OCR
Oxford Cambridge and RSA

Thursday 16 January 2025 – Afternoon

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2501

C305/2501

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

05823/23

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

Date of birth | D | D | M | M | Y | Y | Y | Y |

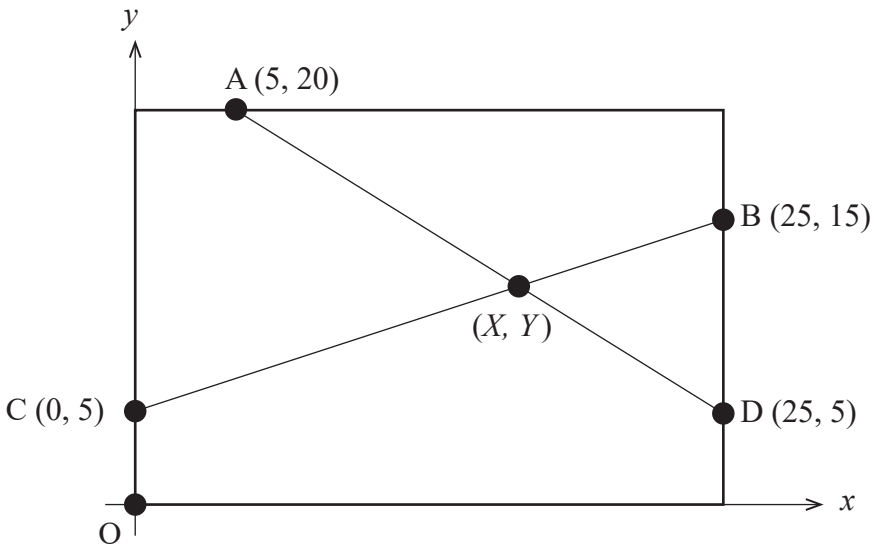
INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined page at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. When a numerical value is needed use g = 9.8 unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A horizontal, rectangular plot of land measuring 25 m by 20 m aligned within a Cartesian axis system (x, y) with the origin at point O is shown below. Surveyors have placed four marker stakes on the plot at points A (5, 20), B (25, 15), C (0, 5) and D (25, 5). Coordinates are given in metres.

Two straight marker ropes join the stakes A and D and the stakes B and C. The point at which these ropes cross has coordinates (X, Y).



Not to scale

(a) Show that X and Y satisfy the following linear simultaneous equations.

$3X + 4Y = 95$

$2X - 5Y = -25$

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(b) Express these equations in matrix notation.

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(c) Use an inverse matrix method to calculate the values X and Y.

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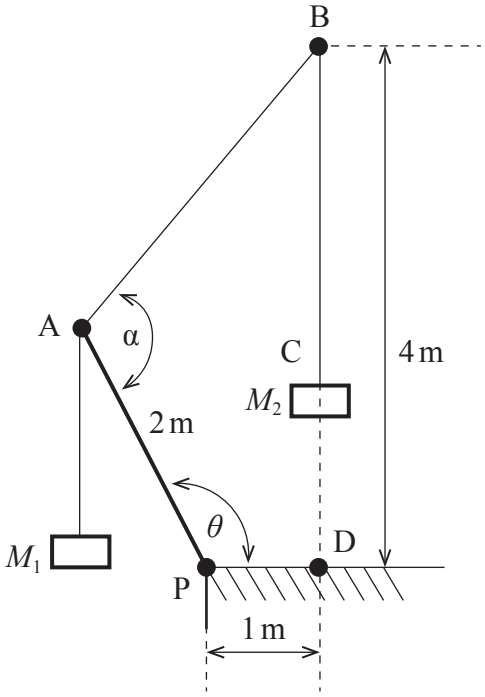
2 The diagram below shows a simple hoist arrangement for supporting a freely hanging mass of M_1 kg. The hoist includes a rigid jib, PA, which can freely rotate about a pivot at point P. The mass M_1 is attached to A by means of an inextensible cable. Another inextensible cable, ABC, attached to the jib at A, passes over a securely mounted frictionless pulley at B and extends to C where a freely hanging counter-balance mass of M_2 kg is attached. You can assume that the masses of the jib and cables can be neglected.

Point D is on the same horizontal level as P and vertically below B, such that DP is 1 m and DB is 4 m. The length of the jib, PA, is 2 m, the angle APD is θ and the angle BAP is α.



Not to scale

(a) By taking moments about P for the equilibrium of the jib, PA, calculate the ratio $M_2 : M_1$ that will allow the jib to remain in equilibrium in a horizontal position with θ equal to 180°. The forces in two cables attached to A and the thrust along PA cause the jib to be in equilibrium.

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(b) Calculate the distance between points P and B.

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For parts **(c)**, **(d)** and **(e)** of this question $\theta = 120°$.

