Qualification Accredited

OCR
Oxford Cambridge and RSA

AS LEVEL

Examiners' report

FURTHER MATHEMATICS A

H235

For first teaching in 2017

Y535/01 Summer 2024 series

Contents

Introduction 4

Paper Y535/01 series overview 5

Question 2 (a) 6

Question 2 (b) 6

Question 3 (a), (b) (i), (b) (ii), (b) (iii) 7

Question 4 (a), (b) 9

Question 5 (a), (b), (c) (i), (ii) 10

Question 5 (d) 12

Question 6 (a) (i), (ii), (b) 12

Question 7 (a), (b), (c), (d) 14

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper Y535/01 series overview

Y535 is one of the optional units for AS Further Mathematics A. This summer candidates seemed to be better prepared for the paper, compared to previous years.

Some questions produced a strong response, for instance Question 1 on number bases, Question 2 (a) on using the vector product to calculate the area of a triangle, Question 3 (a) on stationary points of a 3-D surface, Question 4 (a) on Fibonacci numbers. Other questions, although less successful, had accessible marks, for example Question 5 on matrix groups and Question 7, the modelling question. There were also questions which candidates found very challenging, like Question 3 (b) (iii) on sketching sections of a 3D surface having calculated partial derivatives in the neighbourhood of a stationary point; Question 4 (b) on induction with a twist (n is a multiple of 4), Question 6 on number theory. Questions requiring an explanation, a justification or a supporting argument or containing elements of problem solving are bound to increase the level of challenge.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• read the questions carefully to make sure that they used the requested method and that they had satisfied all requirements • were able to use and apply standard techniques with confidence • made good use of Cayley tables in group questions • were efficient and effective in applying their knowledge, choosing the simplest method to arrive at the solution • communicated well using mathematical language correctly • understood the issues in the modelling question (Question 7) • used calculators appropriately and efficiently.	• made arithmetic/algebraic errors which prevented them from gaining the easiest marks in the paper • did not show sufficient working to justify their solutions in 'determine', 'show that' or 'prove' questions • showed gaps in their knowledge of standard techniques.

Question 2 (a)

2 The points A and B have position vectors **a** and **b** relative to the origin O. It is given that

$\mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ 3\lambda \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} \lambda \\ -4 \\ 6 \end{pmatrix}$, where λ is a real parameter.

(a) In the case when $\lambda = 3$, determine the area of triangle OAB. **[4]**

In this question, which was about forming a vector product in component form and using it to find the area of a triangle, the vast majority of candidates gained 3 or 4 marks. Occasionally, incorrect computation of the vector product led to the loss of an accuracy mark. Candidates often forgot to divide the magnitude of their vector product by two to produce the area of a triangle.

Misconception

A significant number of candidates thought that the magnitude of the vector product gives the area of a triangle – or did not read the question carefully.

Question 2 (b)

(b) Determine the value of λ for which $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. **[2]**

The majority of candidates were able to gain 1 mark in this part question by either calculating a vector product directly or by deducing that the two given vectors are parallel. In either case, only about a third of the candidates remembered or knew to check that the relationship found was valid for all three components of the vector(s).

Question 3 (a), (b) (i), (b) (ii), (b) (iii)

3 The surface S has equation $z = \mathrm{f}(x, y)$, where $\mathrm{f}(x, y) = 4x^2y - 6xy^2 - \frac{1}{12}x^4$ for all real values of x and y. You are given that S has a stationary point at the origin, O, and a second stationary point at the point $P(a, b, c)$, where $c = \mathrm{f}(a, b)$.

(a) Determine the values of a, b and c. **[6]**

(b) Throughout this part, take the values of a and b to be those found in part **(a)**.

(i) Evaluate f_x at the points $U_1(a-0.1, b, \mathrm{f}(a-0.1, b))$ and $U_2(a+0.1, b, \mathrm{f}(a+0.1, b))$. **[2]**

(ii) Evaluate f_y at the points $V_1(a, b-0.1, \mathrm{f}(a, b-0.1))$ and $V_2(a, b+0.1, \mathrm{f}(a, b+0.1))$. **[2]**

