac{2\pi k}{n} j)$  for  $k = 0, 1, 2, \dots, n-1$

**Finite series**

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1) \quad \sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

**Infinite series**

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^r f^{(r)}(a)}{r!} + \dots$$

$$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!} f''(a) + \dots + \frac{x^r}{r!} f^{(r)}(a) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots, \text{ all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots, -1 < x \leq 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots, \text{ all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots, \text{ all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots, -1 \leq x \leq 1$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots, \text{ all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots, \text{ all } x$$

$$\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{(2r+1)} + \dots, -1 < x < 1$$

**Hyperbolic functions**

$$\cosh^2 x - \sinh^2 x = 1, \quad \sinh 2x = 2 \sinh x \cosh x, \quad \cosh 2x = \cosh^2 x + \sinh^2 x$$

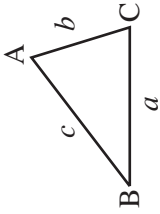
$$\operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1}), \quad \operatorname{arcosh} x = \ln(x + \sqrt{x^2 - 1}), x \geq 1$$

$$\operatorname{artanh} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right), |x| < 1$$

**Matrices**

Anticlockwise rotation through angle  $\theta$ , centre O:  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line  $y = x \tan \theta$ :  $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$



**Cosine rule**  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$  (etc.)  
 $a^2 = b^2 + c^2 - 2bc \cos A$  (etc.)

**Trigonometry**

$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$   
 $\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$   
 $\tan(\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}$ ,  $[(\theta \pm \phi) \neq (k + \frac{1}{2})\pi]$   
 For  $t = \tan \frac{1}{2} \theta$ :  $\sin \theta = \frac{2t}{1+t^2}$ ,  $\cos \theta = \frac{1-t^2}{1+t^2}$   
 $\sin \theta + \sin \phi = 2 \sin \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$   
 $\sin \theta - \sin \phi = 2 \cos \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi)$   
 $\cos \theta + \cos \phi = 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$   
 $\cos \theta - \cos \phi = -2 \sin \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi)$

**Vectors and 3-D coordinate geometry**

(The position vectors of points A, B, C are **a**, **b**, **c**.)  
 The position vector of the point dividing AB in the ratio  $\lambda:\mu$  is  $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{(\lambda + \mu)}$

Line: Cartesian equation of line through A in direction **u** is  $\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3} (= t)$

The resolved part of **a** in the direction **u** is  $\frac{\mathbf{a} \cdot \mathbf{u}}{|\mathbf{u}|}$

Plane: Cartesian equation of plane through A with normal **n** is  $n_1x + n_2y + n_3z + d = 0$  where  $d = -\mathbf{a} \cdot \mathbf{n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + s(\mathbf{b} - \mathbf{a}) + t(\mathbf{c} - \mathbf{a}) = (1 - s - t)\mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The plane through A parallel to **u** and **v** has equation

$\mathbf{r} = \mathbf{a} + s\mathbf{u} + t\mathbf{v}$

**Perpendicular distance of a point from a line and a plane**

Line:  $(x_1, y_1)$  from  $ax + by + c = 0$ :  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Plane:  $(\alpha, \beta, \gamma)$  from  $n_1x + n_2y + n_3z + d = 0$ :  $\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{(n_1^2 + n_2^2 + n_3^2)}}$

**Vector product**

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

**Conics**

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric form	$(a \cos \theta, b \sin \theta)$	$(at^2, 2at)$	$(a \sec \theta, b \tan \theta)$	$(ct, \frac{c}{t})$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm c\sqrt{2}, \pm c\sqrt{2})$
Directrices	$x = \pm \frac{a}{e}$	$x = -a$	$x = \pm \frac{a}{e}$	$x + y = \pm c\sqrt{2}$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

Any of these conics can be expressed in polar coordinates (with the focus as the origin) as:  $\frac{l}{r} = 1 + e \cos \theta$  where  $l$  is the length of the semi-latus rectum.

**Mensuration**

Sphere : Surface area =  $4\pi r^2$   
 Cone : Curved surface area =  $\pi r \times$  slant height

<p><b>Differentiation</b> <math>f(x)</math></p> <p> <math>\tan kx</math>  <math>\sec x \tan x</math>  <math>\cot x</math>  <math>\operatorname{cosec} x</math>  <math>\arcsin x</math>  <math>\arccos x</math>  <math>\arctan x</math>  <math>\sinh x</math>  <math>\cosh x</math>  <math>\tanh x</math>  <math>\operatorname{arsinh} x</math>  <math>\operatorname{arcosh} x</math>  <math>\operatorname{artanh} x</math> </p>	<p><math>f(x)</math></p> <p> <math>k \sec^2 kx</math>  <math>\sec x \tan x</math>  <math>-\operatorname{cosec}^2 x</math>  <math>-\operatorname{cosec} x \cot x</math>  <math>\frac{1}{\sqrt{1-x^2}}</math>  <math>\frac{-1}{\sqrt{1-x^2}}</math>  <math>\frac{1}{1+x^2}</math>  <math>\cosh x</math>  <math>\sinh x</math>  <math>\operatorname{sech}^2 x</math>  <math>\frac{1}{\sqrt{1+x^2}}</math>  <math>\frac{1}{\sqrt{x^2-1}}</math>  <math>\frac{1}{(1-x^2)}</math> </p>
<p><b>Integration</b> <math>f(x)</math></p> <p> <math>\sec^2 kx</math>  <math>\tan x</math>  <math>\cot x</math>  <math>\operatorname{cosec} x</math>  <math>\sec x</math>  <math>\frac{1}{x^2-a^2}</math>  <math>\frac{1}{\sqrt{a^2-x^2}}</math>  <math>\frac{1}{a^2+x^2}</math>  <math>\frac{1}{a^2-x^2}</math>  <math>\sinh x</math>  <math>\cosh x</math>  <math>\tanh x</math>  <math>\frac{1}{\sqrt{a^2+x^2}}</math>  <math>\frac{1}{\sqrt{x^2-a^2}}</math> </p>	<p><math>\int f(x) dx</math> (+ a constant)</p> <p> <math>(1/k) \tan kx</math>  <math>\ln  \sec x </math>  <math>\ln  \sin x </math>  <math>-\ln  \operatorname{cosec} x + \cot x  = \ln \left  \tan \frac{x}{2} \right </math>  <math>\ln  \sec x + \tan x  = \ln \left  \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right </math>  <math>\frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right </math>  <math>\arcsin \left( \frac{x}{a} \right),  x  &lt; a</math>  <math>\frac{1}{a} \arctan \left( \frac{x}{a} \right)</math>  <math>\frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  = \frac{1}{a} \operatorname{artanh} \left( \frac{x}{a} \right),  x  &lt; a</math>  <math>\cosh x</math>  <math>\sinh x</math>  <math>\ln \cosh x</math>  <math>\operatorname{arsinh} \left( \frac{x}{a} \right)</math> or <math>\ln (x + \sqrt{x^2 + a^2})</math>,  <math>\operatorname{arcosh} \left( \frac{x}{a} \right)</math> or <math>\ln (x + \sqrt{x^2 - a^2}), x &gt; a, a &gt; 0</math> </p>
<p><b>Quotient rule</b> <math>y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}</math></p> <p><b>Trapezium rule</b> <math>\int_a^b y dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}</math>, where <math>h = \frac{b-a}{n}</math></p> <p><b>Integration by parts</b> <math>\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx</math></p> <p><b>Area of a sector</b> <math>A = \frac{1}{2} \int r^2 d\theta</math> (polar coordinates)</p> <p><math>A = \frac{1}{2} \int (xy - yx) dt</math> (parametric form)</p> <p><b>Arc length</b> <math>s = \int \sqrt{x^2 + y^2} dt</math> (parametric form)</p> <p><math>s = \int \sqrt{1 + \left[ \frac{dy}{dx} \right]^2} dx</math> (cartesian coordinates)</p> <p><math>s = \int \sqrt{r^2 + \left[ \frac{dr}{d\theta} \right]^2} d\theta</math> (polar coordinates)</p>	<p><b>Surface area of revolution</b> <math>S_x = 2\pi \int y ds = 2\pi \int y \sqrt{x^2 + y^2} dt</math></p> <p><math>S_y = 2\pi \int x ds = 2\pi \int x \sqrt{x^2 + y^2} dt</math></p> <p><b>Curvature</b> <math>\kappa = \frac{d\psi}{ds} = \frac{\dot{x}\dot{y} - \dot{y}\dot{x}}{(x^2 + y^2)^{3/2}} = \frac{\frac{d^2y}{dx^2}}{\left(1 + \left[\frac{dy}{dx}\right]^2\right)^{3/2}}</math></p> <p>Radius of curvature <math>\rho = \frac{1}{\kappa}</math>, Centre of curvature <math>\mathbf{c} = \mathbf{r} + \rho \hat{\mathbf{n}}</math></p> <p><b>L'Hôpital's rule</b>              If <math>f(a) = g(a) = 0</math> and <math>g'(a) \neq 0</math> then <math>\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}</math></p> <p><b>Multi-variable calculus</b>  <math>\operatorname{grad} g = \begin{pmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{pmatrix}</math> For <math>w = g(x, y, z), \delta w = \frac{\partial w}{\partial x} \delta x + \frac{\partial w}{\partial y} \delta y + \frac{\partial w}{\partial z} \delta z</math></p>

**Centre of mass (uniform bodies)**

- Triangular lamina:  $\frac{2}{3}$  along median from vertex
- Solid hemisphere of radius  $r$ :  $\frac{3}{8} r$  from centre
- Hemispherical shell of radius  $r$ :  $\frac{1}{2} r$  from centre
- Solid cone or pyramid of height  $h$ :  $\frac{1}{4} h$  above the base on the line from centre of base to vertex
- Sector of circle, radius  $r$ , angle  $2\theta$ :  $\frac{2r \sin \theta}{3\theta}$  from centre
- Arc of circle, radius  $r$ , angle  $2\theta$  at centre:  $\frac{r \sin \theta}{\theta}$  from centre
- Conical shell, height  $h$ :  $\frac{1}{3} h$  above the base on the line from the centre of base to the vertex

**Motion in polar coordinates**

- Motion in a circle
- Transverse velocity:  $v = r\dot{\theta}$
- Radial acceleration:  $-\dot{r}\dot{\theta}^2 = -\frac{v^2}{r}$
- Transverse acceleration:  $\dot{v} = r\ddot{\theta}$
- General motion
- Radial velocity:  $\dot{r}$
- Transverse velocity:  $r\dot{\theta}$
- Radial acceleration:  $\ddot{r} - r\dot{\theta}^2$
- Transverse acceleration:  $r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r} \frac{d}{dt} (r^2\dot{\theta})$

**Moments as vectors**

The moment about O of  $\mathbf{F}$  acting at  $\mathbf{r}$  is  $\mathbf{r} \times \mathbf{F}$

**Moments of inertia (uniform bodies, mass M)**

- Thin rod, length  $2l$ , about perpendicular axis through centre:  $\frac{1}{3} Ml^2$
- Rectangular lamina about axis in plane bisecting edges of length  $2l$ :  $\frac{1}{3} Ml^2$
- Thin rod, length  $2l$ , about perpendicular axis through end:  $\frac{4}{3} Ml^2$
- Rectangular lamina about edge perpendicular to edges of length  $2l$ :  $\frac{4}{3} Ml^2$
- Rectangular lamina, sides  $2a$  and  $2b$ , about perpendicular axis through centre:  $\frac{1}{3} M(a^2 + b^2)$
- Hoop or cylindrical shell of radius  $r$  about perpendicular axis through centre:  $Mr^2$
- Hoop of radius  $r$  about a diameter:  $\frac{1}{2} Mr^2$
- Disc or solid cylinder of radius  $r$  about axis:  $\frac{1}{2} Mr^2$
- Disc of radius  $r$  about a diameter:  $\frac{1}{4} Mr^2$
- Solid sphere of radius  $r$  about a diameter:  $\frac{2}{5} Mr^2$
- Spherical shell of radius  $r$  about a diameter:  $\frac{2}{3} Mr^2$
- Parallel axes theorem:  $I_A = I_G + M(AG)^2$
- Perpendicular axes theorem:  $I_z = I_x + I_y$  (for a lamina in the  $(x, y)$  plane)

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A)}$$

$$\text{Bayes' Theorem: } P(A_j|B) = \frac{P(A_j)P(B|A_j)}{\sum P(A_i)P(B|A_i)}$$

**Populations**

**Discrete distributions**

$X$  is a random variable taking values  $x_i$  in a discrete distribution with

$$P(X = x_i) = P_i$$

Expectation:  $\mu = E(X) = \sum x_i P_i$

Variance:  $\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 P_i = \sum x_i^2 P_i - \mu^2$

For a function  $g(X)$ :  $E[g(X)] = \sum g(x_i) P_i$

**Continuous distributions**

$X$  is a continuous variable with probability density function (p.d.f.)  $f(x)$

Expectation:  $\mu = E(X) = \int x f(x) dx$

Variance:  $\sigma^2 = \text{Var}(X) = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$

For a function  $g(X)$ :  $E[g(X)] = \int g(x) f(x) dx$

Cumulative

distribution function  $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

**Correlation and regression** For a sample of  $n$  pairs of observations  $(x_i, y_i)$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n},$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

**Covariance**  $\frac{S_{xy}}{n} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i - \bar{x} \bar{y}}{n}$

**Product-moment correlation: Pearson's coefficient**

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left[ \sum (x_i - \bar{x})^2 \right] \left[ \sum (y_i - \bar{y})^2 \right]}} = \frac{\frac{\sum x_i y_i - \bar{x} \bar{y}}{n}}{\sqrt{\left[ \left( \frac{\sum x_i^2}{n} - \bar{x}^2 \right) \left( \frac{\sum y_i^2}{n} - \bar{y}^2 \right) \right]}}$$

**Rank correlation: Spearman's coefficient**

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

**Regression**

Least squares regression line of  $y$  on  $x$ :  $y - \bar{y} = b(x - \bar{x})$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\frac{\sum x_i y_i - \bar{x} \bar{y}}{n}}{\frac{\sum x_i^2 - \bar{x}^2}{n}}$$

**Estimates**

**Unbiased estimates from a single sample**

$\bar{X}$  for population mean  $\mu$ ;  $\text{Var } \bar{X} = \frac{\sigma^2}{n}$

$S^2$  for population variance  $\sigma^2$  where  $S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 f_i$

**Probability generating functions**

For a discrete distribution

$$G(t) = E(t^X)$$

$$E(X) = G'(1); \text{Var}(X) = G''(1) + \mu - \mu^2$$

$G_{X+Y}(t) = G_X(t) G_Y(t)$  for independent  $X, Y$

**Moment generating functions:**

$$M_X(\theta) = E(e^{\theta X})$$

$$E(X) = M'(0) = \mu; \quad E(X^n) = M^{(n)}(0)$$

$$\text{Var}(X) = M''(0) - \{M'(0)\}^2$$

$$M_{X+Y}(\theta) = M_X(\theta) M_Y(\theta) \text{ for independent } X, Y$$



**Markov Chains**

$$\mathbf{P}_{n+1} = \mathbf{P}_n \mathbf{P}$$

Long run proportion  $\mathbf{p} = \mathbf{pP}$

**Bivariate distributions**

Covariance  $\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X \mu_Y$

Product-moment correlation coefficient  $\rho = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

Sum and difference

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$$

If X, Y are independent:  $\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$

$$E(XY) = E(X) E(Y)$$

Coding

$$\left. \begin{aligned} X &= aX' + b \\ Y &= cY' + d \end{aligned} \right\} \Rightarrow \text{Cov}(X, Y) = ac \text{Cov}(X', Y')$$

**Analysis of variance**

One-factor model:  $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ , where  $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$SS_B = \sum_i n_i (\bar{x}_i - \bar{x})^2 = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$$

$$SS_T = \sum_{i,j} (x_{ij} - \bar{x})^2 = \sum_{i,j} x_{ij}^2 - \frac{T^2}{n}$$

**Regression**

$Y_i$	RSS	No. of parameters, $p$
$\alpha + \beta x_i + \varepsilon_i$	$\sum (y_i - a - bx_i)^2$	2
$\alpha + \beta f(x_i) + \varepsilon_i$	$\sum (y_i - a - bf(x_i))^2$	2
$\alpha + \beta x_i + \gamma z_i + \varepsilon_i$	$\sum (y_i - a - bx_i - cz_i)^2$	3

$\varepsilon_i \sim N(0, \sigma^2)$   $a, b, c$  are estimates for  $\alpha, \beta, \gamma$ .  $\hat{\sigma}^2 = \frac{\text{RSS}}{n-p}$

For the model  $Y_i = \alpha + \beta x_i + \varepsilon_i$ ,

$$b = \frac{S_{xy}}{S_{xx}}, b \sim N\left(\beta, \frac{\sigma^2}{S_{xx}}\right), \frac{b - \beta}{\sqrt{\hat{\sigma}^2 / S_{xx}}} \sim t_{n-2}$$

$$a = \bar{y} - b\bar{x}, a \sim N\left(\alpha, \frac{\sigma^2 \sum x_i^2}{n S_{xx}}\right)$$

$$a + bx_0 \sim N\left(\alpha + \beta x_0, \sigma^2 \left\{ \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right\}\right)$$

$$\text{RSS} = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = S_{yy}(1 - r^2)$$

**Randomised response technique**

$$E(\hat{p}) = \frac{\frac{y}{n} - (1 - \theta)}{(2\theta - 1)} \quad \text{Var}(\hat{p}) = \frac{[(2\theta - 1)p + (1 - \theta)][\theta - (2\theta - 1)p]}{n(2\theta - 1)^2}$$

**Factorial design**

Interaction between 1st and 2nd of 3 treatments

$$(-) \left\{ \frac{(Abc - abc) + (AbC - abc)}{2} - \frac{(Abc - aBC) + (ABC - aBC)}{2} \right\}$$

**Exponential smoothing**

$$\hat{y}_{n+1} = \alpha y_n + \alpha(1 - \alpha)y_{n-1} + \alpha(1 - \alpha)^2 y_{n-2} + \dots + \alpha(1 - \alpha)^{n-1} y_1 + (1 - \alpha)^n y_0$$

$$\hat{y}_{n+1} = \hat{y}_n + \alpha(y_n - \hat{y}_n)$$

$$\hat{y}_{n+1} = \alpha y_n + (1 - \alpha) \hat{y}_n$$

**STATISTICS: HYPOTHESIS TESTS**

Description	Test statistic	Distribution
t-test for the difference in the means of 2 samples	$\frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ <p>where <math>s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}</math></p>	$t_{n_1 + n_2 - 2}$
Wilcoxon single sample test	A statistic $T$ is calculated from the ranked data.	See tables
Wilcoxon Rank-sum (or Mann-Whitney) 2-Sample test	<p>Samples size <math>m, n; m \leq n</math></p> <p>Wilcoxon  <math>W</math> = sum of ranks of sample size <math>m</math>                      Mann-Whitney  <math>T = W - \frac{1}{2} m(m + 1)</math></p>	See tables
Normal test on binomial proportion	$\frac{p - \theta}{\sqrt{\left(\frac{\theta(1 - \theta)}{n}\right)}}$	$N(0, 1)$
$\chi^2$ test for variance	$\frac{(n - 1)s^2}{\sigma^2}$	$\chi^2_{n - 1}$
F-test on ratio of two variances	$\frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2}, \quad s_1^2 > s_2^2$	$F_{n_1 - 1, n_2 - 1}$

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \frac{\sum x_i y_i - \bar{x} \bar{y}}{\sqrt{\left[ \left( \frac{\sum x_i^2}{n} - \bar{x}^2 \right) \left( \frac{\sum y_i^2}{n} - \bar{y}^2 \right) \right]}}$	
Spearman rank correlation test	$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$	
Normal test for a mean	$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	$N(0, 1)$
t-test for a mean	$\frac{\bar{x} - \mu}{s / \sqrt{n}}$	$t_{n - 1}$
$\chi^2$ test	$\sum \frac{(f_o - f_e)^2}{f_e}$	$\chi^2_v$
t-test for paired sample	$\frac{(\bar{x}_1 - \bar{x}_2) - \mu}{s / \sqrt{n}}$	$t$ with $(n - 1)$ degrees of freedom
Normal test for the difference in the means of 2 samples with different variances	$\frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$N(0, 1)$

**STATISTICS: DISTRIBUTIONS**

<b>Name</b>	<b>Function</b>	<b>Mean</b>	<b>Variance</b>	<b>p.g.f. G(t) (discrete) m.g.f. M(θ) (continuous)</b>
Binomial B(n, p) <i>Discrete</i>	$P(X = r) = {}^n C_r q^{n-r} p^r$ , for $r = 0, 1, \dots, n$ , $0 < p < 1$ , $q = 1 - p$	$np$	$npq$	$G(t) = (q + pt)^n$
Poisson (λ) <i>Discrete</i>	$P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$ , for $r = 0, 1, \dots$ , $\lambda > 0$	$\lambda$	$\lambda$	$G(t) = e^{\lambda(t-1)}$
Normal N(μ, σ <sup>2</sup> ) <i>Continuous</i>	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ , $-\infty < x < \infty$	$\mu$	$\sigma^2$	$M(\theta) = \exp(\mu\theta + \frac{1}{2}\sigma^2\theta^2)$
Uniform (Rectangular) on [a, b] <i>Continuous</i>	$f(x) = \frac{1}{b-a}$ , $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$	$M(\theta) = \frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta}$
Exponential <i>Continuous</i>	$f(x) = \lambda e^{-\lambda x}$ , $x \geq 0$ , $\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$M(\theta) = \frac{\lambda}{\lambda - \theta}$
Geometric <i>Discrete</i>	$P(X = r) = q^{r-1} p$ , $r = 1, 2, \dots$ , $0 < p < 1$ , $q = 1 - p$	$\frac{1}{p}$	$\frac{q}{p^2}$	$G(t) = \frac{pt}{1 - qt}$
Negative binomial <i>Discrete</i>	$P(X = r) = {}^{r-1} C_{n-1} q^{r-n} p^n$ , $r = n, n+1, \dots$ , $0 < p < 1$ , $q = 1 - p$	$\frac{n}{p}$	$\frac{nq}{p^2}$	$G(t) = \left(\frac{pt}{1 - qt}\right)^n$

**Numerical Solution of Equations**

The Newton-Raphson iteration for solving  $f(x) = 0 : x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Numerical integration**

The trapezium rule

$$\int_a^b y dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}, \text{ where } h = \frac{b-a}{n}$$

The mid-ordinate rule

$$\int_a^b y dx \approx h(y_{\frac{1}{2}} + y_{1\frac{1}{2}} + \dots + y_{n-1\frac{1}{2}} + y_{n-\frac{1}{2}}), \text{ where } h = \frac{b-a}{n}$$

Simpson's rule

for  $n$  even

$$\int_a^b y dx \approx \frac{1}{3} h \{ (y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) \},$$

where  $h = \frac{b-a}{n}$

The Gaussian 2-point integration rule

$$\int_{-h}^h f(x) dx \approx h \left[ f\left(\frac{-h}{\sqrt{3}}\right) + f\left(\frac{h}{\sqrt{3}}\right) \right]$$

**Interpolation/finite differences**

Lagrange's polynomial :  $P_n(x) = \sum L_r(x)f(x_r)$  where  $L_r(x) = \prod_{\substack{i=0 \\ i \neq r}}^n \frac{x - x_i}{x_r - x_i}$

Newton's forward difference interpolation formula

$$f(x) = f(x_0) + \frac{(x - x_0)}{h} \Delta f(x_0) + \frac{(x - x_0)(x - x_1)}{2!h^2} \Delta^2 f(x_0) + \dots$$

Newton's divided difference interpolation formula

$$f(x) = f[x_0] + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + \dots$$

**Numerical differentiation**

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

**Taylor polynomials**

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \text{error}$$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a + \xi), \quad 0 < \xi < h$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \text{error}$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(\eta), \quad a < \eta < x$$

**Numerical solution of differential equations**

For  $\frac{dy}{dx} = f(x, y)$ :

Euler's method :  $y_{r+1} = y_r + hf(x_r, y_r)$ ;  $x_{r+1} = x_r + h$

Runge-Kutta method (order 2) (modified Euler method)

$$y_{r+1} = y_r + \frac{1}{2} (k_1 + k_2)$$

where  $k_1 = hf(x_r, y_r)$ ,  $k_2 = hf(x_r + h, y_r + k_1)$

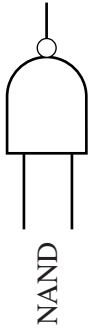
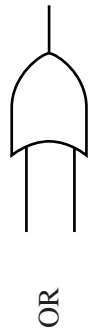
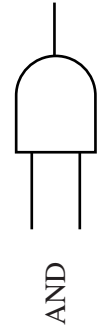
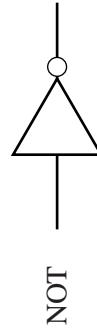
Runge-Kutta method, order 4:

$$y_{r+1} = y_r + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4),$$

where  $k_1 = hf(x_r, y_r)$        $k_2 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1)$

$$k_3 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2) \quad k_4 = hf(x_r + h, y_r + k_3).$$

**Logic gates**



# Statistical Tables

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CUMULATIVE BINOMIAL PROBABILITY

The Binomial distribution: cumulative probabilities

$$P(X \leq x) = \sum_{r=0}^x {}^n C_r (1-p)^{n-r} p^r$$

$n \setminus \begin{matrix} p \\ x \end{matrix}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950	
1	0.9500	0.9000	0.8500	0.8333	0.8000	0.7500	0.7000	0.6667	0.6500	0.6000	0.5500	0.5000	0.4500	0.4000	0.3500	0.3333	0.3000	0.2500	0.2000	0.1667	0.1500	0.1000	0.0500	0.0000
2	0.9025	0.8100	0.7225	0.6944	0.6400	0.5625	0.4900	0.4444	0.4225	0.3600	0.3025	0.2500	0.2025	0.1600	0.1225	0.1111	0.0900	0.0625	0.0400	0.0278	0.0225	0.0100	0.0025	0.0000
3	0.8574	0.7290	0.6141	0.5787	0.5120	0.4219	0.3430	0.2963	0.2746	0.2160	0.1664	0.1250	0.0911	0.0640	0.0429	0.0370	0.0270	0.0156	0.0080	0.0046	0.0034	0.0010	0.0001	0.0000
4	0.8145	0.6561	0.5220	0.4823	0.4096	0.3164	0.2401	0.1975	0.1785	0.1296	0.0915	0.0625	0.0410	0.0256	0.0150	0.0123	0.0081	0.0039	0.0016	0.0008	0.0005	0.0001	0.0000	0.0000
5	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000
6	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

CUMULATIVE BINOMIAL PROBABILITY

$n$	$\frac{p}{x}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950		
8	0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000	0.0000	0.0000
	4	1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2587	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004	0.0000	0.0000
	5	1.0000	0.9998	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058	0.0000	0.0000
	6	1.0000	0.9999	0.9998	0.9997	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572	0.0000	0.0000
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366	0.0000
9	0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	0.0000	0.0000	0.0000
	4	1.0000	0.9991	0.9944	0.9911	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000	0.0000	0.0000
	5	1.0000	0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006	0.0000	0.0000
	6	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084	0.0000	0.0000
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9991	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712	0.0000	0.0000
10	0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
	4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000	0.0000	0.0000
	5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001	0.0000	0.0000
	6	1.0000	1.0000	1.0000	1.0000	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010	0.0000	0.0000
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115	0.0000
11	0	0.5688	0.3138	0.1673	0.1346	0.0859	0.0422	0.0198	0.0116	0.0088	0.0036	0.0014	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.8981	0.6974	0.4922	0.4307	0.3221	0.1971	0.1130	0.0751	0.0606	0.0302	0.0139	0.0059	0.0022	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	2	0.9848	0.9104	0.7788	0.7268	0.6174	0.4552	0.3127	0.2341	0.2001	0.1189	0.0652	0.0327	0.0148	0.0059	0.0020	0.0014	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9984	0.9815	0.9306	0.9044	0.8389	0.7133	0.5696	0.4726	0.4256	0.2963	0.1911	0.1133	0.0610	0.0293	0.0122	0.0088	0.0043	0.0012	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9999	0.9972	0.9841	0.9755	0.9496	0.8854	0.7897	0.7110	0.6683	0.5328	0.3971	0.2744	0.1738	0.0994	0.0501	0.0386	0.0216	0.0076	0.0020	0.0006	0.0003	0.0000	0.0000	0.0000	0.0000
	5	1.0000	0.9997	0.9973	0.9954	0.9883	0.9657	0.9218	0.8779	0.8513	0.7535	0.6331	0.5000	0.3669	0.2465	0.1487	0.1221	0.0782	0.0343	0.0117	0.0046	0.0027	0.0003	0.0000	0.0000	0.0000
	6	1.0000	1.0000	1.0000	1.0000	0.9980	0.9924	0.9784	0.9614	0.9499	0.9006	0.8262	0.7256	0.6029	0.4672	0.3317	0.2890	0.2103	0.1146	0.0504	0.0245	0.0159	0.0028	0.0001	0.0000	0.0000
	7	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9912	0.9878	0.9707	0.9390	0.8867	0.8089	0.7037	0.5744	0.5274	0.4304	0.2867	0.1611	0.0956	0.0694	0.0185	0.0016	0.0000	0.0000
11	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9986	0.9980	0.9941	0.9852	0.9673	0.9348	0.8811	0.7999	0.7659	0.6873	0.5448	0.3826	0.2732	0.2212	0.0896	0.0152	0.0000	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9993	0.9978	0.9941	0.9861	0.9698	0.9394	0.9249	0.8870	0.8029	0.6779	0.5693	0.5078	0.3026	0.1019	0.0000	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9986	0.9964	0.9912	0.9884	0.9802	0.9578	0.9141	0.8654	0.8327	0.6862	0.4312	0.0000	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

CUMULATIVE BINOMIAL PROBABILITY

$n$	$\frac{p}{x}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950		
12	0	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9998	0.9957	0.9761	0.9637	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000	0.0000	
	7	1.0000	0.9999	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002	0.0000	
	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022	0.0000	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196	0.0000	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9999	0.9989	0.9968	0.9912	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184	0.0000	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596	
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
13	0	0.5133	0.2542	0.1209	0.0935	0.0550	0.0238	0.0097	0.0051	0.0037	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.8646	0.6213	0.3983	0.3365	0.2336	0.1267	0.0637	0.0385	0.0296	0.0126	0.0049	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	2	0.9755	0.8661	0.6920	0.6281	0.5017	0.3326	0.2025	0.1387	0.1132	0.0579	0.0269	0.0112	0.0041	0.0013	0.0003	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	3	0.9969	0.9658	0.8820	0.8419	0.7473	0.5843	0.4206	0.3224	0.2783	0.1686	0.0929	0.0461	0.0203	0.0078	0.0025	0.0016	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	4	0.9997	0.9935	0.9658	0.9488	0.9009	0.7940	0.6543	0.5520	0.5005	0.3530	0.2279	0.1334	0.0698	0.0321	0.0126	0.0088	0.0040	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	
	5	1.0000	0.9991	0.9925	0.9873	0.9700	0.9198	0.8346	0.7587	0.7159	0.5744	0.4268	0.2905	0.1788	0.0977	0.0462	0.0347	0.0182	0.0056	0.0012	0.0003	0.0000	0.0000	0.0000	0.0000	
	6	1.0000	0.9999	0.9987	0.9976	0.9930	0.9757	0.9376	0.8965	0.8705	0.7712	0.6437	0.5000	0.3563	0.2288	0.1295	0.1035	0.0624	0.0243	0.0070	0.0024	0.0013	0.0001	0.0000	0.0000	
	7	1.0000	0.9998	0.9997	0.9997	0.9988	0.9944	0.9818	0.9653	0.9538	0.9023	0.8212	0.7095	0.5732	0.4256	0.2841	0.2413	0.1654	0.0802	0.0300	0.0127	0.0075	0.0009	0.0000	0.0000	
	8	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9912	0.9874	0.9679	0.9302	0.8666	0.7721	0.6470	0.4995	0.4480	0.3457	0.2060	0.0991	0.0512	0.0342	0.0065	0.0003	0.0000	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9984	0.9975	0.9922	0.9797	0.9539	0.9071	0.8314	0.7217	0.6776	0.5794	0.4157	0.2527	0.1581	0.1180	0.0342	0.0031	0.0000	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9987	0.9959	0.9888	0.9731	0.9421	0.8868	0.8613	0.7975	0.6674	0.4983	0.3719	0.3080	0.1339	0.0245	0.0000	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9983	0.9951	0.9874	0.9704	0.9615	0.9363	0.8733	0.86635	0.8017	0.6635	0.4983	0.3719	0.3080	0.1339	0.0245	0.0000	
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9996	0.9987	0.9963	0.9949	0.9903	0.9762	0.9450	0.9065	0.8791	0.7458	0.4867		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	0	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	
	6	1.0000	0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	0.0000	0.0000	
	7	1.0000	0.9997	0.9993	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	0.0000	0.0000	
	8	1.0000	0.9999	0.9999	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000	0.0000	
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004	0.0000	
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042	0.0000	
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301	
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9991	0.9971	0.9919														



CUMULATIVE BINOMIAL PROBABILITY

$n$	$\frac{p}{x}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950			
15	0	0.4633	0.2059	0.0874	0.0649	0.0352	0.0134	0.0047	0.0023	0.0016	0.0005	0.0001	0.0000	0.0000													
	1	0.8290	0.5490	0.3186	0.2596	0.1671	0.0802	0.0353	0.0194	0.0142	0.0052	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000									
	2	0.9638	0.8159	0.6042	0.5322	0.3980	0.2361	0.1268	0.0794	0.0617	0.0271	0.0107	0.0037	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000								
	3	0.9945	0.9444	0.8227	0.7685	0.6482	0.4613	0.2969	0.2092	0.1727	0.0905	0.0424	0.0176	0.0063	0.0019	0.0005	0.0003	0.0003	0.0001	0.0000							
	4	0.9994	0.9873	0.9383	0.9102	0.8358	0.6865	0.5155	0.4041	0.3519	0.2173	0.1204	0.0592	0.0255	0.0093	0.0028	0.0018	0.0018	0.0007	0.0001	0.0000						
	5	0.9999	0.9978	0.9832	0.9726	0.9389	0.8516	0.7216	0.6184	0.5643	0.4032	0.2608	0.1509	0.0769	0.0338	0.0124	0.0085	0.0037	0.0008	0.0001	0.0000						
	6	1.0000	0.9997	0.9964	0.9934	0.9819	0.9434	0.8689	0.7970	0.7548	0.6098	0.4522	0.3036	0.1818	0.0950	0.0422	0.0308	0.0152	0.0042	0.0008	0.0002						
	7	1.0000	0.9994	0.9987	0.9958	0.9827	0.9500	0.9118	0.8868	0.8868	0.7869	0.6535	0.5000	0.3465	0.2131	0.1132	0.0682	0.0500	0.0173	0.0042	0.0013						
	8	0.9999	0.9998	0.9998	0.9992	0.9958	0.9848	0.9692	0.9578	0.9578	0.9050	0.8182	0.6964	0.5478	0.3902	0.2452	0.2030	0.1311	0.0566	0.0181	0.0066						
	9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9915	0.9876	0.9876	0.9662	0.9231	0.8491	0.7392	0.5968	0.4357	0.3816	0.2784	0.1484	0.0611	0.0274						
	10	1.0000	0.9999	0.9993	0.9985	0.9972	0.9907	0.9745	0.9408	0.9372	0.9907	0.9745	0.9408	0.8796	0.7827	0.6481	0.5959	0.4845	0.3135	0.1642	0.0898						
	11	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315						
	12	1.0000	0.9999	0.9997	0.9989	0.9963	0.9933	0.9206	0.8732	0.9893	0.9929	0.9383	0.9206	0.9983	0.9948	0.9858	0.9806	0.9647	0.9198	0.8329	0.7404						
	13	1.0000	0.9999	0.9999	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000						
	14	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315						
15	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315							
16	0	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000													
	1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000									
	2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000								
	3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000	0.0000								
	4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000								
	5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000							
	6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000						
	7	0.9999	0.9998	0.9979	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004						
	8	1.0000	0.9998	0.9996	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021						
	9	1.0000	1.0000	1.0000	0.9999	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101						
	10	1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378						
	11	1.0000	0.9997	0.9992	0.9992	0.9987	0.9951	0.9851	0.9616	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134						
	12	1.0000	0.9999	0.9999	0.9999	0.9998	0.9991	0.9965	0.9894	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709						
	13	1.0000	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000						
	14	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315						
15	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315							
16	1.0000	0.9999	0.9999	0.9997	0.9995	0.9981	0.9937	0.9824	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7908	0.7031	0.5387	0.3518	0.2315							
17	0	0.4181	0.1668	0.0631	0.0451	0.0225	0.0075	0.0023	0.0010	0.0007	0.0002	0.0000	0.0000	0.0000													
	1	0.7922	0.4818	0.2525	0.1983	0.1182	0.0501	0.0193	0.0096	0.0067	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000									
	2	0.9497	0.7618	0.5198	0.4435	0.3096	0.1637	0.0774	0.0442	0.0327	0.0123	0.0041	0.0012	0.0003	0.0001	0.0000	0.0000	0.0000									
	3	0.9912	0.9174	0.7556	0.6887	0.5489	0.3530	0.2019	0.1304	0.1028	0.0464	0.0184	0.0064	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000								
	4	0.9988	0.9779	0.9013	0.8604	0.7582	0.5739	0.3887	0.2814	0.2348	0.1260	0.0596	0.0245	0.0086	0.0025	0.0006	0.0003	0.0001	0.0000								
	5	0.9999	0.9953	0.9681	0.9496	0.8943	0.7653	0.5968	0.4777	0.4197	0.2639	0.1471	0.0717	0.0301	0.0106	0.0030	0.0019	0.0007	0.0001	0.0000							
	6	1.0000	0.9992	0.9917	0.9853	0.9623	0.8929	0.7752	0.6739	0.6188	0.4478	0.2902	0.1662	0.0826	0.0348	0.0120	0.0080	0.0032	0.0006	0.0001	0.0000						
	7	0.9999	0.9983	0.9965	0.9965	0.9891	0.9598	0.8954	0.8281	0.7872	0.6405	0.4743	0.3145	0.1834	0.0919	0.0383	0.0273	0.0127	0.0031	0.0005	0.0001						
	8	1.0000	0.9997	0.9993	0.9974	0.9876	0.9597	0.9245	0.9006	0.8011	0.6626	0.5000	0.3374	0.1989	0.0994	0.0755	0.0403	0.0124	0.0026	0.0007	0.0003						
	9	1.0000	0.9999	0.9999	0.9995	0.9969	0.9873	0.9727	0.9617	0.9081	0.8166	0.6855	0.5257	0.3595	0.2128	0.1719	0.1046	0.0402	0.0109	0.0035	0.0017						
	10	1.0000	0.9999	0.9999	0.9999	0.9999	0.9994	0.9968	0.9920	0.9880	0.9652	0.9174	0.8338	0.7098	0.5522	0.3812	0.3261	0.2248	0.1071	0.0377	0.0147						
	11	1.0000	0.9999	0.9993																							

CUMULATIVE BINOMIAL PROBABILITY

$n$	$\frac{p}{x}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950		
18	0	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
	1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9419	0.7338	0.4794	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.9998	0.9973	0.9947	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	8	1.0000	0.9995	0.9989	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	9	0.9999	0.9998	0.9998	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000	0.0000	0.0000
	10	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000	0.0000	0.0000
	11	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	12	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	13	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	14	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	15	0.9998	0.9973	0.9947	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	16	1.0000	0.9995	0.9989	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	17	0.9999	0.9998	0.9998	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000	0.0000	0.0000
18	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000	0.0000	0.0000	
19	0.3774	0.1351	0.0456	0.0313	0.0144	0.0042	0.0011	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
0	0.7547	0.4203	0.1985	0.1502	0.0829	0.0310	0.0104	0.0047	0.0031	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
1	0.9335	0.7054	0.4413	0.3643	0.2369	0.1113	0.0462	0.0240	0.0170	0.0055	0.0015	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.9868	0.8850	0.6841	0.6070	0.4551	0.2631	0.1332	0.0787	0.0591	0.0230	0.0077	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
3	0.9980	0.9648	0.8556	0.8011	0.6733	0.4654	0.2822	0.1879	0.1500	0.0696	0.0280	0.0096	0.0028	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
4	0.9998	0.9914	0.9463	0.9176	0.8369	0.6678	0.4739	0.3519	0.2968	0.1629	0.0777	0.0318	0.0109	0.0031	0.0007	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
5	1.0000	0.9983	0.9837	0.9719	0.9324	0.8251	0.6655	0.5431	0.4812	0.3081	0.1727	0.0835	0.0342	0.0116	0.0031	0.0019	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
6	0.9997	0.9959	0.9921	0.9921	0.9767	0.9225	0.8180	0.7207	0.6656	0.4878	0.3169	0.1796	0.0871	0.0352	0.0114	0.0074	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
7	1.0000	0.9992	0.9982	0.9982	0.9933	0.9713	0.9161	0.8538	0.8145	0.6675	0.4940	0.3238	0.1841	0.0885	0.0347	0.0241	0.0105	0.0023	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
8	0.9999	0.9996	0.9996	0.9996	0.9984	0.9911	0.9674	0.9352	0.9125	0.8139	0.6710	0.5000	0.3290	0.1861	0.0875	0.0648	0.0326	0.0089	0.0016	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	
9	1.0000	0.9999	0.9999	0.9999	0.9997	0.9977	0.9895	0.9759	0.9653	0.9115	0.8159	0.6762	0.5060	0.3325	0.1855	0.1462	0.0839	0.0287	0.0067	0.0018	0.0008	0.0000	0.0000	0.0000	0.0000	
10	0.9997	0.9959	0.9921	0.9921	0.9767	0.9225	0.8180	0.7207	0.6656	0.4878	0.3169	0.1796	0.0871	0.0352	0.0114	0.0074	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
11	1.0000	0.9992	0.9982	0.9982	0.9933	0.9713	0.9161	0.8538	0.8145	0.6675	0.4940	0.3238	0.1841	0.0885	0.0347	0.0241	0.0105	0.0023	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
12	0.9999	0.9994	0.9994	0.9994	0.9999	0.9994	0.9981	0.9981	0.9969	0.9884	0.9658	0.9165	0.8273	0.6919	0.5188	0.4569	0.3345	0.1749	0.0676	0.0281	0.0163	0.0017	0.0000	0.0000	0.0000	
13	1.0000	0.9999	0.9999	0.9999	1.0000	0.9999	0.9996	0.9996	0.9993	0.9969	0.9891	0.9682	0.9223	0.8371	0.7032	0.6481	0.5261	0.3322	0.1631	0.0824	0.0537	0.0086	0.0002	0.0000	0.0000	
14	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9994	0.9972	0.9904	0.9920	0.9304	0.8500	0.7178	0.5346	0.3267	0.1989	0.1444	0.0352	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
15	0.9998	0.9983	0.9837	0.9719	0.9324	0.8251	0.6655	0.5431	0.4812	0.3081	0.1727	0.0835	0.0342	0.0116	0.0031	0.0019	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
16	0.9997	0.9959	0.9921	0.9921	0.9767	0.9225	0.8180	0.7207	0.6656	0.4878	0.3169	0.1796	0.0871	0.0352	0.0114	0.0074	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
17	1.0000	0.9992	0.9982	0.9982	0.9933	0.9713	0.9161	0.8538	0.8145	0.6675	0.4940	0.3238	0.1841	0.0885	0.0347	0.0241	0.0105	0.0023	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
18	0.9999	0.9996	0.9996	0.9996	0.9984																					

CUMULATIVE BINOMIAL PROBABILITY

$n$	$\frac{p}{x}$	0.050	0.100	0.150	1/6	0.200	0.250	0.300	1/3	0.350	0.400	0.450	0.500	0.550	0.600	0.650	2/3	0.700	0.750	0.800	5/6	0.850	0.900	0.950	
20	0	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000	0.0000												
	1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000												
	2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000											
	3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000										
	4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000								
	5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000							
	6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000						
	7		0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000					
	8		0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000				
	9		1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000			
	10			1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000		
	11				1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001		
	12					1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004		
	13						1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000	
	14							1.0000	0.9998	0.9998	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003	
	15								1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026	
	16									1.0000	0.9997	0.9987	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159	
	17										1.0000	0.9998	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755	
	18											1.0000	1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642	
	19												1.0000	1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415	
	20													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITY

The Poisson distribution: cumulative probabilities

$$P(X \leq x) = \sum_{r=0}^x e^{-\lambda} \frac{\lambda^r}{r!}$$

$\lambda \backslash x$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

$\lambda \backslash x$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4	.....	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5	.....	.....	.....	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000
7	.....	.....	.....	.....	.....	.....	.....	.....	.....

$\lambda \backslash x$	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
0	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

$\lambda \backslash x$	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
0	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9793	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	.....	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	0.9999
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

$\lambda \backslash x$	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
0	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

$\lambda \backslash x$	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
0	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

CUMULATIVE POISSON PROBABILITY

$x \backslash \lambda$	7.00	7.10	7.20	7.30	7.40	7.50	7.60	7.70	7.80	7.90
0	0.0009	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004
1	0.0073	0.0067	0.0061	0.0056	0.0051	0.0047	0.0043	0.0039	0.0036	0.0033
2	0.0296	0.0275	0.0255	0.0236	0.0219	0.0203	0.0188	0.0174	0.0161	0.0149
3	0.0818	0.0767	0.0719	0.0674	0.0632	0.0591	0.0554	0.0518	0.0485	0.0453
4	0.1730	0.1641	0.1555	0.1473	0.1395	0.1321	0.1249	0.1181	0.1117	0.1055
5	0.3007	0.2881	0.2759	0.2640	0.2526	0.2414	0.2307	0.2203	0.2103	0.2006
6	0.4497	0.4349	0.4204	0.4060	0.3920	0.3782	0.3646	0.3514	0.3384	0.3257
7	0.5987	0.5838	0.5689	0.5541	0.5393	0.5246	0.5100	0.4956	0.4812	0.4670
8	0.7291	0.7160	0.7027	0.6892	0.6757	0.6620	0.6482	0.6343	0.6204	0.6065
9	0.8305	0.8202	0.8096	0.7988	0.7877	0.7764	0.7649	0.7531	0.7411	0.7290
10	0.9015	0.8942	0.8867	0.8788	0.8707	0.8622	0.8535	0.8445	0.8352	0.8257
11	0.9467	0.9420	0.9371	0.9319	0.9265	0.9208	0.9148	0.9085	0.9020	0.8952
12	0.9730	0.9703	0.9673	0.9642	0.9609	0.9573	0.9536	0.9496	0.9454	0.9409
13	0.9872	0.9857	0.9841	0.9824	0.9805	0.9784	0.9762	0.9739	0.9714	0.9687
14	0.9943	0.9935	0.9927	0.9918	0.9908	0.9897	0.9886	0.9873	0.9859	0.9844
15	0.9976	0.9972	0.9969	0.9964	0.9959	0.9954	0.9948	0.9941	0.9934	0.9926
16	0.9990	0.9989	0.9987	0.9985	0.9983	0.9980	0.9978	0.9974	0.9971	0.9967
17	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9989	0.9988	0.9986
18	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994
19	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998
20	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
21	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000

$x \backslash \lambda$	8.00	8.10	8.20	8.30	8.40	8.50	8.60	8.70	8.80	8.90
0	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001
1	0.0030	0.0028	0.0025	0.0023	0.0021	0.0019	0.0018	0.0016	0.0015	0.0014
2	0.0138	0.0127	0.0118	0.0109	0.0100	0.0093	0.0086	0.0079	0.0073	0.0068
3	0.0424	0.0396	0.0370	0.0346	0.0323	0.0301	0.0281	0.0262	0.0244	0.0228
4	0.0996	0.0940	0.0887	0.0837	0.0789	0.0744	0.0701	0.0660	0.0621	0.0584
5	0.1912	0.1822	0.1736	0.1653	0.1573	0.1496	0.1422	0.1352	0.1284	0.1219
6	0.3134	0.3013	0.2896	0.2781	0.2670	0.2562	0.2457	0.2355	0.2256	0.2160
7	0.4530	0.4391	0.4254	0.4119	0.3987	0.3856	0.3728	0.3602	0.3478	0.3357
8	0.5925	0.5786	0.5647	0.5507	0.5369	0.5231	0.5094	0.4958	0.4823	0.4689
9	0.7166	0.7041	0.6915	0.6788	0.6659	0.6530	0.6400	0.6269	0.6137	0.6006
10	0.8159	0.8058	0.7955	0.7850	0.7743	0.7634	0.7522	0.7409	0.7294	0.7178
11	0.8881	0.8807	0.8731	0.8652	0.8571	0.8487	0.8400	0.8311	0.8220	0.8126
12	0.9362	0.9313	0.9261	0.9207	0.9150	0.9091	0.9029	0.8965	0.8898	0.8829
13	0.9658	0.9628	0.9595	0.9561	0.9524	0.9486	0.9445	0.9403	0.9358	0.9311
14	0.9827	0.9810	0.9791	0.9771	0.9749	0.9726	0.9701	0.9675	0.9647	0.9617
15	0.9918	0.9908	0.9898	0.9887	0.9875	0.9862	0.9848	0.9832	0.9816	0.9798
16	0.9963	0.9958	0.9953	0.9947	0.9941	0.9934	0.9926	0.9918	0.9909	0.9899
17	0.9984	0.9982	0.9979	0.9977	0.9973	0.9970	0.9966	0.9962	0.9957	0.9952
18	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9985	0.9983	0.9981	0.9978
19	0.9997	0.9997	0.9997	0.9996	0.9995	0.9995	0.9994	0.9993	0.9992	0.9991
20	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996
21	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
22	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
23	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.0000

$x \backslash \lambda$	5.00	5.10	5.20	5.30	5.40	5.50	5.60	5.70	5.80	5.90
0	0.0067	0.0061	0.0055	0.0050	0.0045	0.0041	0.0037	0.0033	0.0030	0.0027
1	0.0404	0.0372	0.0342	0.0314	0.0289	0.0266	0.0244	0.0224	0.0206	0.0189
2	0.1247	0.1165	0.1088	0.1016	0.0948	0.0884	0.0824	0.0768	0.0715	0.0666
3	0.2650	0.2513	0.2381	0.2254	0.2133	0.2017	0.1906	0.1800	0.1700	0.1604
4	0.4405	0.4231	0.4061	0.3895	0.3733	0.3575	0.3422	0.3272	0.3127	0.2987
5	0.6160	0.5984	0.5809	0.5635	0.5461	0.5289	0.5119	0.4950	0.4783	0.4619
6	0.7622	0.7474	0.7324	0.7171	0.7017	0.6860	0.6703	0.6544	0.6384	0.6224
7	0.8666	0.8560	0.8449	0.8335	0.8217	0.8095	0.7970	0.7841	0.7710	0.7576
8	0.9319	0.9252	0.9181	0.9106	0.9027	0.8944	0.8857	0.8766	0.8672	0.8574
9	0.9682	0.9644	0.9603	0.9559	0.9512	0.9462	0.9409	0.9352	0.9292	0.9228
10	0.9863	0.9844	0.9823	0.9800	0.9775	0.9747	0.9718	0.9686	0.9651	0.9614
11	0.9945	0.9937	0.9927	0.9916	0.9904	0.9890	0.9875	0.9859	0.9841	0.9821
12	0.9980	0.9976	0.9972	0.9967	0.9962	0.9955	0.9949	0.9941	0.9932	0.9922
13	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983	0.9980	0.9977	0.9973	0.9969
14	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991	0.9990	0.9988
15	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9996	0.9996
16	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
17	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$x \backslash \lambda$	6.00	6.10	6.20	6.30	6.40	6.50	6.60	6.70	6.80	6.90
0	0.0025	0.0022	0.0020	0.0018	0.0017	0.0015	0.0014	0.0012	0.0011	0.0010
1	0.0174	0.0159	0.0146	0.0134	0.0123	0.0113	0.0103	0.0095	0.0087	0.0080
2	0.0620	0.0577	0.0536	0.0498	0.0463	0.0430	0.0400	0.0371	0.0344	0.0320
3	0.1512	0.1425	0.1342	0.1264	0.1189	0.1118	0.1052	0.0988	0.0928	0.0871
4	0.2851	0.2719	0.2592	0.2469	0.2351	0.2237	0.2127	0.2022	0.1920	0.1823
5	0.4457	0.4298	0.4141	0.3988	0.3837	0.3690	0.3547	0.3406	0.3270	0.3137
6	0.6063	0.5902	0.5742	0.5582	0.5423	0.5265	0.5108	0.4953	0.4799	0.4647
7	0.7440	0.7301	0.7160	0.7017	0.6873	0.6728	0.6581	0.6433	0.6285	0.6136
8	0.8472	0.8367	0.8259	0.8148	0.8033	0.7916	0.7796	0.7673	0.7548	0.7420
9	0.9161	0.9090	0.9016	0.8939	0.8858	0.8774	0.8686	0.8596	0.8502	0.8405
10	0.9574	0.9531	0.9486	0.9437	0.9386	0.9332	0.9274	0.9214	0.9151	0.9084
11	0.9799	0.9776	0.9750	0.9723	0.9693	0.9661	0.9627	0.9591	0.9552	0.9510
12	0.9912	0.9900	0.9887	0.9873	0.9857	0.9840	0.9821	0.9801	0.9779	0.9755
13	0.9964	0.9958	0.9952	0.9945	0.9937	0.9929	0.9920	0.9909	0.9898	0.9885
14	0.9986	0.9984	0.9981	0.9978	0.9974	0.9970	0.9966	0.9961	0.9956	0.9950
15	0.9995	0.9994	0.9993	0.9992	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979
16	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9993	0.9992
17	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997	0.9997
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
19	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITY

$\lambda \backslash x$	10.00	10.10	10.20	10.30	10.40	10.50	10.60	10.70	10.80	10.90
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0005	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002
2	0.0028	0.0026	0.0023	0.0022	0.0020	0.0018	0.0016	0.0016	0.0014	0.0013
3	0.0103	0.0096	0.0089	0.0083	0.0077	0.0071	0.0066	0.0062	0.0057	0.0053
4	0.0293	0.0274	0.0257	0.0241	0.0225	0.0211	0.0197	0.0185	0.0173	0.0162
5	0.0671	0.0634	0.0599	0.0566	0.0534	0.0504	0.0475	0.0448	0.0423	0.0398
6	0.1301	0.1240	0.1180	0.1123	0.1069	0.1016	0.0966	0.0918	0.0872	0.0828
7	0.2202	0.2113	0.2027	0.1944	0.1863	0.1785	0.1710	0.1636	0.1566	0.1498
8	0.3328	0.3217	0.3108	0.3001	0.2896	0.2794	0.2694	0.2597	0.2502	0.2410
9	0.4579	0.4455	0.4332	0.4210	0.4090	0.3971	0.3854	0.3739	0.3626	0.3515
10	0.5830	0.5705	0.5580	0.5456	0.5331	0.5207	0.5084	0.4961	0.4840	0.4719
11	0.6968	0.6853	0.6738	0.6622	0.6505	0.6387	0.6269	0.6150	0.6031	0.5912
12	0.7916	0.7820	0.7722	0.7623	0.7522	0.7420	0.7316	0.7210	0.7104	0.6996
13	0.8645	0.8571	0.8494	0.8416	0.8336	0.8253	0.8169	0.8083	0.7995	0.7905
14	0.9165	0.9112	0.9057	0.9000	0.8940	0.8879	0.8815	0.8750	0.8682	0.8612
15	0.9513	0.9477	0.9440	0.9400	0.9359	0.9317	0.9272	0.9225	0.9177	0.9126
16	0.9730	0.9707	0.9684	0.9658	0.9632	0.9604	0.9574	0.9543	0.9511	0.9477
17	0.9857	0.9844	0.9830	0.9815	0.9799	0.9781	0.9763	0.9744	0.9723	0.9701
18	0.9928	0.9921	0.9913	0.9904	0.9895	0.9885	0.9874	0.9863	0.9850	0.9837
19	0.9965	0.9962	0.9957	0.9953	0.9948	0.9942	0.9936	0.9930	0.9923	0.9915
20	0.9984	0.9982	0.9980	0.9978	0.9975	0.9972	0.9969	0.9966	0.9962	0.9958
21	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986	0.9984	0.9982	0.9980
22	0.9997	0.9997	0.9996	0.9996	0.9995	0.9994	0.9994	0.9993	0.9992	0.9991
23	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996
24	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
25	.....	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
26	.....	.....	.....	.....	.....	.....	.....	.....	1.0000	1.0000