(c) Show that length AB is approximately 3.024 m.

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(d) Calculate angle α.

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(e) By taking moments about P for the equilibrium of the jib, PA, calculate the value of M_2 that will allow the jib to remain supported when $M_1 = 50$.

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(a) A body travelling round a circular path with a radius of r m has a constant linear speed of $v\,\text{m}\,\text{s}^{-1}$ and a constant angular speed of $\omega\,\text{rad}\,\text{s}^{-1}$ (radians per second).

Express v in terms r and ω.

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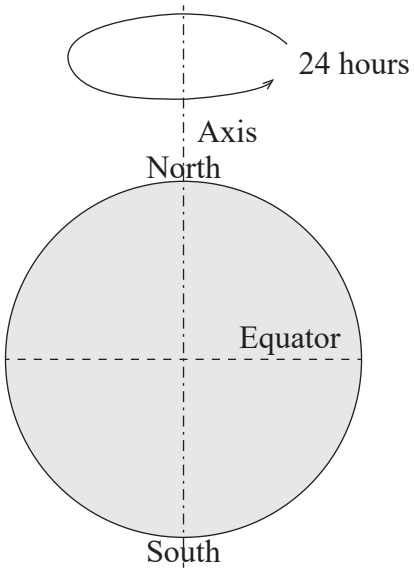
For the rest of this question you should assume that the Earth is a perfect sphere with a radius r_e of 6371 km. You should also assume that the Earth makes one complete revolution about its axis in 24 hours as shown below.



(b) A low orbit satellite travelling in a circular path with a constant linear speed at a height of 700 km above the Earth requires 100 minutes to make a complete orbit.

Calculate the linear speed of this satellite.

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(c) A GPS satellite travelling in a circular path with a linear speed of 14 000 km per hour makes a complete orbit about the Earth in 12 hours.

Calculate the satellite's height above the Earth.

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(d) Gravitational acceleration towards the centre of the Earth diminishes as the height above the Earth's surface increases. At a height of h km above the Earth's surface, gravitational acceleration, g_h km s^{-2}, can be approximated by

$$g_h \approx g \times 10^{-3} \left(\frac{r_e}{r_e + h} \right)^2,$$

where $g = 9.8$ and $r_e = 6371$.

For a satellite moving in a circular path at a height of h km above the Earth with a linear speed of v km s^{-1}, the acceleration, a km s^{-2}, towards the centre of the Earth is given by

$$a = \frac{v^2}{r_e + h}.$$

A geostationary satellite follows a circular path and remains directly above a fixed point on the Earth's equator. The gravitational acceleration of the satellite is equal to the acceleration towards the centre of the Earth, that is $g_h = a$.

For a geostationary satellite show that

$$h = 0.1 \times \sqrt[3]{g \left(\frac{r_e}{\omega} \right)^2} - r_e,$$

where ω is the angular speed of both the satellite and the Earth about its axis.

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4 The passive impedances, $Z\ \Omega$, of a resistor and an inductor connected to an alternating input voltage with a frequency of ω radians per second are given below.

Resistor: $Z_R = R$ where $R\ \Omega$ is resistance

Inductor: $Z_L = \omega L\text{j}$ where L H is inductance

where $\text{j} = \sqrt{-1}$.

When these components are connected in parallel the total passive impedance, Z_T, satisfies the relationship

$$\frac{1}{Z_T} = \frac{1}{Z_R} + \frac{1}{Z_L}.$$

(a) When the alternating voltage has a frequency of 50 Hz, show that $\omega = 314.16$.

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(b) Show that

$$Z_T = \frac{R\omega L\left(\omega L + R\text{j}\right)}{(\omega L)^2 + R^2}.$$

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(c) Given that $R = 5$ and $L = 6\times10^{-3}$, express Z_T in the form $a + b$j where a and b are real values.

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(d) Represent Z_T on an Argand diagram.

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(e) Express Z_T in the form $r(\cos\theta + \text{j}\sin\theta)$, where r is a real value and θ is an angle in degrees.

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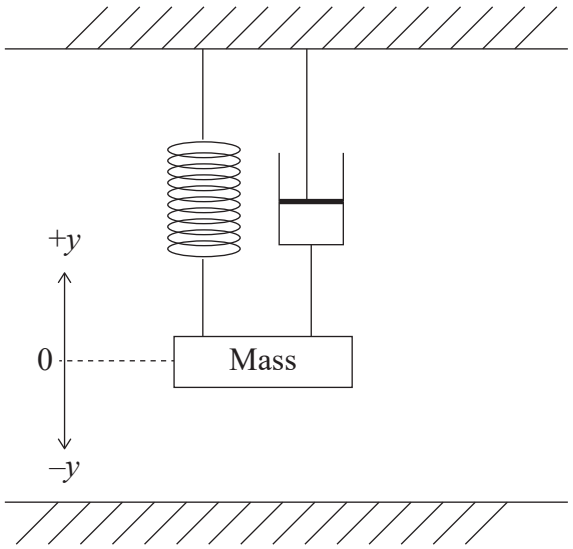
5 The diagram below shows a mass, spring and damper mechanism supported in a vertical plane. The mechanism shown is situated at rest with the centre of the mass aligned with the origin of a y-axis.