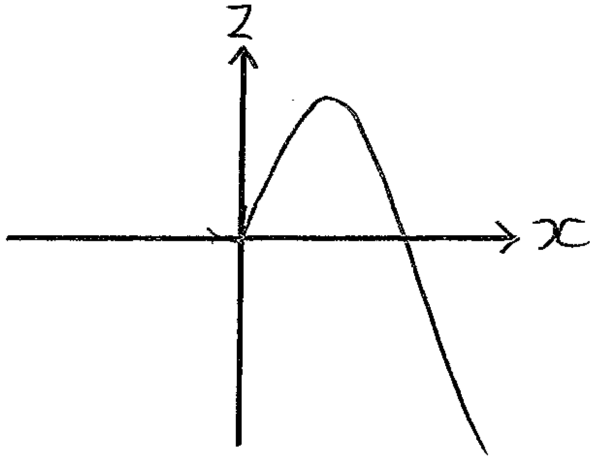
(iii) Use the answers to parts **(b)(i)** and **(b)(ii)** to sketch the portions of the sections of S, given by

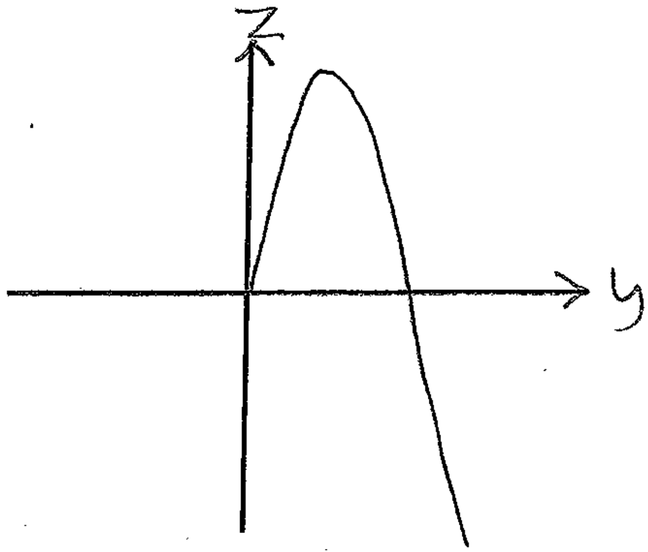
- $z = \mathrm{f}(x, b)$, for $|x-a| \leqslant 0.1$,
- $z = \mathrm{f}(a, y)$, for $|y-b| \leqslant 0.1$. **[2]**

Part (a) was a very well answered, with the vast majority of candidates achieving full marks by calculating first partial derivatives and equating them to zero. There was a noticeable improvement compared to a similar question in June 2023. On the other hand, in parts (b) (i) and (b) (ii), fewer than a half gained full marks and most obtained no mark. Similarly, very few candidates were able to sketch the correct sections in part (b) (iii), with many leaving this part completely empty.

Exemplar 1

3(b)(iii)





In Exemplar 1 the candidate gains both available marks. Both (portions of) sections are sketched as negative curvature quadratics and axes are properly labelled. Their position is not required to be accurate.

Assessment for learning

Candidates would benefit from further practice on sketching sections and contours (OCR Ref. 8.05c).

Misconception

A significant number of candidates drew circles instead of quadratics.

Question 4 (a), (b)

4 The first five terms of the Fibonacci sequence, $\{F_n\}$, where $n \geqslant 1$, are $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3$ and $F_5 = 5$.

(a) Use the recurrence definition of the Fibonacci sequence, $F_{n+1} = F_n + F_{n-1}$, to express F_{n+4} in terms of F_n and F_{n-1}. **[2]**

(b) Hence prove by induction that F_n is a multiple of 3 when n is a multiple of 4. **[3]**

In part (a), candidates demonstrated agility in working with Fibonacci numbers and sequences, with approximately three quarters achieving 2 marks. In part (b) on induction, about two thirds of responses gained at least 1 mark and one in ten candidates gained full marks. Responses showed some difficulty in dealing with n as a multiple of 4 and a tendency to consider the recurrence relation for n+1 rather than 4k+4 or 4(k+1). Often, responses did not receive the mark for the basic case as the conclusion that F_4 or F_8 is a multiple of 3 was missing.

This question assesses three strands of AO2 (Reason, interpret and communicate mathematically): construct rigorous mathematical arguments (including proofs), make deductions and explain their reasoning, so a certain level of rigour was required.

Question 5 (a), (b), (c) (i), (ii)

5 The set S consists of all 2×2 matrices having determinant 1 or −1. For instance, the matrices $\mathbf{P} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$, $\mathbf{Q} = -\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are elements of S. It is given that $\times_{\mathbf{M}}$ is the operation of matrix multiplication.