$\lambda \backslash x$	9.00	9.10	9.20	9.30	9.40	9.50	9.60	9.70	9.80	9.90
0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
1	0.0012	0.0011	0.0010	0.0009	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005
2	0.0062	0.0058	0.0053	0.0049	0.0045	0.0042	0.0038	0.0035	0.0033	0.0030
3	0.0212	0.0198	0.0184	0.0172	0.0160	0.0149	0.0138	0.0129	0.0120	0.0111
4	0.0550	0.0517	0.0486	0.0456	0.0429	0.0403	0.0378	0.0355	0.0333	0.0312
5	0.1157	0.1098	0.1041	0.0986	0.0935	0.0885	0.0838	0.0793	0.0750	0.0710
6	0.2068	0.1978	0.1892	0.1808	0.1727	0.1649	0.1574	0.1502	0.1433	0.1366
7	0.3239	0.3123	0.3010	0.2900	0.2792	0.2687	0.2584	0.2485	0.2388	0.2294
8	0.4557	0.4426	0.4296	0.4168	0.4042	0.3918	0.3796	0.3676	0.3558	0.3442
9	0.5874	0.5742	0.5611	0.5479	0.5349	0.5218	0.5089	0.4960	0.4832	0.4705
10	0.7060	0.6941	0.6820	0.6699	0.6576	0.6453	0.6329	0.6205	0.6080	0.5955
11	0.8030	0.7932	0.7832	0.7730	0.7626	0.7520	0.7412	0.7303	0.7193	0.7081
12	0.8758	0.8684	0.8607	0.8529	0.8448	0.8364	0.8279	0.8191	0.8101	0.8009
13	0.9261	0.9210	0.9156	0.9100	0.9042	0.8981	0.8919	0.8853	0.8786	0.8716
14	0.9585	0.9552	0.9517	0.9480	0.9441	0.9400	0.9357	0.9312	0.9265	0.9216
15	0.9780	0.9760	0.9738	0.9715	0.9691	0.9665	0.9638	0.9609	0.9579	0.9546
16	0.9889	0.9878	0.9865	0.9852	0.9838	0.9823	0.9806	0.9789	0.9770	0.9751
17	0.9947	0.9941	0.9934	0.9927	0.9919	0.9911	0.9902	0.9892	0.9881	0.9870
18	0.9976	0.9973	0.9969	0.9966	0.9962	0.9957	0.9952	0.9947	0.9941	0.9935
19	0.9989	0.9988	0.9986	0.9985	0.9983	0.9980	0.9978	0.9975	0.9972	0.9969
20	0.9996	0.9995	0.9994	0.9993	0.9992	0.9991	0.9990	0.9989	0.9987	0.9986
21	0.9998	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0.9994
22	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9997
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
24	.....	.....	.....	.....	.....	1.0000	1.0000	1.0000	1.0000	1.0000

Critical values for the product moment correlation coefficient,  $r$

$n$	5%		2 1/2%		1%		1/2%	
	10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999				
4	0.9000	0.9500	0.9800	0.9900				
5	0.8054	0.8783	0.9343	0.9587				
6	0.7293	0.8114	0.8822	0.9172				
7	0.6694	0.7545	0.8329	0.8745				
8	0.6215	0.7067	0.7887	0.8343				
9	0.5822	0.6664	0.7498	0.7977				
10	0.5494	0.6319	0.7155	0.7646				
11	0.5214	0.6021	0.6851	0.7348				
12	0.4973	0.5760	0.6581	0.7079				
13	0.4762	0.5529	0.6339	0.6835				
14	0.4575	0.5324	0.6120	0.6614				
15	0.4409	0.5140	0.5923	0.6411				
16	0.4259	0.4973	0.5742	0.6226				
17	0.4124	0.4821	0.5577	0.6055				
18	0.4000	0.4683	0.5425	0.5897				
19	0.3887	0.4555	0.5285	0.5751				
20	0.3783	0.4438	0.5155	0.5614				
21	0.3687	0.4329	0.5034	0.5487				
22	0.3598	0.4227	0.4921	0.5368				
23	0.3515	0.4132	0.4815	0.5256				
24	0.3438	0.4044	0.4716	0.5151				
25	0.3365	0.3961	0.4622	0.5052				
26	0.3297	0.3882	0.4534	0.4958				
27	0.3233	0.3809	0.4451	0.4869				
28	0.3172	0.3739	0.4372	0.4785				
29	0.3115	0.3673	0.4297	0.4705				
30	0.3061	0.3610	0.4226	0.4629				
31	0.3009	0.3550	0.4158	0.4556				
32	0.2960	0.3494	0.4093	0.4487				
33	0.2913	0.3440	0.4032	0.4421				
34	0.2869	0.3388	0.3972	0.4357				
35	0.2826	0.3338	0.3916	0.4296				
36	0.2785	0.3291	0.3862	0.4238				
37	0.2746	0.3246	0.3810	0.4182				
38	0.2709	0.3202	0.3760	0.4128				
39	0.2673	0.3160	0.3712	0.4076				
40	0.2638	0.3120	0.3665	0.4026				
41	0.2605	0.3081	0.3621	0.3978				
42	0.2573	0.3044	0.3578	0.3932				
43	0.2542	0.3008	0.3536	0.3887				
44	0.2512	0.2973	0.3496	0.3843				
45	0.2483	0.2940	0.3457	0.3801				
46	0.2455	0.2907	0.3420	0.3761				
47	0.2429	0.2876	0.3384	0.3721				
48	0.2403	0.2845	0.3348	0.3683				
49	0.2377	0.2816	0.3314	0.3646				
50	0.2353	0.2787	0.3281	0.3610				
51	0.2329	0.2759	0.3249	0.3575				
52	0.2306	0.2732	0.3218	0.3542				
53	0.2284	0.2706	0.3188	0.3509				
54	0.2262	0.2681	0.3158	0.3477				
55	0.2241	0.2656	0.3129	0.3445				
56	0.2221	0.2632	0.3102	0.3415				
57	0.2201	0.2609	0.3074	0.3385				
58	0.2181	0.2586	0.3048	0.3357				
59	0.2162	0.2564	0.3022	0.3328				
60	0.2144	0.2542	0.2997	0.3301				

Critical values for Spearman's rank correlation coefficient,  $r_s$

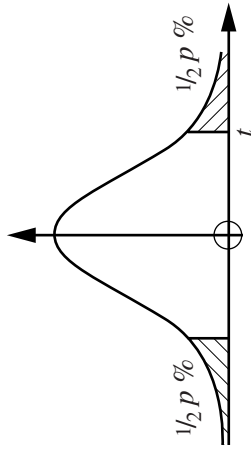
$n$	5%		2 1/2%		1%		1/2%	
	10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-
4	1.0000	-	-	-	-	-	-	-
5	0.9000	1.0000	1.0000	-	-	-	-	-
6	0.8286	0.8857	0.9429	1.0000				
7	0.7143	0.7857	0.8929	0.9286				
8	0.6429	0.7381	0.8333	0.8810				
9	0.6000	0.7000	0.7833	0.8333				
10	0.5636	0.6485	0.7455	0.7939				
11	0.5364	0.6182	0.7091	0.7545				
12	0.5035	0.5874	0.6783	0.7273				
13	0.4835	0.5604	0.6484	0.7033				
14	0.4637	0.5385	0.6264	0.6791				
15	0.4464	0.5214	0.6036	0.6536				
16	0.4294	0.5029	0.5824	0.6353				
17	0.4142	0.4877	0.5662	0.6176				
18	0.4014	0.4716	0.5501	0.5996				
19	0.3912	0.4596	0.5351	0.5842				
20	0.3805	0.4466	0.5218	0.5699				
21	0.3701	0.4364	0.5091	0.5558				
22	0.3608	0.4252	0.4975	0.5438				
23	0.3528	0.4160	0.4862	0.5316				
24	0.3443	0.4070	0.4757	0.5209				
25	0.3369	0.3977	0.4662	0.5108				
26	0.3306	0.3901	0.4571	0.5009				
27	0.3242	0.3828	0.4487	0.4915				
28	0.3180	0.3755	0.4401	0.4828				
29	0.3118	0.3685	0.4325	0.4749				
30	0.3063	0.3624	0.4251	0.4670				
31	0.3012	0.3560	0.4185	0.4593				
32	0.2962	0.3504	0.4117	0.4523				
33	0.2914	0.3449	0.4054	0.4455				
34	0.2871	0.3396	0.3995	0.4390				
35	0.2829	0.3347	0.3936	0.4328				
36	0.2788	0.3300	0.3882	0.4268				
37	0.2748	0.3253	0.3829	0.4211				
38	0.2710	0.3209	0.3778	0.4155				
39	0.2674	0.3168	0.3729	0.4103				
40	0.2640	0.3128	0.3681	0.4051				
41	0.2606	0.3087	0.3636	0.4002				
42	0.2574	0.3051	0.3594	0.3955				
43	0.2543	0.3014	0.3550	0.3908				
44	0.2513	0.2978	0.3511	0.3865				
45	0.2484	0.2945	0.3470	0.3822				
46	0.2456	0.2913	0.3433	0.3781				
47	0.2429	0.2880	0.3396	0.3741				
48	0.2403	0.2850	0.3361	0.3702				
49	0.2378	0.2820	0.3326	0.3664				
50	0.2353	0.2791	0.3293	0.3628				
51	0.2329	0.2764	0.3260	0.3592				
52	0.2307	0.2736	0.3228	0.3558				
53	0.2284	0.2710	0.3198	0.3524				
54	0.2262	0.2685	0.3168	0.3492				
55	0.2242	0.2659	0.3139	0.3460				
56	0.2221	0.2636	0.3111	0.3429				
57	0.2201	0.2612	0.3083	0.3400				
58	0.2181	0.2589	0.3057	0.3370				
59	0.2162	0.2567	0.3030	0.3342				
60	0.2144	0.2545	0.3005	0.3314				





PERCENTAGE POINTS OF  $\chi^2$  AND  $t$  - DISTRIBUTIONS

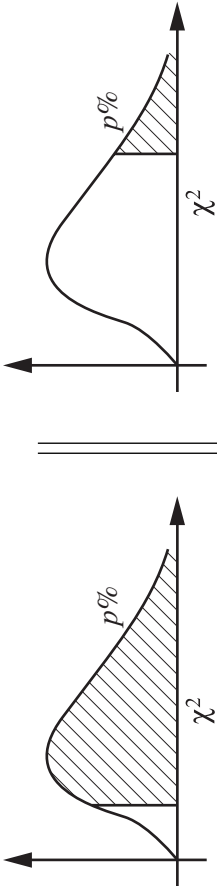
Percentage points of the  $t$ -distribution



$p\%$	10	5	2	1
$\nu = 1$	6.314	12.71	31.82	63.66
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
20	1.725	2.086	2.528	2.845
30	1.697	2.042	2.457	2.750
50	1.676	2.009	2.403	2.678
100	1.660	1.984	2.364	2.626
$\infty$	1.645	1.960	2.326	2.576

= Percentage points of the Normal distribution  $N(0, 1)$

Percentage points of the  $\chi^2$  (chi-squared) distribution



$p\%$	99	97.5	95	90	10	5.0	2.5	1.0	0.5
$\nu = 1$	.0001	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

**CRITICAL VALUES FOR F - TEST**

**2 1/2% points of the F-distribution**

$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
1	648	800	864	900	922	937	948	957	969	977	997	1018
2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.5	39.5
3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.4	14.3	14.1	13.9
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.84	8.75	8.51	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.62	6.52	6.28	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.46	5.37	5.12	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.76	4.67	4.42	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.30	4.20	3.95	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	3.96	3.87	3.61	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.72	3.62	3.37	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.53	3.43	3.17	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.37	3.28	3.02	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.25	3.15	2.89	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.15	3.05	2.79	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.06	2.96	2.70	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.99	2.89	2.63	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.92	2.82	2.56	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.87	2.77	2.50	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.82	2.72	2.45	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.77	2.68	2.41	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.73	2.64	2.37	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.70	2.60	2.33	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.67	2.57	2.30	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.64	2.54	2.27	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.61	2.51	2.24	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.59	2.49	2.22	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.57	2.47	2.19	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.55	2.45	2.17	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.53	2.43	2.15	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.51	2.41	2.14	1.79
32	5.53	4.15	3.56	3.22	3.00	2.84	2.72	2.62	2.48	2.38	2.10	1.75
34	5.50	4.12	3.53	3.19	2.97	2.81	2.69	2.59	2.45	2.35	2.08	1.72
36	5.47	4.09	3.51	3.17	2.94	2.79	2.66	2.57	2.43	2.33	2.05	1.69
38	5.45	4.07	3.48	3.15	2.92	2.76	2.64	2.55	2.41	2.31	2.03	1.66
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.39	2.29	2.01	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.27	2.17	1.88	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.16	2.05	1.76	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.05	1.94	1.64	1.00

**5% points of the F-distribution**

$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.0	254.3
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.5	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.64	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.77	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.53	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.84	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.41	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.12	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	2.90	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.74	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.85	2.79	2.61	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.51	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.67	2.60	2.42	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.60	2.53	2.35	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.29	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.49	2.42	2.24	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.45	2.38	2.19	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.41	2.34	2.15	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.38	2.31	2.11	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.08	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.32	2.25	2.05	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.30	2.23	2.03	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.27	2.20	2.00	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	1.98	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.24	2.16	1.96	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.22	2.15	1.95	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.20	2.13	1.93	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.19	2.12	1.91	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.18	2.10	1.90	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	1.89	1.62
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.14	2.07	1.86	1.59
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.12	2.05	1.84	1.57
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.11	2.03	1.82	1.55
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.09	2.02	1.81	1.53
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.08	2.00	1.79	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.70	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.91	1.83	1.61	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.83	1.75	1.52	1.00

**CRITICAL VALUES FOR F - TEST**

**0.1% points of the F-distribution**

$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
1	4053	5000	5404	5625	5764	5859	5929	5981	6056	6107	6235	6366
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.4	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.2	128.3	125.9	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.05	47.41	45.77	44.05
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.92	26.42	25.14	23.79
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	15.75
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	11.70
8	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.34
9	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	9.87	9.57	8.72	7.81
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.74	8.44	7.64	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	7.92	7.63	6.85	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.25	5.42
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.80	6.52	5.78	4.97
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.40	6.13	5.41	4.60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.08	5.81	5.10	4.31
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.81	5.55	4.85	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.58	5.32	4.63	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.39	5.13	4.45	3.67
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.22	4.97	4.29	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.08	4.82	4.15	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	4.95	4.70	4.03	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.83	4.58	3.92	3.15
23	14.19	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.73	4.48	3.82	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.64	4.39	3.74	2.97
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.56	4.31	3.66	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.48	4.24	3.59	2.82
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.41	4.17	3.52	2.75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.35	4.11	3.46	2.69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.29	4.05	3.41	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.24	4.00	3.36	2.59
32	13.12	8.64	6.94	6.01	5.43	5.02	4.72	4.48	4.14	3.91	3.27	2.50
34	12.97	8.52	6.83	5.92	5.34	4.93	4.63	4.40	4.06	3.83	3.19	2.42
36	12.83	8.42	6.74	5.84	5.26	4.86	4.56	4.33	3.99	3.76	3.12	2.35
38	12.71	8.33	6.66	5.76	5.19	4.79	4.49	4.26	3.93	3.70	3.06	2.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	3.87	3.64	3.01	2.23
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.54	3.32	2.69	1.89
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.24	3.02	2.40	1.54
$\infty$	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	2.96	2.74	2.13	1.00

**1% points of the F-distribution**

$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
1	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.6	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	13.9	13.5
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.05	9.89	9.47	9.02
6	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.31	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.07	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.28	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.73	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.33	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.54	4.40	4.02	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	3.78	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.10	3.96	3.59	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	3.94	3.80	3.43	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.80	3.67	3.29	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.69	3.55	3.18	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.59	3.46	3.08	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.51	3.37	3.00	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.43	3.30	2.92	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.37	3.23	2.86	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.31	3.17	2.80	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.26	3.12	2.75	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.21	3.07	2.70	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.17	3.03	2.66	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.13	2.99	2.62	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.09	2.96	2.58	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.06	2.93	2.55	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.03	2.90	2.52	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.00	2.87	2.49	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.98	2.84	2.47	2.01
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	2.93	2.80	2.42	1.96
34	7.45	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.90	2.76	2.38	1.91
36	7.40	5.25	4.38	3.89	3.58	3.35	3.18	3.05	2.86	2.72	2.35	1.87
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.83	2.69	2.32	1.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.80	2.66	2.29	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.63	2.50	2.12	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.47	2.34	1.95	1.38
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.32	2.18	1.79	1.00

**CRITICAL VALUES FOR THE MANN-WHITNEY TEST**

**Critical Values for the Mann-Whitney Test**

The critical values in these tables are for the Mann-Whitney test statistic,  $T$ . Critical values for the Wilcoxon test statistic,  $W$ , may be derived by adding  $\frac{1}{2}m(m+1)$  (where  $m$  is the size of the sample from which the rank sum has been obtained). These values are tabulated on pages 28 and 29.

1 - tail 2 - tail	$m$	$n$	5%		2 1/2%		1%		1/2%	
			10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1 - tail	8	8	15	13	9	7				
2 - tail	8	9	18	15	11	9				
	8	10	20	17	13	11				
	8	11	23	19	15	13				
	8	12	26	22	17	15				
	8	13	28	24	20	17				
	8	14	31	26	22	18				
	8	15	33	29	24	20				
	8	16	36	31	26	22				
	8	17	39	34	28	24				
	8	18	41	36	30	26				
	8	19	44	38	32	28				
	8	20	47	41	34	30				
	8	21	49	43	36	32				
	8	22	52	45	38	34				
	8	23	54	48	40	35				
	8	24	57	50	42	37				
	8	25	60	53	45	39				
	9	9	21	17	14	11				
	9	10	24	20	16	13				
	9	11	27	23	18	16				
	9	12	30	26	21	18				
	9	13	33	28	23	20				
	9	14	36	31	26	22				
	9	15	39	34	28	24				
	9	16	42	37	31	27				
	9	17	45	39	33	29				
	9	18	48	42	35	31				
	9	19	51	45	38	33				
	9	20	54	48	40	36				
	9	21	57	50	43	38				
	9	22	60	53	45	40				
	9	23	63	56	48	43				
	9	24	66	59	50	45				
	9	25	69	62	53	47				

1 - tail 2 - tail	$m$	$n$	5%		2 1/2%		1%		1/2%	
			10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1 - tail	6	10	14	11	8	6				
2 - tail	6	11	16	13	9	7				
	6	12	17	14	11	9				
	6	13	19	16	12	10				
	6	14	21	17	13	11				
	6	15	23	19	15	12				
	6	16	25	21	16	13				
	6	17	26	22	18	15				
	6	18	28	24	19	16				
	6	19	30	25	20	17				
	6	20	32	27	22	18				
	6	21	34	29	23	19				
	6	22	36	30	24	21				
	6	23	37	32	26	22				
	6	24	39	33	27	23				
	6	25	41	35	29	24				
	7	7	11	8	6	4				
	7	8	13	10	7	6				
	7	9	15	12	9	7				
	7	10	17	14	11	9				
	7	11	19	16	12	10				
	7	12	21	18	14	12				
	7	13	24	20	16	13				
	7	14	26	22	17	15				
	7	15	28	24	19	16				
	7	16	30	26	21	18				
	7	17	33	28	23	19				
	7	18	35	30	24	21				
	7	19	37	32	26	22				
	7	20	39	34	28	24				
	7	21	41	36	30	25				
	7	22	44	38	31	27				
	7	23	46	40	33	29				
	7	24	48	42	35	30				
	7	25	50	44	36	32				

1 - tail 2 - tail	$m$	$n$	5%		2 1/2%		1%		1/2%	
			10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1 - tail	5	5	4	2	1	0				
2 - tail	5	6	5	3	2	1				
	5	7	6	5	3	1				
	5	8	8	6	4	2				
	5	9	9	7	5	3				
	5	10	11	8	6	4				
	5	11	12	9	7	5				
	5	12	13	11	8	6				
	5	13	15	12	9	7				
	5	14	16	13	10	7				
	5	15	18	14	11	8				
	5	16	19	15	12	9				
	5	17	20	17	13	10				
	5	18	22	18	14	11				
	5	19	23	19	15	12				
	5	20	25	20	16	13				
	5	21	26	22	17	14				
	5	22	28	23	18	14				
	5	23	29	24	19	15				
	5	24	30	25	20	16				
	5	25	32	27	21	17				
	6	6	7	5	3	2				
	6	7	8	6	4	3				
	6	8	10	8	6	4				
	6	9	12	10	7	5				

1 - tail 2 - tail	$m$	$n$	5%		2 1/2%		1%		1/2%	
			10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1 - tail	3	14	7	5	2	1				
2 - tail	3	15	7	5	3	2				
	3	16	8	6	3	2				
	3	17	9	6	4	2				
	3	18	9	7	4	2				
	3	19	10	7	4	3				
	3	20	11	8	5	3				
	3	21	11	8	5	3				
	3	22	12	9	6	4				
	3	23	13	9	6	4				
	3	24	13	10	6	4				
	3	25	14	10	7	5				
	4	4	1	0	-	-				
	4	5	2	1	0	-				
	4	6	3	2	1	0				
	4	7	4	3	1	0				
	4	8	5	4	2	1				
	4	9	6	4	3	1				
	4	10	7	5	3	2				
	4	11	8	6	4	2				
	4	12	9	7	5	3				
	4	13	10	8	5	3				
	4	14	11	9	6	4				
	4	15	12	10	7	5				
	4	16	14	11	7	5				
	4	17	15	11	8	6				
	4	18	16	12	9	6				
	4	19	17	13	9	7				
	4	20	18	14	10	8				
	4	21	19	15	11	8				
	4	22	20	16	11	9				
	4	23	21	17	12	9				
	4	24	22	17	13	10				
	4	25	23	18	13	10				

1 - tail 2 - tail	$m$	$n$	5%		2 1/2%		1%		1/2%	
			10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
1 - tail	2	2	-	-	-	-				
2 - tail	2	3	-	-	-	-				
	2	4	-	-	-	-				
	2	5	0	-	-	-				
	2	6	0	-	-	-				
	2	7	0	-	-	-				
	2	8	1	0	-	-				
	2	9	1	0	-	-				
	2	10	1	0	-	-				
	2	11	1	0	-	-				
	2	12	2	1	0	-				
	2	13	2	1	0	-				
	2	14	3	1	0	-				
	2	15	3	1	0	-				
	2	16	3	1	0	-				
	2	17	3	2	0	-				
	2	18	4	2	0	-				
	2	19	4	2	1	0				
	2	20	4	2	1	0				
	2	21	5	3	1	0				
	2	22	5	3	1	0				
	2	23	5	3	1	0				
	2	24	6	3	1	0				
	2	25	6	3	1	0				
	3	3	0	-	-	-				
	3	4	0	-	-	-				
	3	5	1	0	-	-				
	3	6	2	1	0	-				
	3	7	2	1	0	-				
	3	8	3	2	0	-				
	3	9	4	2	1	0				
	3	10	4	3	1	0				
	3	11	5	3	1	0				
	3	12	5	4	2	1				
	3	13	6	4	2	1				

**CRITICAL VALUES FOR THE MANN-WHITNEY TEST**

1 - tail 2 - tail	m	n	5%		2 1/2%		1%		1/2%	
			10%	5%	10%	5%	2%	1%	2%	1%
1 - tail	19	19	123	113	101	93				
2 - tail	19	19	130	119	107	99				
1 - tail	19	21	138	126	113	105				
2 - tail	19	22	145	133	120	111				
1 - tail	19	23	152	140	126	117				
2 - tail	19	24	160	147	133	123				
1 - tail	19	25	167	154	139	129				
1 - tail	20	20	138	127	114	105				
2 - tail	20	21	146	134	121	112				
1 - tail	20	22	154	141	127	118				
2 - tail	20	23	161	149	134	125				
1 - tail	20	24	169	156	141	131				
2 - tail	20	25	177	163	148	138				
1 - tail	21	21	154	142	128	118				
2 - tail	21	22	162	150	135	125				
1 - tail	21	23	170	157	142	132				
2 - tail	21	24	179	165	150	139				
1 - tail	21	25	187	173	157	146				
1 - tail	22	22	171	158	143	133				
2 - tail	22	23	179	166	150	140				
1 - tail	22	24	188	174	158	147				
2 - tail	22	25	197	182	166	155				
1 - tail	23	23	189	175	158	148				
2 - tail	23	24	198	183	167	155				
1 - tail	23	25	207	192	175	163				
1 - tail	24	24	207	192	175	164				
2 - tail	24	25	217	201	184	172				
1 - tail	25	25	227	211	192	180				

1 - tail 2 - tail	m	n	5%		2 1/2%		1%		1/2%	
			10%	5%	10%	5%	2%	1%	2%	1%
1 - tail	16	19	101	92	82	74				
2 - tail	16	20	107	98	87	79				
1 - tail	16	21	113	103	92	84				
2 - tail	16	22	119	109	97	89				
1 - tail	16	23	125	115	102	94				
2 - tail	16	24	131	120	108	99				
1 - tail	16	25	137	126	113	104				
1 - tail	17	17	96	87	77	70				
2 - tail	17	18	102	93	82	75				
1 - tail	17	19	109	99	88	81				
2 - tail	17	20	115	105	93	86				
1 - tail	17	21	121	111	99	91				
2 - tail	17	22	128	117	105	96				
1 - tail	17	23	134	123	110	102				
2 - tail	17	24	141	129	116	107				
1 - tail	17	25	147	135	122	112				
1 - tail	18	18	109	99	88	81				
2 - tail	18	19	116	106	94	87				
1 - tail	18	20	123	112	100	92				
2 - tail	18	21	130	119	106	98				
1 - tail	18	22	136	125	112	104				
2 - tail	18	23	143	132	118	109				
1 - tail	18	24	150	138	124	115				
2 - tail	18	25	157	145	130	121				

1 - tail 2 - tail	m	n	5%		2 1/2%		1%		1/2%	
			10%	5%	10%	5%	2%	1%	2%	1%
1 - tail	14	14	61	55	47	42				
2 - tail	14	15	66	59	51	46				
1 - tail	14	16	71	64	56	50				
2 - tail	14	17	77	69	60	54				
1 - tail	14	18	82	74	65	58				
2 - tail	14	19	87	78	69	63				
1 - tail	14	20	92	83	73	67				
2 - tail	14	21	97	88	78	71				
1 - tail	14	22	102	93	82	75				
2 - tail	14	23	107	98	87	79				
1 - tail	14	24	113	102	91	83				
2 - tail	14	25	118	107	95	87				
1 - tail	15	15	72	64	56	51				
2 - tail	15	16	77	70	61	55				
1 - tail	15	17	83	75	66	60				
2 - tail	15	18	88	80	70	64				
1 - tail	15	19	94	85	75	69				
2 - tail	15	20	100	90	80	73				
1 - tail	15	21	105	96	85	78				
2 - tail	15	22	111	101	90	82				
1 - tail	15	23	116	106	94	87				
2 - tail	15	24	122	111	99	91				
1 - tail	15	25	128	117	104	96				
1 - tail	16	16	83	75	66	60				
2 - tail	16	17	89	81	71	65				
1 - tail	16	18	95	86	76	70				

1 - tail 2 - tail	m	n	5%		2 1/2%		1%		1/2%	
			10%	5%	10%	5%	2%	1%	2%	1%
1 - tail	12	12	42	37	31	27				
2 - tail	12	13	47	41	35	31				
1 - tail	12	14	51	45	38	34				
2 - tail	12	15	55	49	42	37				
1 - tail	12	16	60	53	46	41				
2 - tail	12	17	64	57	49	44				
1 - tail	12	18	68	61	53	47				
2 - tail	12	19	72	65	56	51				
1 - tail	12	20	77	69	60	54				
2 - tail	12	21	81	73	64	58				
1 - tail	12	22	85	77	67	61				
2 - tail	12	23	90	81	71	64				
1 - tail	12	24	94	85	75	68				
2 - tail	12	25	98	89	78	71				
1 - tail	13	13	51	45	39	34				
2 - tail	13	14	56	50	43	38				
1 - tail	13	15	61	54	47	42				
2 - tail	13	16	65	59	51	45				
1 - tail	13	17	70	63	55	49				
2 - tail	13	18	75	67	59	53				
1 - tail	13	19	80	72	63	57				
2 - tail	13	20	84	76	67	60				
1 - tail	13	21	89	80	71	64				
2 - tail	13	22	94	85	75	68				
1 - tail	13	23	98	89	79	72				
2 - tail	13	24	103	94	83	75				
1 - tail	13	25	108	98	87	79				

1 - tail 2 - tail	m	n	5%		2 1/2%		1%		1/2%	
			10%	5%	10%	5%	2%	1%	2%	1%
1 - tail	10	10	27	23	19	16				
2 - tail	10	11	31	26	22	18				
1 - tail	10	12	34	29	24	21				
2 - tail	10	13	37	33	27	24				
1 - tail	10	14	41	36	30	26				
2 - tail	10	15	44	39	33	29				
1 - tail	10	16	48	42	36	31				
2 - tail	10	17	51	45	38	34				
1 - tail	10	18	55	48	41	37				
2 - tail	10	19	58	52	44	39				
1 - tail	10	20	62	55	47	42				
2 - tail	10	21	65	58	50	44				
1 - tail	10	22	68	61	53	47				
2 - tail	10	23	72	64	55	50				
1 - tail	10	24	75	67	58	52				
2 - tail	10	25	79	71	61	55				
1 - tail	11	11	34	30	25	21				
2 - tail	11	12	38	33	28	24				
1 - tail	11	13	42	37	31	27				
2 - tail	11	14	46	40	34	30				
1 - tail	11	15	50	44	37	33				
2 - tail	11	16	54	47	41	36				
1 - tail	11	17	57	51	44	39				
2 - tail	11	18	61	55	47	42				
1 - tail	11	19	65	58	50	45				
2 - tail	11	20	69	62	53	48				
1 - tail	11	21	73	65	57	51				
2 - tail	11	22	77	69	60	54				
1 - tail	11	23	81	73	63	57				
2 - tail	11	24	85	76	66	60				
1 - tail	11	25	89	80	70	63				

For larger values of  $m, n$  it is usually adequate to use a Normal approximation with continuity correction, with mean  $\frac{1}{2}mn$  and variance  $\frac{1}{12}mn(m+n+1)$ .

**CRITICAL VALUES FOR THE WILCOXON RANK SUM 2-SAMPLE TEST**

**Critical Values for the Wilcoxon Rank Sum 2-Sample Test**

1 - tail 2 - tail	m	n	5%		2 1/2%		1%	
			10%	5%	5%	2 1/2%	2%	1%
1 - tail	8	8	51	49	45	43	45	43
2 - tail	8	9	54	51	47	45	47	45
1 - tail	8	10	56	53	49	47	49	47
2 - tail	8	11	59	55	51	49	51	49
1 - tail	8	12	62	58	53	51	53	51
2 - tail	8	13	64	60	56	53	56	53
1 - tail	8	14	67	62	58	54	58	54
2 - tail	8	15	69	65	60	56	60	56
1 - tail	8	16	72	67	62	58	62	58
2 - tail	8	17	75	70	64	60	64	60
1 - tail	8	18	77	72	66	62	66	62
2 - tail	8	19	80	74	68	64	68	64
1 - tail	8	20	83	77	70	66	70	66
2 - tail	8	21	85	79	72	68	72	68
1 - tail	8	22	88	81	74	70	74	70
2 - tail	8	23	90	84	76	71	76	71
1 - tail	8	24	93	86	78	73	78	73
2 - tail	8	25	96	89	81	75	81	75
1 - tail	9	9	66	62	59	56	59	56
2 - tail	9	10	69	65	61	58	61	58
1 - tail	9	11	72	68	63	61	63	61
2 - tail	9	12	75	71	66	63	66	63
1 - tail	9	13	78	73	68	65	68	65
2 - tail	9	14	81	76	71	67	71	67
1 - tail	9	15	84	79	73	69	73	69
2 - tail	9	16	87	82	76	72	76	72
1 - tail	9	17	90	84	78	74	78	74
2 - tail	9	18	93	87	80	76	80	76
1 - tail	9	19	96	90	83	78	83	78
2 - tail	9	20	99	93	85	81	85	81
1 - tail	9	21	102	95	88	83	88	83
2 - tail	9	22	105	98	90	85	90	85
1 - tail	9	23	108	101	93	88	93	88
2 - tail	9	24	111	104	95	90	95	90
1 - tail	9	25	114	107	98	92	98	92

1 - tail 2 - tail	m	n	5%		2 1/2%		1%	
			10%	5%	5%	2 1/2%	2%	1%
1 - tail	6	10	35	32	29	27	29	27
2 - tail	6	11	37	34	30	28	30	28
1 - tail	6	12	38	35	32	30	32	30
2 - tail	6	13	40	37	33	31	33	31
1 - tail	6	14	42	38	34	32	34	32
2 - tail	6	15	44	40	36	33	36	33
1 - tail	6	16	46	42	37	34	37	34
2 - tail	6	17	47	43	39	36	39	36
1 - tail	6	18	49	45	40	37	40	37
2 - tail	6	19	51	46	41	38	41	38
1 - tail	6	20	53	48	43	39	43	39
2 - tail	6	21	55	50	44	40	44	40
1 - tail	6	22	57	51	45	42	45	42
2 - tail	6	23	58	53	47	43	47	43
1 - tail	6	24	60	54	48	44	48	44
2 - tail	6	25	62	56	50	45	50	45
1 - tail	7	7	39	36	34	32	34	32
2 - tail	7	8	41	38	35	34	35	34
1 - tail	7	9	43	40	37	35	37	35
2 - tail	7	10	45	42	39	37	39	37
1 - tail	7	11	47	44	40	38	40	38
2 - tail	7	12	49	46	42	40	42	40
1 - tail	7	13	52	48	44	41	44	41
2 - tail	7	14	54	50	45	43	45	43
1 - tail	7	15	56	52	47	44	47	44
2 - tail	7	16	58	54	49	46	49	46
1 - tail	7	17	61	56	51	47	51	47
2 - tail	7	18	63	58	52	49	52	49
1 - tail	7	19	65	60	54	50	54	50
2 - tail	7	20	67	62	56	52	56	52
1 - tail	7	21	69	64	58	53	58	53
2 - tail	7	22	72	66	59	55	59	55
1 - tail	7	23	74	68	61	57	61	57
2 - tail	7	24	76	70	63	58	63	58
1 - tail	7	25	78	72	64	60	64	60

The critical values in these tables are for the Wilcoxon Rank Sum 2-sample test statistic,  $W$ . Critical values for the Mann-Whitney test statistic,  $T$ , may be derived by subtracting  $\frac{1}{2}m(m+1)$  (where  $m$  is the size of the sample from which the rank sum has been obtained).

1 - tail 2 - tail	m	n	5%		2 1/2%		1%	
			10%	5%	5%	2 1/2%	2%	1%
1 - tail	5	5	19	17	16	15	16	15
2 - tail	5	6	20	18	17	16	17	16
1 - tail	5	7	21	20	18	16	18	16
2 - tail	5	8	23	21	19	17	19	17
1 - tail	5	9	24	22	20	18	20	18
2 - tail	5	10	26	23	21	19	21	19
1 - tail	5	11	27	24	22	20	22	20
2 - tail	5	12	28	26	23	21	23	21
1 - tail	5	13	30	27	24	22	24	22
2 - tail	5	14	31	28	25	22	25	22
1 - tail	5	15	33	29	26	23	26	23
2 - tail	5	16	34	30	27	24	27	24
1 - tail	5	17	35	32	28	25	28	25
2 - tail	5	18	37	33	29	26	29	26
1 - tail	5	19	38	34	30	27	30	27
2 - tail	5	20	40	35	31	28	31	28
1 - tail	5	21	41	37	32	29	32	29
2 - tail	5	22	43	38	33	29	33	29
1 - tail	5	23	44	39	34	30	34	30
2 - tail	5	24	45	40	35	31	35	31
1 - tail	5	25	47	42	36	32	36	32
2 - tail	6	6	28	26	24	23	24	23
1 - tail	6	7	29	27	25	24	25	24
2 - tail	6	8	31	29	27	25	27	25
1 - tail	6	9	33	31	28	26	28	26

1 - tail 2 - tail	m	n	5%		2 1/2%		1%	
			10%	5%	5%	2 1/2%	2%	1%
1 - tail	3	14	13	11	8	7	8	7
2 - tail	3	15	13	11	9	8	9	8
1 - tail	3	16	14	12	9	8	9	8
2 - tail	3	17	15	12	10	8	10	8
1 - tail	3	18	15	13	10	8	10	8
2 - tail	3	19	16	13	10	9	10	9
1 - tail	3	20	17	14	11	9	11	9
2 - tail	3	21	17	14	11	9	11	9
1 - tail	3	22	18	15	12	10	12	10
2 - tail	3	23	19	15	12	10	12	10
1 - tail	3	24	19	16	12	10	12	10
2 - tail	3	25	20	16	13	11	13	11
1 - tail	4	4	11	10	-	-	-	-
2 - tail	4	5	12	11	10	-	-	-
1 - tail	4	6	13	12	11	10	11	10
2 - tail	4	7	14	13	11	10	11	10
1 - tail	4	8	15	14	12	11	12	11
2 - tail	4	9	16	14	13	11	13	11
1 - tail	4	10	17	15	13	12	13	12
2 - tail	4	11	18	16	14	12	14	12
1 - tail	4	12	19	17	15	13	15	13
2 - tail	4	13	20	18	15	13	15	13
1 - tail	4	14	21	19	16	14	16	14
2 - tail	4	15	22	20	16	14	16	14
1 - tail	4	16	24	21	17	15	17	15
2 - tail	4	17	25	21	18	16	18	16
1 - tail	4	18	26	22	19	16	19	16
2 - tail	4	19	27	23	19	17	19	17
1 - tail	4	20	28	24	20	18	20	18
2 - tail	4	21	29	25	21	18	21	18
1 - tail	4	22	30	26	21	19	21	19
2 - tail	4	23	31	27	22	19	22	19
1 - tail	4	24	32	27	23	20	23	20
2 - tail	4	25	33	28	23	20	23	20

1 - tail 2 - tail	m	n	5%		2 1/2%		1%	
			10%	5%	5%	2 1/2%	2%	1%
1 - tail	2	2	-	-	-	-	-	-
2 - tail	2	3	-	-	-	-	-	-
1 - tail	2	4	-	-	-	-	-	-
2 - tail	2	5	3	-	-	-	-	-
1 - tail	2	6	3	-	-	-	-	-
2 - tail	2	7	3	-	-	-	-	-
1 - tail	2	8	4	3	-	-	-	-
2 - tail	2	9	4	3	-	-	-	-
1 - tail	2	10	4	3	-	-	-	-
2 - tail	2	11	4	3	-	-	-	-
1 - tail	2	12	5	4	-	-	-	-
2 - tail	2	13	5	4	3	-	-	-
1 - tail	2	14	6	4	3	-	-	-
2 - tail	2	15	6	4	3	-	-	-
1 - tail	2	16	6	4	3	-	-	-
2 - tail	2	17	6	5	3	-	-	-
1 - tail	2	18	7	5	3	-	-	-
2 - tail	2	19	7	5	4	3	-	-
1 - tail	2	20	7	5	4	3	-	-
2 - tail	2	21	8	6	4	3	-	-
1 - tail	2	22	8	6	4	3	-	-
2 - tail	2	23	8	6	4	3	-	-
1 - tail	2	24	9	6	4	3	-	-
2 - tail	2	25	9	6	4	3	-	-
1 - tail	3	3	6	-	-	-	-	-
2 - tail	3	4	6	-	-	-	-	-
1 - tail	3	5	7	6	-	-	-	-
2 - tail	3	6	8	7	-	-	-	-
1 - tail	3	7	8	7	6	-	-	-
2 - tail	3	8	9	8	6	-	-	-
1 - tail	3	9	10	8	7	6	-	-
2 - tail	3	10	10	9	7	6	-	-
1 - tail	3	11	11	9	7	6	-	-
2 - tail	3	12	11	10	8	7	-	-
1 - tail	3	13	12	10	8	7	-	-

CRITICAL VALUES FOR THE WILCOXON RANK SUM 2-SAMPLE TEST

1 - tail		5%		2 1/2%		1%		1/2%	
10%		5%		2 1/2%		1%		1%	
1 - tail	2 - tail	10%	5%	2 1/2%	1%	2 1/2%	1%	10%	5%
<i>m</i>	<i>n</i>								
19	19	313	303	291	283				
19	20	320	309	297	289				
19	21	328	316	303	295				
19	22	335	323	310	301				
19	23	342	330	316	307				
19	24	350	337	323	313				
19	25	357	344	329	319				
20	20	348	337	324	315				
20	21	356	344	331	322				
20	22	364	351	337	328				
20	23	371	359	344	335				
20	24	379	366	351	341				
20	25	387	373	358	348				
21	21	385	373	359	349				
21	22	393	381	366	356				
21	23	401	388	373	363				
21	24	410	396	381	370				
21	25	418	404	388	377				
22	22	424	411	396	386				
22	23	432	419	403	393				
22	24	441	427	411	400				
22	25	450	435	419	408				
23	23	465	451	434	424				
23	24	474	459	443	431				
23	25	483	468	451	439				
24	24	507	492	475	464				
24	25	517	501	484	472				
25	25	552	536	517	505				

1 - tail		5%		2 1/2%		1%		1/2%	
10%		5%		2 1/2%		1%		1%	
1 - tail	2 - tail	10%	5%	2 1/2%	1%	2 1/2%	1%	10%	5%
<i>m</i>	<i>n</i>								
16	19	237	228	218	210				
16	20	243	234	223	215				
16	21	249	239	228	220				
16	22	255	245	233	225				
16	23	261	251	238	230				
16	24	267	256	244	235				
16	25	273	262	249	240				
17	17	249	240	230	223				
17	18	255	246	235	228				
17	19	262	252	241	234				
17	20	268	258	246	239				
17	21	274	264	252	244				
17	22	281	270	258	249				
17	23	287	276	263	255				
17	24	294	282	269	260				
17	25	300	288	275	265				
18	18	280	270	259	252				
18	19	287	277	265	258				
18	20	294	283	271	263				
18	21	301	290	277	269				
18	22	307	296	283	275				
18	23	314	303	289	280				
18	24	321	309	295	286				
18	25	328	316	301	292				

1 - tail		5%		2 1/2%		1%		1/2%	
10%		5%		2 1/2%		1%		1%	
1 - tail	2 - tail	10%	5%	2 1/2%	1%	2 1/2%	1%	10%	5%
<i>m</i>	<i>n</i>								
14	14	166	160	152	147				
14	15	171	164	156	151				
14	16	176	169	161	155				
14	17	182	174	165	159				
14	18	187	179	170	163				
14	19	192	183	174	168				
14	20	197	188	178	172				
14	21	202	193	183	176				
14	22	207	198	187	180				
14	23	212	203	192	184				
14	24	218	207	196	188				
14	25	223	212	200	192				
15	15	192	184	176	171				
15	16	197	190	181	175				
15	17	203	195	186	180				
15	18	208	200	190	184				
15	19	214	205	195	189				
15	20	220	210	200	193				
15	21	225	216	205	198				
15	22	231	221	210	202				
15	23	236	226	214	207				
15	24	242	231	219	211				
15	25	248	237	224	216				
16	16	219	211	202	196				
16	17	225	217	207	201				
16	18	231	222	212	206				

1 - tail		5%		2 1/2%		1%		1/2%	
10%		5%		2 1/2%		1%		1%	
1 - tail	2 - tail	10%	5%	2 1/2%	1%	2 1/2%	1%	10%	5%
<i>m</i>	<i>n</i>								
12	12	120	115	109	105				
12	13	125	119	113	109				
12	14	129	123	116	112				
12	15	133	127	120	115				
12	16	138	131	124	119				
12	17	142	135	127	122				
12	18	146	139	131	125				
12	19	150	143	134	129				
12	20	155	147	138	132				
12	21	159	151	142	136				
12	22	163	155	145	139				
12	23	168	159	149	142				
12	24	172	163	153	146				
12	25	176	167	156	149				
13	13	142	136	130	125				
13	14	147	141	134	129				
13	15	152	145	138	133				
13	16	156	150	142	136				
13	17	161	154	146	140				
13	18	166	158	150	144				
13	19	171	163	154	148				
13	20	175	167	158	151				
13	21	180	171	162	155				
13	22	185	176	166	159				
13	23	189	180	170	163				
13	24	194	185	174	166				
13	25	199	189	178	170				

1 - tail		5%		2 1/2%		1%		1/2%	
10%		5%		2 1/2%		1%		1%	
1 - tail	2 - tail	10%	5%	2 1/2%	1%	2 1/2%	1%	10%	5%
<i>m</i>	<i>n</i>								
10	10	82	78	74	71				
10	11	86	81	77	73				
10	12	89	84	79	76				
10	13	92	88	82	79				
10	14	96	91	85	81				
10	15	99	94	88	84				
10	16	103	97	91	86				
10	17	106	100	93	89				
10	18	110	103	96	92				
10	19	113	107	99	94				
10	20	117	110	102	97				
10	21	120	113	105	99				
10	22	123	116	108	102				
10	23	127	119	110	105				
10	24	130	122	113	107				
10	25	134	126	116	110				
11	11	100	96	91	87				
11	12	104	99	94	90				
11	13	108	103	97	93				
11	14	112	106	100	96				
11	15	116	110	103	99				
11	16	120	113	107	102				
11	17	123	117	110	105				
11	18	127	121	113	108				
11	19	131	124	116	111				
11	20	135	128	119	114				
11	21	139	131	123	117				
11	22	143	135	126	120				
11	23	147	139	129	123				
11	24	151	142	132	126				
11	25	155	146	136	129				

For larger values of  $m, n$  it is usually adequate to use a Normal approximation, with continuity correction, with mean  $\frac{1}{2}mn + \frac{1}{2}m(m+1)$  and variance  $\frac{1}{12}mn(m+n+1)$

**CRITICAL VALUES FOR THE WILCOXON SINGLE SAMPLE AND PAIRED SAMPLE TESTS  
SHEWHART CHART: ACTION AND WARNING LINES**

**Action and Warning lines for Shewhart Chart for Ranges**

Group Size <i>n</i>	Action Lines		Warning Lines	
	<i>D</i> <sub>1</sub>	<i>D</i> <sub>2</sub>	<i>D</i> <sub>3</sub>	<i>D</i> <sub>4</sub>
2	0.00	4.12	0.04	2.81
3	0.04	2.99	0.18	2.18
4	0.10	2.58	0.29	1.94
5	0.16	2.36	0.37	1.80
6	0.21	2.22	0.42	1.72
7	0.26	2.12	0.46	1.66
8	0.29	2.05	0.50	1.62
9	0.33	1.99	0.52	1.58
10	0.35	1.94	0.54	1.55

The action and warning lines are obtained by multiplying the values in the table by the mean range of the values obtained from the process.

**Critical values for the Wilcoxon Single Sample and Paired Sample tests**

1 – tail 2 – tail	<i>n</i>	5%		2 1/2%		1%		1/2%	
		10%	5%	5%	2 1/2%	2%	1%	1%	1/2%
	26		110	98	84	75			
	27		119	107	92	83			
	28		130	116	101	91			
	29		140	126	110	100			
	30		151	137	120	109			
	31		163	147	130	118			
	32		175	159	140	128			
	33		187	170	151	138			
	34		200	182	162	148			
	35		213	195	173	159			
	36		227	208	185	171			
	37		241	221	198	182			
	38		256	235	211	194			
	39		271	249	224	207			
	40		286	264	238	220			
	41		302	279	252	233			
	42		319	294	266	247			
	43		336	310	281	261			
	44		353	327	296	276			
	45		371	343	312	291			
	46		389	361	328	307			
	47		407	378	345	322			
	48		426	396	362	339			
	49		446	415	379	355			
	50		466	434	397	373			
	21		67	58	49	42			
	22		75	65	55	48			
	23		83	73	62	54			
	24		91	81	69	61			
	25		100	89	76	68			

For larger values of *n*, the Normal approximation with mean  $\frac{n(n+1)}{4}$ ,

variance  $\frac{n(n+1)(2n+1)}{24}$  should be used for  $T = \min [P, Q]$ .



**RANDOM NUMBERS AND RANDOM PERMUTATIONS  
ESTIMATION OF STANDARD DEVIATION FROM RANGE**

**Random permutations (size 4)**

3 1 2 4	2 4 3 1	4 3 2 1	4 3 2 1
2 3 1 4	4 3 1 2	3 1 4 2	3 1 4 2
4 2 3 1	3 1 2 4	1 4 3 2	1 4 3 2
1 3 2 4	1 4 2 3	4 3 2 1	4 3 2 1
2 4 3 1	1 2 4 3	1 3 2 4	1 3 2 4
4 3 1 2	2 4 3 1	3 4 1 2	3 4 1 2
2 1 4 3	4 1 2 3	3 4 1 2	3 4 1 2
4 3 1 2	3 1 2 4	3 2 1 4	3 2 1 4
2 3 4 1	3 4 2 1	1 4 2 3	1 4 2 3
3 2 1 4	3 4 2 1	3 4 1 2	3 4 1 2
1 4 3 2	4 3 2 1	2 1 4 3	2 1 4 3
1 4 2 3	2 3 1 4	1 4 2 3	1 4 2 3
2 3 1 4	3 1 4 2	4 2 3 1	4 2 3 1
4 1 3 2	2 4 1 3	4 2 3 1	4 2 3 1
2 1 4 3	1 3 4 2	3 2 4 1	3 2 4 1
2 3 1 4	1 3 4 2	2 3 1 4	2 3 1 4
2 3 4 1	2 4 3 1	3 2 1 4	3 2 1 4
3 1 4 2	3 1 2 4	4 1 2 3	4 1 2 3
4 3 2 1	4 2 1 3	4 2 3 1	4 2 3 1
2 3 4 1	4 1 2 3	2 3 4 1	2 3 4 1
1 4 2 3	2 4 3 1	4 1 3 2	4 1 3 2
2 4 3 1	4 1 2 3	1 3 4 2	1 3 4 2
1 2 4 3	1 2 3 4	4 3 2 1	4 3 2 1
2 3 4 1	1 3 4 2	2 4 3 1	2 4 3 1
4 3 1 2	4 3 2 1	1 4 3 2	1 4 3 2
1 4 2 3	3 2 1 4	2 1 4 3	2 1 4 3
3 4 1 2	1 4 3 2	1 4 2 3	1 4 2 3
2 4 1 3	4 2 3 1	3 4 2 1	3 4 2 1
1 4 3 2	4 3 2 1	2 3 1 4	2 3 1 4
1 3 2 4	4 2 1 3	4 2 1 3	4 2 1 3
1 4 3 2	3 2 1 4	4 1 2 3	4 1 2 3
2 4 1 3	3 4 1 2	4 3 1 2	4 3 1 2
3 4 1 2	4 3 1 2	3 4 2 1	3 4 2 1
4 2 1 3	2 3 4 1	4 3 1 2	4 3 1 2
3 2 4 1	4 3 2 1	2 3 4 1	2 3 4 1
3 2 4 1	4 3 1 2	3 2 1 4	3 2 1 4
1 2 3 4	2 4 1 3	1 3 4 2	1 3 4 2
4 3 1 2	3 4 1 2	4 1 3 2	4 1 3 2
1 2 4 3	1 4 3 2	2 1 4 3	2 1 4 3
4 1 2 3	3 1 2 4	4 1 2 3	4 1 2 3

**Random Numbers**

68236	35335	71329	96803	24413
62385	36545	59305	59948	17232
64058	80195	30914	16664	50818
64822	68554	90952	64984	92295
17716	22164	05161	04412	59002
03928	22379	92325	79920	99070
11021	08533	83855	37723	77339
01830	68554	86787	90447	54796
36782	73208	93548	77405	58355
58158	45059	83980	40176	40737
91239	10532	27993	11516	61327
27073	98804	60544	12133	01422
81501	00633	62681	84319	03374
64374	26598	54466	94768	19144
29896	26739	30871	29795	13472
38996	72151	65746	16513	62796
73936	81751	00149	99126	23117
18795	93118	84105	18307	49807
76816	99822	92314	45035	43490
12091	60413	90467	42457	50490
41538	19059	69055	94355	84262
12909	04950	14986	08205	53582
49185	94608	87317	37725	66450
37771	48526	14939	32848	77677
22532	13814	69092	78342	37774
60132	24386	10989	54346	41531
23784	56693	45902	33406	53867
03081	20189	77226	89923	67301
51273	64049	19919	45518	43243
03281	40214	60679	68712	71636

**Estimation of standard deviation from range**

$n$	$a_n$	$n$	$a_n$	$n$	$a_n$	$n$	$a_n$
2	0.8862	5	0.4299	8	0.3512	11	0.3152
3	0.5908	6	0.3946	9	0.3367	12	0.3069
4	0.4857	7	0.3698	10	0.3249	13	0.2998

**RANDOM PERMUTATIONS**

**Random permutations (size 10)**

5 8 1 6 7 9 2 10 3 4	8 2 5 7 9 10 1 4 6 3
7 2 4 8 6 1 3 10 5 9	3 5 8 6 1 9 2 10 7 4
10 1 2 4 9 3 7 5 6 8	9 1 3 10 7 4 6 5 8 2
5 2 6 7 1 3 10 9 4 8	6 3 10 9 7 4 5 1 2 8
5 1 8 4 9 6 3 10 7 2	8 4 10 3 9 5 7 6 1 2
3 2 7 6 10 8 5 1 4 9	6 9 2 5 8 3 10 4 7 1
10 6 5 9 7 4 3 1 2 8	9 6 1 4 2 5 10 7 3 8
8 4 7 9 10 6 3 1 2 5	9 1 6 8 2 3 10 5 7 4
5 9 2 4 3 7 1 6 8 10	4 6 10 8 1 9 7 5 3 2
10 8 7 4 5 9 3 6 2 1	5 6 2 7 1 4 8 9 3 10
6 9 5 7 2 10 8 3 4 1	10 6 8 2 9 5 4 1 7 3
10 8 2 6 3 7 4 9 5 1	7 3 6 5 2 8 9 10 4 1
6 3 4 8 5 10 2 9 7 1	9 6 1 3 4 8 10 2 5 7
9 4 7 10 6 2 1 5 8 3	2 6 5 1 9 8 7 3 4 10
9 3 7 8 2 5 4 6 1 10	8 4 7 9 1 10 5 6 2 3
3 2 9 4 1 6 10 7 8 5	8 1 9 10 3 5 4 2 6 7
7 2 10 9 1 4 3 5 8 6	9 10 8 1 4 2 6 7 3 5
1 10 3 4 6 2 9 8 7 5	2 9 8 4 5 6 1 10 3 7
7 9 2 1 6 3 10 4 8 5	10 2 1 7 4 9 8 5 3 6
1 5 6 9 2 8 3 7 4 10	10 6 3 1 8 9 7 2 4 5
10 4 7 5 8 3 1 6 2 9	4 8 7 9 6 10 3 5 2 1
9 10 6 5 7 3 1 8 4 2	2 3 1 8 9 5 6 10 4 7
4 8 2 9 10 1 7 5 3 6	9 2 8 1 10 6 3 5 7 4
3 8 5 2 9 7 4 6 10 1	10 5 8 2 6 9 4 1 7 3
9 3 4 7 1 6 10 2 5 8	3 4 1 2 9 5 8 10 7 6
1 4 7 3 8 10 5 6 9 2	1 3 4 2 6 10 9 5 8 7
3 7 8 9 5 4 2 6 10 1	3 4 5 8 9 7 10 1 2 6
4 9 10 2 3 1 8 5 6 7	7 5 10 2 1 8 6 9 3 4
9 5 10 4 1 6 7 8 3 2	2 5 7 3 6 1 8 10 4 9
9 3 6 4 7 2 5 1 10 8	2 4 9 3 5 1 7 8 6 10
5 1 3 6 7 8 9 10 2 4	3 5 8 1 2 4 7 6 9 10
2 8 5 4 6 10 1 3 7 9	2 4 6 8 7 1 9 3 10 5
9 10 8 6 7 2 3 1 4 5	8 5 6 4 1 7 10 2 3 9
5 8 10 3 6 9 1 7 4 2	7 3 10 5 6 4 2 9 8 1
5 10 7 2 4 8 3 1 9 6	9 7 8 3 6 5 1 4 10 2
6 4 3 2 10 5 7 8 9 1	9 8 1 10 5 3 2 7 4 6
4 7 8 1 6 10 2 9 5 3	8 10 7 4 6 3 5 2 9 1
1 7 8 9 3 4 2 6 5 10	2 10 4 1 9 7 6 3 8 5
6 9 1 3 7 2 5 8 4 10	7 9 3 6 4 10 1 2 5 8
5 9 2 7 10 3 4 6 1 8	8 1 9 2 3 7 4 10 5 6

**Random permutations (size 5)**

5 2 3 4 1	4 2 3 5 1	3 1 5 4 2
2 5 1 3 4	3 1 2 4 5	5 3 2 4 1
4 5 3 2 1	2 1 4 3 5	2 1 5 4 3
2 5 3 4 1	1 5 3 4 2	1 4 3 2 5
5 2 3 1 4	5 3 4 1 2	2 5 4 3 1
3 5 1 4 2	5 4 3 2 1	5 1 4 3 2
2 3 4 1 5	4 5 2 3 1	2 5 3 4 1
1 2 5 4 3	2 4 5 3 1	3 4 1 2 5
2 4 1 5 3	1 2 3 5 4	4 1 2 5 3
2 5 1 3 4	3 5 2 1 4	5 4 2 1 3
3 4 1 5 2	5 2 3 1 4	3 2 1 5 4
2 1 5 3 4	3 1 4 2 5	1 4 5 2 3
2 4 1 3 5	3 1 5 2 4	1 2 3 5 4
5 1 3 2 4	4 2 3 5 1	4 5 1 3 2
3 2 4 1 5	1 5 3 4 2	1 3 5 2 4
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3 4 1 5 2	4 3 5 1 2	1 5 2 4 3
2 5 1 3 4	1 2 3 5 4	2 3 5 1 4