When the mass is lifted above the origin and allowed to fall it will pass the origin and then oscillate about the position where $y = 0$ with ever decreasing amplitude. The position of the centre of the mass, y, at time t after the mass is released is given by

$y = e^{-2t}(A \cos 2t + B \sin 2t)$, for $t \geqslant 0$,

where A and B are constants.

(a) By differentiation find a formula for the speed of the mass at time t.

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(b) The mass is lifted to the position where $y = 1$. When $t = 0$ the mass is released from rest.

Use this information to calculate the values of A and B.

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(c) Using the equation for y given above together with the values for A and B you found in part **(b)**, show that

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 0.$$

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(d) Calculate the furthest distance the centre of the mass will fall below the y origin after it is released.

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(e) With reference to your calculations, explain whether the mechanism is heavily damped or lightly damped.

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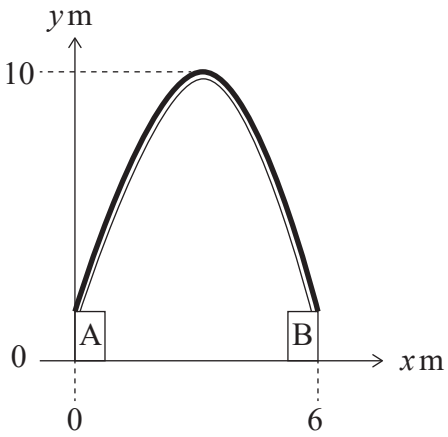
6 The diagram below shows a symmetrical, ceremonial arch constructed on top of two equally high support plinths, A and B. The plinths are mounted on horizontal ground and the points where the arch rests on the plinths are 6 m apart.



Not to scale

The shape of the arch is aligned within a Cartesian coordinate system (x, y) and modelled by the equation

$$2\frac{dy}{dx} = e^{-(x-3)} - e^{(x-3)} \text{ for } 0 \leqslant x \leqslant 6,$$

where x m is the horizontal distance from left side of plinth A and y m is the height of the arch above ground level. The maximum height of the arch above ground level is 10 m.

(a) Determine the equation of the arch in the form $y = f(x)$.

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(b) Calculate the height of each of the two plinths.

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(c) Calculate the **two** values of x for which the arch is 5 m above the ground.

You may use the substitution $X = e^{x-3}$.

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Distance 1 .....

Distance 2 .....

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(a) Show that

$$\int x \cos ax \, \mathrm{d}x = \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} + C$$

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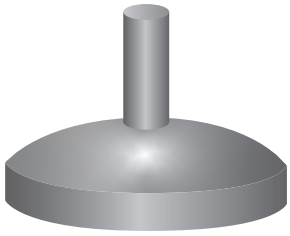
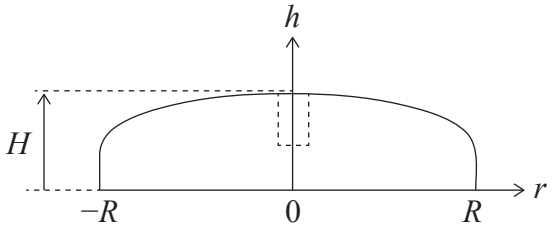
A hollow cylindrical support for a sunshade together with part of the pole is shown below. The support is filled with water.



The support, which is made of plastic, has a flat base and an outside radius of R mm as shown in the side elevation view below. The side elevation also shows Cartesian axes r and h, with the origin at the centre of the base of the support.



The support has a height of h mm at a distance of r mm from its centre. The value of h is given by

$$h = H \cos\left(\frac{r}{A}\right), \qquad \text{for } -R \leqslant r \leqslant R,$$

where H and A are constants, both measured in millimetres.

Ignoring the space in the support occupied by the pole and the thickness of the plastic, the maximum volume of water in the support, V mm^3, is given by

$$V = 2\pi \int_0^R rh\mathrm{d}r = 2\pi H \int_0^R r\cos\left(\frac{r}{A}\right)\mathrm{d}r.$$

(b) Calculate V when R =150, H =120 and A =150. Give your answer in litres.

For this calculation you are advised to set your calculator in radian mode. One cubic metre is equivalent to 1000 litres.

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END OF QUESTION PAPER

EXTRA ANSWER SPACE

If you need extra space use this lined page. You must write the question numbers clearly in the margin.