(a) State the identity element of S under $\times_{\mathbf{M}}$. **[1]**

The group G is generated by **P**, under $\times_{\mathbf{M}}$.

(b) Determine the order of G. **[2]**

The group H is generated by **Q** and **R**, also under $\times_{\mathbf{M}}$.

(c) **(i)** By finding each element of H, determine the order of H. **[4]**

(ii) List all the proper subgroups of H. **[3]**

Candidates found this question on groups rather challenging in general. Two strands of AO2 (Reason, interpret and communicate mathematically) are assessed, explain reasoning and make deductions. In part (a), almost every candidate stated that the identity matrix of order 2 is the identity of *S* and in part (b) the majority gained 2 marks out of two by giving the correct order with justification, correctly interpreting the command word 'Determine'. In part (c) (i), the vast majority gained at least 1 mark, but no candidate gained 4 marks, as in responses did not check whether some of the matrices generated via matrix products are repeated. In question (c) (ii), approximately one in ten students listed all the proper subgroups of *H* and just under a half of the candidates could not list any (correct) subgroups.

Assessment for learning

Thorough practice with different types of groups and their properties can help. Visual aids like Cayley tables can be useful for understanding the structure of groups and subgroups.

Exemplar 2

5(c)(i)	$Q = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ $R = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $QR = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$
	$RQ = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ ~~(Q)(R)~~ $QRQR = RQ$ $QQR = RQ$ $QRQ = R$
	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $RQR = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$
	$RRQ = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$

5(c)(i)	**(continued)**
	Q R QR RQ I
	~~6 elements~~ ~~order ≠ 6~~
	6 elements order 6

Exemplar 2 gains the first B mark for finding $\mathbf{Q}^2$ (labelled **RQR**), the second B mark for finding **QR** and the fourth B mark for stating the order of *H* and giving all the six elements. **RQ** is shown equal to $\mathbf{Q}^2\mathbf{R}$, but **QR** is not shown equal to $\mathbf{RQ}^2$; thus, the third B mark is not achieved.

Question 5 (d)

(d) State whether each of the following statements is true or false. Give a reason for each of your answers.

- G is abelian
- G is cyclic
- H is abelian
- H is cyclic **[4]**

In part (d) only a few candidates gave all four correct statements with correct justifications, but more than a half were able to gain either 1 or 2 marks. Here, two strands of AO2 ('Reason, interpret and communicate mathematically') are assessed, explain reasoning and use mathematical language and notation correctly, so generic statements like 'H is not abelian since matrix product is not commutative' are insufficient.

Question 6 (a) (i), (ii), (b)

6 For positive integers n, let $f(n) = 1 + 2^n + 4^n$.

(a) (i) Given that n is a multiple of 3, but **not** of 9, use the division algorithm to write down the two possible forms that n can take. **[1]**

(ii) Show that when n is a multiple of 3, but **not** of 9, $f(n)$ is a multiple of 73. **[6]**

(b) Determine the value of $f(n)$, modulo 73, in the case when n is a multiple of 9. **[2]**

Candidates found this problem-solving question (Translate problems in mathematical contexts into mathematical processes) generally very challenging.

In part (a) (i), on the division algorithm, just above one quarter gained the available mark; in parts (a) (ii) and (b), on finite modular arithmetic, very few candidates were able to show that f(n) is a multiple of 73 by recognising that they need to systematically check each different case, use index properties and work modulus 73.

Exemplar 3

6(a)(ii)

$f(n) = 1 + 2^n + 4^n$

$f(9k+3) = 1 + 2^{9k+3} + 4^{9k+3}$

$= 1 + 2^{9k} \times 2^3 + 4^{9k} \times 4^3$

$= 1 + 512^k \times 8 + 262144^k \times 64$

$= 1 + (7 \times 73 + 1)^k \times 8 + (3591 \times 73 + 1)^k \times 64$

~~$\therefore f(9k+3) = 1 + 8 \times (\text{multiple of } 73 + 1)^k + 64 \times (\text{multiple of } 73 + 1)^k$~~

$f(9k+3) \equiv 1 + 8 + 64 \pmod{73}$

$\equiv 73 \pmod{73}$

$\equiv 0 \pmod{73}$

$f(9q+6) = 1 + 2^{9q+6} + 4^{9q+6}$

$= 1 + 64(512)^k + 4096(262144)^k$

$= 1 + 64(7 \times 73 + 1)^k + 4096(3591 \times 73 + 1)^k$

$\equiv 1 + 64 \times 1 + 4096 \times 1 \pmod{73}$

$\equiv 4161$

$\equiv 73 \times 57 \equiv 0 \bmod (73)$

Exemplar 3 is an example of a response gaining full marks. Both cases are analysed and powers of 2 which are equivalent to 1 mod 73 expediently factored out.

Misconception

Often candidates parametrised *n* (a multiple of 3, but not of 9) as *n*=3*k*.

OCR support

For support material on sequences and series, see OCR Delivery Guides Section 8.02 Number Theory (AS and Stage 1), check-in tests and activities.

Question 7 (a), (b), (c), (d)

7 In a long-running biochemical experiment, an initial amount of 1200 mg of an enzyme is placed into a mixture. The model for the amount of enzyme present in the mixture suggests that, at the end of each **hour**, one-eighth of the amount of enzyme that was present at the start of that hour is used up due to chemical reactions within the mixture.

To compensate for this, at the end of each **six-hour** period of time, a further 500 mg of the enzyme is added to the mixture.

(a) Let n be the number of **six-hour** periods that have elapsed since the experiment began.

Explain how the amount of enzyme, E_n mg, in the mixture is given by the recurrence system $E_0 = 1200$ and $E_{n+1} = \left(\frac{7}{8}\right)^6 E_n + 500$ for $n \geqslant 0$. **[2]**

(b) Solve the recurrence system given in part **(a)** to obtain an exact expression for E_n in terms of n. **[5]**

(c) Hence determine, in the long term, the amount of enzyme in the mixture. Give your answer correct to **3** significant figures. **[2]**

(d) **In this question you must show detailed reasoning.**

The long-running experiment is then repeated. This time a new requirement is added that the amount of enzyme present in the mixture must always be at least 500 mg.

Show that the new requirement ceases to be satisfied before 12 hours have elapsed. **[3]**

In general candidates did well in this modelling question. In part (a) (AO 3.3: translate situations in context into mathematical models) the majority gained two marks. Then in part (b), more than a half of the candidates achieved full marks, demonstrating good skills in solving recurrence relations and using the mathematical model (AO 3.4). However, some struggled with justifying the long-term behaviour of the system in part (c), where they are supposed to explain their reasoning (AO 2.4) and make inferences (AO 2.2b).

Finally, most candidates found part (d) challenging and fewer than a half gained any mark. Those who were successful were able to iterate their hourly sequence (by calculator) and make the jump to the start of the second six-hour period by adding 500 (AO 3.1a: translate problems in mathematical contexts into mathematical processes). They would then repeat the process for the second six-hour stretch (AO 3.5a: evaluate the outcomes of models in context) and conclude that the requirement is not met (AO 3.5b recognise the limitations of models).

Assessment for learning

Students should practise solving different types of recurrence relations and understanding their implications. Visualising the long-term behaviour through graphs or tables can aid comprehension.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don't have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students' results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

We've made it easier for Exams Officers to download copies of your candidates' completed papers or 'scripts'. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular professional development courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is a free test-building platform, providing unlimited users exclusively for staff at OCR centres with an Interchange account.

Choose from a large bank of questions to build personalised tests and custom mark schemes, with the option to add custom cover pages to simulate real examinations. You can also edit and download complete past papers.

Find out more.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals (examined units only).

Find out more.

You will need an Interchange account to access our digital products. If you do not have an Interchange account please contact your centre administrator (usually the Exams Officer) to request a username, or nominate an existing Interchange user in your department.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223 553998

Alternatively, you can email us on
support@ocr.org.uk

For more information visit

- **ocr.org.uk/qualifications/resource-finder**
- **ocr.org.uk**
- **facebook.com/ocrexams**
- **twitter.com/ocrexams**
- **instagram.com/ocrexaminations**
- **linkedin.com/company/ocr**
- **youtube.com/ocrexams**

We really value your feedback

Click to send us an autogenerated email about this resource. Add comments if you want to. Let us know how we can improve this resource or what else you need. Your email address will not be used or shared for any marketing purposes.

I like this **I dislike this**

Please note – web links are correct at date of publication but other websites may change over time. If you have any problems with a link you may want to navigate to that organisation's website for a direct search.

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT