Qualification Accredited

OCR
Oxford Cambridge and RSA

Examiners' report

A LEVEL

MATHEMATICS A

OCR Level 3 Advanced GCE in Mathematics A

H240

For first teaching in 2017

H240/03 Summer 2025 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper 3 series overview

This is the third examination component for the A Level examination for GCE Mathematics A. It is a two-hour paper consisting of 100 marks which tests content from Pure Mathematics (Section A, 50 marks) and Mechanics (Section B, 50 marks). Pure Mathematics content is tested on all three papers, and any topic could be tested on any of the three papers. Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper, as part of their linear A Level Mathematics qualification, produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on both the pure and mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

OCR support

Section 2d in the Mathematics A H240 specification provides guidance on the command words used in examination questions, with additional guidance provided in a series of blogs written by the Subject Advisor team.

Students can download a useful summary sheet from the qualification website.

There was one full question (Question 9) and two-part questions (Questions 3 (c) and 4 (b)) on this paper which began with the demand '**In this question you must show detailed reasoning**'; to quote the specification that 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain enough detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method.' The specification then considers several examples which centres should consider so that future candidates understand exactly what is required when this request appears in future series.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word features in Questions 10 (c), 11 (b), 11 (c) and 12 (b).

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 4 (a), 5 (a), 5 (b), 5 (c), 6 (a), 6 (b), 6 (d), 10 (a), 11 (a), 12 (a) and 12 (c).

While there is no specific level of working needed to justify answers to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• made efficient use of calculators • understood the level of response required for command words used in the questions • read questions carefully and provided the answers that were requested • used formal mathematical notation and language correctly.	• made careless mistakes in algebraic manipulation • used imprecise notation or language • provided mathematical working that was correct but did not answer the specific question that was being asked • did not give sufficient evidence in 'Show that', 'Determine' and 'Detailed Reasoning' questions.

Section A overview

Content from the Pure section of the specification may be assessed on any of the three papers of H240. Most candidates appeared well prepared for the pure content, with method marks being given, but there were several places where a more concise use of algebraic notation and language may have led to more of the corresponding accuracy marks being available.

Question 1 (a)

1 **(a)** Express $3x^2 - 12x + 17$ in the form $a(x-b)^2 + c$ where a, b and c are constants. **[3]**

This part was answered extremely well with most candidates scoring at least 2 marks for expressing the given quadratic as $3(x-2)^2 + \cdots$. The most common error was the lack of a suitable bracket so often leading to a value of 13 for *c* rather than the correct 5. Unfortunately, some candidates gave answers that were not in the form requested (so missing the squared off the bracket or including the squared inside the bracket).

Question 1 (b)

(b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. **[1]**

Most candidates stated the coordinates of the minimum point correctly as (2, 5) (either from using their answer to part (a) or restarting in this part). Others gained the mark here on the follow through from their completed the square form seen in part (a). The two most common errors seen were $(-2, 5)$ and $(2, -5)$.

Question 1 (c)

(c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. **[1]**

Many candidates struggled to realise that the equation of the normal to the curve at its minimum point was equivalent to asking for the equation of the line of symmetry. Many candidates unsuccessfully attempted to use the result $y - y_1 = -\frac{1}{f'(x_1)}(x - x_1)$ to find this equation (even though the command was to 'state' and there only being 1 mark available). The most common incorrect answer seen by examiners was $y = 5$ (which was the equation of the tangent to the curve at its minimum point).

Question 2 (a)

2 **(a)** The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x-direction.

Write down the equation of the curve after it has been translated. **[2]**

This part was answered extremely well with many correct answers seen. The most common errors included $y = \frac{1}{(x+4)^3}$, $y = \frac{1}{x^3} \pm 4$. The question has explicitly asked for 'the equation' so simply stating the expression $\frac{1}{(x-4)^3}$ only gained the method mark.

Question 2 (b)

(b) The curve $y = 5x^2$ is stretched parallel to the y-axis with scale factor 4.

The point on the curve $y = 5x^2$ with x-coordinate 3 is transformed to the point P.

Write down the coordinates of P. **[2]**

Like part (a) this was answered extremely well with many candidates giving the correct answer of (3, 180). While most candidates correctly realised that the *x*-coordinate was invariant, many struggled to correctly state the corresponding *y*-coordinate. The question asked for coordinates, but examiners could award the marks if candidates made it explicitly clear what values of *x* and *y* were being declaring in this part.

Question 3 (a)

3 **(a)** Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet. **[2]**

Most candidates drew the correct V-shape with the cusp on the positive *x*-axis. The most common errors included: the cusp appearing on the negative *x*-axis, the graph not extending into the second quadrant (so stopping on the positive *y*-axis) or not labelling either or both of the *x*- and *y*-intercepts.

Question 3 (b)

(b) Solve the inequality $|2x - 5| < 1$. **[2]**

The part was answered extremely well although there were a significant minority of candidates that struggled with manipulating negative terms in an inequality with their solution of $5 - 2x < 1$ incorrectly stated as $x < 2$ or obtaining an inequality with a negative numerical value.

Question 3 (c)

(c) In this question you must show detailed reasoning.

Hence find **all** the possible integer values N that satisfy the inequality

$\left|2e^{0.1N} - 5\right| < 1.$ **[3]**

Although most candidates realised that importance of their answer to part (b) and started by stating that $2 < e^{01.N} < 3$, many re-started in this part or did not see the significance of the previous part. While many candidates correctly stated that $10\ln 2 < N < 10\ln 3$ many did not read the question carefully and either left this as their final answer or gave an answer in a form that did not make it explicitly clear what positive integers satisfied the given inequality (e.g. an answer of $7 \le N \le 10$ was far too common). Finally, as this question required detailed reasoning (DR) many went straight from $10\ln 2 < N < 10\ln 3$ to *N* = 7, 8, 9 and 10 without first showing the critical values of 6.93… and 10.98….

Question 4 (a)

4 Functions f and g are defined for all real values of x by

$f(x) = \frac{x-k}{2}$ and $g(x) = x^2 + kx + 5$, where k is a constant.

You are given that the equation $f^{-1}g(x) = 4 - 2kx$ has real distinct roots.

(a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. **[5]**

Most candidates correctly found the inverse of f and applied the correct order of operations to obtain $f^{-1}g(x) = 2x^2 + 2kx + 10 + k$ although some candidates incorrectly found $fg(x)$ and then attempted to find the inverse of this, or simply equated $fg(x)$ to $4 - 2kx$. A significant number of candidates incorrectly considered the equation $f^{-1}g(x) = 0$. Of those that did consider $f^{-1}g(x) = 4 - 2kx$ most obtained the correct three-term quadratic in x and were successful in deriving the given inequality by considering the discriminant > 0. As this was a given answer any incorrect working seen (or setting the discriminant > 0 after dividing by 8) meant that the final accuracy mark could not be awarded.

Question 4 (b)

(b) In this question you must show detailed reasoning.

Hence find the set of values of k. Give your answer in set notation. **[3]**

While many candidates scored full marks in this part, some did not show a correct method for solving this quadratic inequality even though the question clearly said that 'detailed reasoning' must be shown. It was also the case that many candidates struggled with the requirement to give their answer in set notation with many either leaving their answer as $k < -\frac{3}{2}, k > 2$, giving an answer using interval notation, or using the symbol for 'intersection' rather than the correct answer of $\left\{k: k < -\frac{3}{2}\right\} \cup \{k: k > 2\}$. It was also noted by examiners that many candidates defaulted to giving their solution in terms of x rather than k.

Question 5 (a)

5 **(a)** Use the substitution $u = \cos x$ to show that

$$\int \frac{1+\cos x}{\sin x} \mathrm{d}x = \ln(1-\cos x) + c.$$ **[4]**

This part was answered relatively well with many candidates scoring the first 3 marks for correctly applying the given substitution and obtaining $-\int \frac{1}{1-u} \mathrm{d}u$ from $-\int \frac{1+u}{1-u^2} \mathrm{d}u$ (by using a difference of two squares). When errors occurred, they were usually down to sign errors from applying the identity $\sin^2 x + \cos^2 x = 1$ when simplifying the denominator, or from incorrect differentiating $\cos x$ as $\sin x$. Those candidates who left the integral as $-\int \frac{1}{1-u} \mathrm{d}u$ were much more successful in obtaining the given answer than those who re-wrote this as $\int \frac{1}{u-1} \mathrm{d}u$. It was common in this part for many candidates to incorrectly claim that $\int \frac{1}{1-u} \mathrm{d}u = \ln(1-u) + c$.

Exemplar 1

5(a)

$$\frac{du}{dx} = -\sin x$$

$$dx = \frac{du}{-\sin x}$$

$$\int \frac{1+u}{(\sin x)(-\sin x)}\,du = \int \frac{1+u}{-\sin^2 x}\,du = \int \frac{1+u}{-(1-\cos^2 x)}\,du$$

$$u^2 = \cos^2 x$$

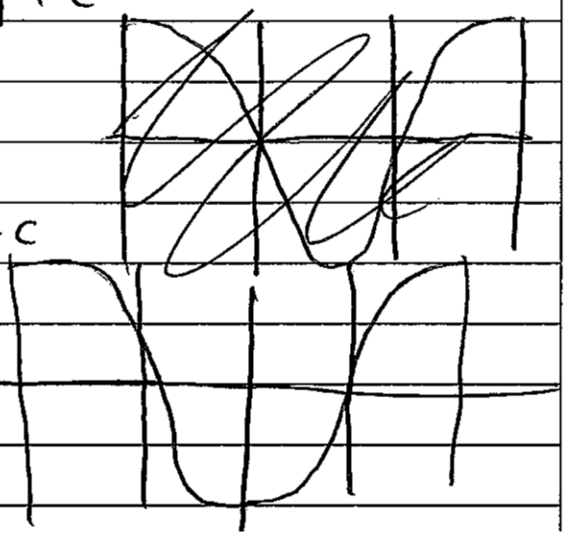
$$\int \frac{1+u}{-(1-u^2)}\,du = \int \frac{1+u}{u^2-1}\,du = \int \frac{\cancel{(1+u)}}{\cancel{(u+1)}(u-1)}\,du$$

$$= \int \frac{1}{u-1}\,du = \ln|u-1| + c$$

$$= \ln|\cos x - 1| + c$$

$$\cancel{= \ln(1-\cos x)} + c$$

$$= \ln(1-\cos x) + c$$



As discussed earlier, this exemplar illustrates the issues faced by candidates if they re-wrote the integral as $\int \frac{1}{u-1}\mathrm{d}u$. It was not acceptable to simply replace $|\ln(\cos x - 1)| + c$ with the given answer without some comment of why it was appropriate to do so.

Question 5 (b)

Fig. 1

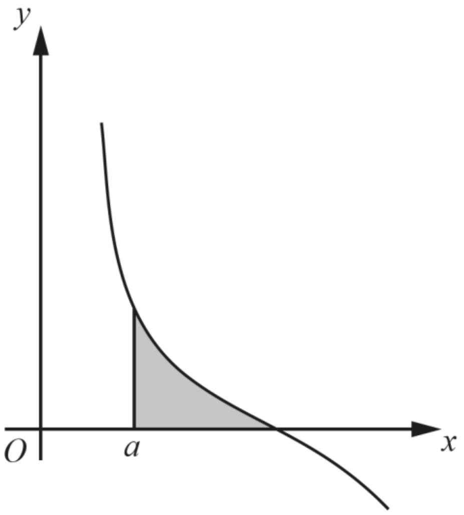


Fig. 1 shows part of the curve $y = \dfrac{1+\cos x}{\sin x}$.

The shaded region is bounded by the curve, the x-axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

(b) Show that the value of a satisfies the equation $a - \cos^{-1}(1 - 2e^{-a}) = 0$. **[4]**

While many candidates correctly found that the curve intersected the x-axis at π, some struggled with applying the limits correctly to the given answer from part (a), and a number of candidates incorrectly thought that the shaded region had an area of a^2. Of those that did correctly obtain the equation $\ln 2 - \ln(1-a) = a$ many struggled with the associated log work required to get the answer in the given form with many incorrect replacing $\ln 2 - \ln(1-a) = a$ with $2 - (1-a) = e^a$.

Question 5 (c)

(c) Show by calculation that the value of a lies between 1.1 and 1.2. **[2]**

Most candidates correctly used the change of sign test to show that *a* was between 1.1. and 1.2 (although it was clear to examiners that a few candidates had their calculators set in degrees rather than radians). The most common errors were to give the value at 1.2 as 0.38… rather than 0.038, not to indicate that a change of sign had occurred, or to not give a conclusion that mentioned that as there was a change of sign this indicated that *a* would lie in between 1.1 and 1.2.

Assessment for learning

We always use radians when considering gradients and enclosed areas for trigonometric functions. Using degrees would involve including extra conversion factor constants which would make the calculations more complex.

Question 5 (d)

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1 - 2\mathrm{e}^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to **2** decimal places. Show the result of each step of the iterative process. **[2]**

This part was answered extremely well with most candidates using the given iterative equation and showing sufficient iterations to obtain the correct value of 1.18 for the value of *a*. Occasionally, some candidates gave answers which implied that their calculator was set in degrees or did not give the answer to the required 2 decimal places.

Question 5 (e)

Fig. 2

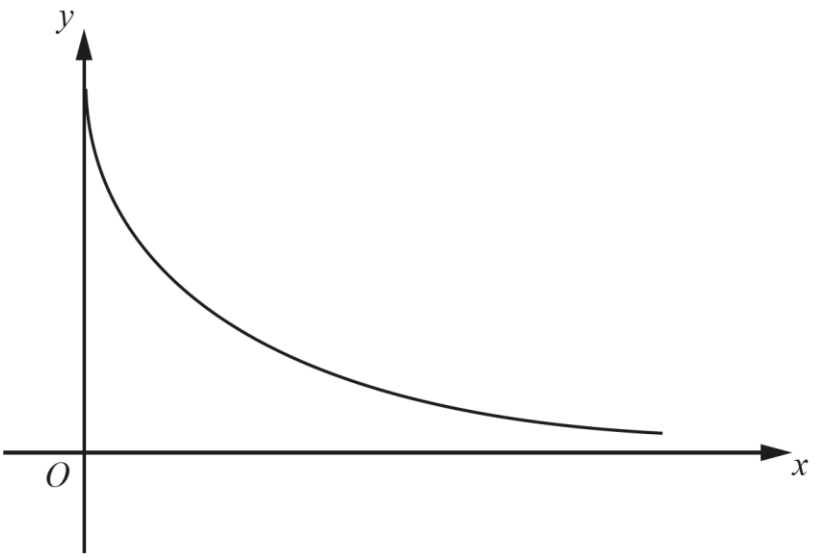


Fig. 2 shows the curve $y = \cos^{-1}(1 - 2\mathrm{e}^{-x})$.

(e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2\mathrm{e}^{-x})$ at the point where $x = a$, where a is the value found in part **(d)**, lies in the interval $(-1, 0)$. **[2]**

Candidates struggled with this part with very few able to give a correct explanation of why the gradient would lie in the given interval. Candidates needed to articulated that the given figure indicated that the curve had a negative gradient for all values of x and as the iterative formula converged to the value of *a* in part (d) this implied that $\left|\frac{\mathrm{d}}{\mathrm{d}x}(\cos^{-1}(1 - 2\mathrm{e}^{-x}))\right|_{x=a} < 1$ and so the gradient at the point $x = a$ would be in the interval $(-1,0)$.

Question 6 (a)

6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

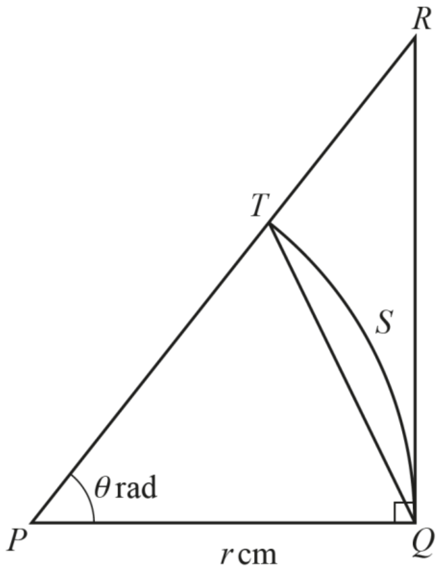
$\sin(A+B) = \sin A \cos B + \cos A \sin B$ and
$\sin(A-B) = \sin A \cos B - \cos A \sin B$.

(a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right).$$ **[1]**

A significant proportion of candidates struggled with deriving the given trigonometric identity with many not showing sufficient detail that $\sin C - \sin D$ was equal to the given result via $2\cos A \sin B$ together with the results that $A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$.

Question 6 (b)



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT, sector $PQST$ and triangle PQR, show that

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}.$$ **[4]**

While most candidates scored at least 1 mark for correctly stating the area of sector *PQST* or the area of triangle *PQT* many struggled with finding the area of triangle PQR with many not realising that $QR = r\tan\theta$. Very few candidates referred directly to the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector of a circle with θ measured in radians, with many either quoting incorrect formulae or having to convert from a corresponding formula with the angle given in degrees. Of those that did have a correct inequality, e.g. $\frac{1}{2}r^2\sin\theta < \frac{1}{2}r^2\theta < \frac{1}{2}r^2\tan\theta$ most then went on to correctly derive the given answer (although some did not show sufficient working or justification for why $\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$).

Assessment for learning

Candidates must be familiar with all the Formulae given in section 5d of the H240 specification (which will not be provided in the question paper).

Question 6 (c) (i)

(c) **(i)** Hence write down a similar inequality interval for the expression $\frac{\sin\theta}{\theta}$. **[1]**

Very few candidates scored the mark in this part with many simply stating that the similar inequality interval was $1 < \frac{\sin\theta}{\theta} < \cos\theta$.

Question 6 (c) (ii)

(ii) Hence state $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$. **[1]**

Although this part was better answered than part (i), most candidates seemed to be unaware of this limit either from previous work on differentiating sine and cosine from first principles, the small angle approximation for sine or from the sandwich inequality in the previous part.

Question 6 (d)

(d) Using parts **(a)** and **(c)(ii)**, show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. **[4]**

Although many candidates scored the first mark in this part for correctly considering $\frac{\sin(x+h)-\sin x}{h}$ many did not read the question carefully and instead used the standard approach of expanding the numerator using the compound angle formula instead of using the result from part (a). Of those that did use part (a) most were successful in correctly showing from first principles that the derivative of $\sin x$ was $\cos x$.

Question 6 (e)

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let	$y = \cos x$, where x is measured in radians.
	$y = \sin\left(\frac{\pi}{2} - x\right)$
so	$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(\frac{\pi}{2} - x\right)$
but	$\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$
therefore	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x$.

(e) Identify the error made by the student. **[1]**

It was pleasing that most candidates in this part could identify the error and most explained too what the correct line of working should have been.

Section B overview

One general point with regards to the answering of certain mechanics questions should be made in this overview. This is that unless told otherwise (so on this paper for Question 10) the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 10 or 9.81 (and this value is stated explicitly on the front cover of the examination paper).

Question 7 (a)

7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i}+3\mathbf{j})$ N and $(a\mathbf{i}+3b\mathbf{j})$ N. The acceleration of P is $(2\mathbf{i}-3\mathbf{j})\,\mathrm{m\,s^{-2}}$.

(a) Find the value of a and the value of b. **[3]**

While most candidates correctly set up a vector equation using **F** = m**a** to find the values of a and b it was surprising how many candidates either misread the second force vector as $(a\mathbf{i}+b\mathbf{j})$ or included gravity as an additional force. Those that did set up the correct equation usually obtained both correct values of a and b.

Question 7 (b)

At time $t = 0$ seconds the velocity of P is $\mathbf{u}\,\mathrm{m\,s^{-1}}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i}-6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{u}$. **[2]**

This part was answered better than part (a) although it many candidates did not spot the significance that the expression for the acceleration of P was independent of t and spent time using a calculus approach when it was much simpler and quicker to apply the constant acceleration formula **v** = **u** + **a**t.

Question 8 (a)

8

Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A.

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of R N and the air resistance acting on B has a constant magnitude of 0.3 N. The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is $9.2\,\mathrm{m\,s^{-2}}$ and the tension in the string is T N.

(a) By applying Newton's second law to B, find the value of T. **[2]**

Although answered well by most candidates some struggled to include all the forces acting on *B* with many forgetting to include the weight of *B*. Of those that did include all three forces acting on *B* many made sign error with the acceleration term which led to a common incorrect answer of 14.9 for the tension in the string.

Question 8 (b)

(b) Hence, or otherwise, find the value of R. **[2]**

Candidates were evenly split between applying Newton's second law to *A* or the system to answer this part. Those who attempted the former were usually more successful as many who attempted a 'system' equation included, in error, the tension in the string (or missed off one of the forces).

Question 9

9 In this question you must show detailed reasoning.

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v\,\mathrm{m\,s^{-1}}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is $15\,\mathrm{m\,s^{-2}}$. **[6]**

This was probably the best answered question in the mechanics section with many candidates scoring all 6 marks. As this part required DR many did not gain a mark for not showing the explicit substituting of $t = 10$ into their integrated expression to find the required distance travelled. The most common errors seen by examiners were those candidates who implied that $s = \frac{\mathrm{d}v}{\mathrm{d}t}$ and/or $a = \int v\,\mathrm{d}t$, and some incorrectly used the equations of constant acceleration even though the acceleration was variable.

Exemplar 2

9

$a = \frac{dv}{dt} = 3 + 1.2t \qquad a = 15 = 3 + 1.2t$

$12 = 1.2t \rightarrow 10 = t$

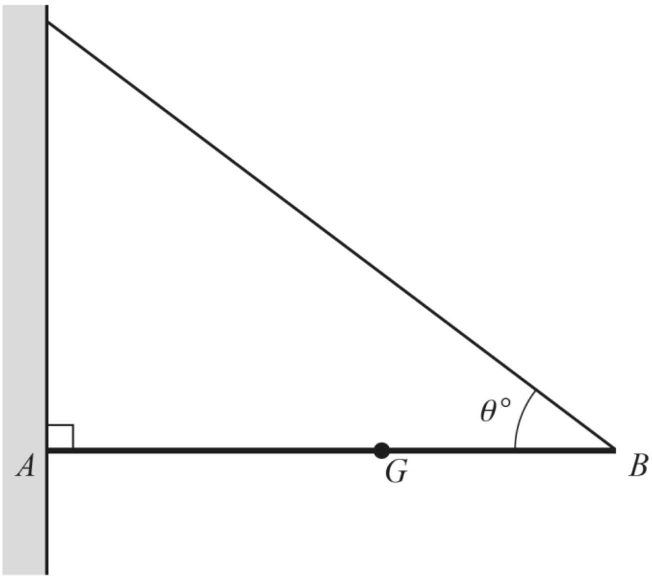
$s = \int v\,dt = \int_0^{10} (2 + 3t + 0.6t^2)\,dt$

$s = \left[2t + \frac{3}{2}t^2 + 0.2t^3\right]_0^{10} = 370 - 0 = \boxed{370\text{ m}}$

Exemplar 2 highlights the point made earlier about not showing sufficient working in a question that required detailed reasoning. The candidate has given no explicit indication of where the answer of 370 came from (as they have not shown the limits of 0 and 10 being applied to the cubic expression) and so 1 of the 6 marks could not be awarded.

Question 10 (a)

10



The diagram shows a **non-uniform** rod AB of mass 4 kg and length 2 m. The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B. The other end of the string is attached to the wall at a point vertically above A. The string is inclined at an angle of $\theta°$ to the horizontal, where $\sin\theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB.

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m.

(a) Show that the distance AG is 1.2 m. **[1]**

Most candidates correctly showed the required distance in this part by considering $AG \times 4g = 47.04$.

Question 10 (b)

(b) Find the tension in the string. **[2]**

While most candidates seem to realise that the most effective way of finding the tension in the string was to take moments about A, many forgot to include a distance or used a wrong angle.

Question 10 (c)

(c) Determine the magnitude of the contact force exerted on the rod by the wall at A. **[5]**

There was a common misconception in this part that the 'magnitude of the contact force exerted on the rod by the wall at *A*' referred only to a force with a horizontal component at the wall and not a vertical component too. Some candidates attempted to take moments again (most commonly about either points *G* or *B*) but these attempts were usually not as successful as those who resolved vertically and horizontally (and then completed the problem by used Pythagoras).

Assessment for learning

Candidates are strongly advised that when taking moments, they should make it clear to the examiners which point (in this case on the rod) they are taking moments about. In this, and the previous part, several candidates attempted to take moments about point *G* and/or *B* together with resolve forces vertically and/or horizontally; these attempts were not always as successful.

Question 11 (a)

11 In this question you should take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$.
The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})$m relative to O.

(a) Show that $y = 6x - 1.25x^2$. **[5]**

Candidates found this part very demanding with many making little progress beyond stating that $x = 2t$. Of those that realised that the initial vertical component of velocity could be found by applying $v = u + at$ with $v = -8, a = -10$ and $t = 2$, most were successful in obtaining the given cartesian equation by applying $s = ut + \frac{1}{2}at^2$ vertically and then eliminating t. Even though the question made it very clear that in this question candidates should use $g = 10$, many still used a value of 9.8. It must also be noted that examiners found some candidates' work extremely hard to follow (throughout the entirety of this question) due to poor notation when it came to dealing with vector and scalar quantities with statements such as $2\mathbf{i} - 8\mathbf{j} = u + 2(-10)$ being sadly all too common.

Question 11 (b)

(b) Determine the maximum height of P above the ground during its motion. **[3]**

It was surprising how many candidates did not use the given result from part (a) and instead went back to the *suvat* equations to find the maximum height of *P* above the ground during its motion. Of those that did use the equation from part (a), those that used calculus rather than completing the square were far more successful.

Question 11 (c)

(c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.

- the speed of P
- the direction of motion of P **[5]**

The responses to this part were mixed with many candidates believing that both the speed and direction of motion could be found by considering the vertical and horizontal displacement of *P* when $x = 2.5$. Of those that did use the velocity components many did not give sufficient detail when it came to the direction of motion; an angle alone was not sufficient as it needed to be clear if the angle was below or above the horizontal (or equivalently to the upward or downward vertical).

Question 11 (d)

In reality P does not move only under the influence of gravity but is also subject to air resistance.

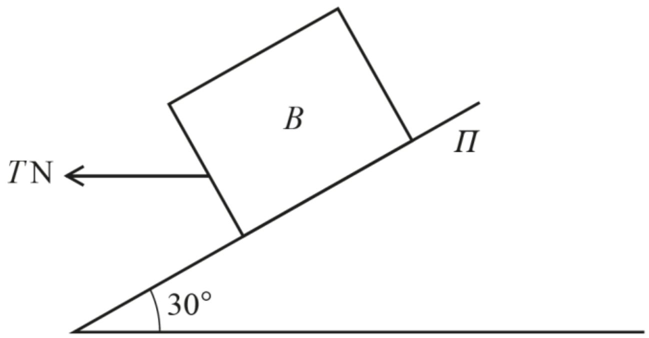
(d) Explain how this would affect your answer to part **(b)**. **[1]**

This part was generally answered well with most candidates correctly stating that the maximum height would be less than the value found in part (b). Surprisingly some candidates explained in detail what impact air resistance would have on all aspects of the ball's motion but did not explicitly say how it would affect the maximum height.

Question 12 (a)

12

Fig. 1



A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude T N, acting above a line of greatest slope, is applied to B (see **Fig. 1**).

The coefficient of friction between B and the plane is 0.8.

(a) Show that the value of T is 14.9, correct to **3** significant figures. **[6]**

Most candidates attempted to resolve parallel and perpendicular to the plane for *B,* but they were not always successful. Aside from the usual sin/cos confusion and sign errors a number of candidates stated the correct expression for the friction component, namely, $T\cos 30 + 10g\sin 30$ but incorrectly stated the normal contact force as $10g\cos 30$ only. Those candidates that annotated the copy of the diagram in the answer space were less likely to make mistakes with the components of each force. Those candidates who did resolve in both directions correctly usually went on to apply $F = \mu R$ correctly and obtained a correct expression (and hence value) for *T*.

While some candidates attempted to resolve vertically and horizontally (rather than perpendicular and parallel to the plane) these responses were rarely as successful.

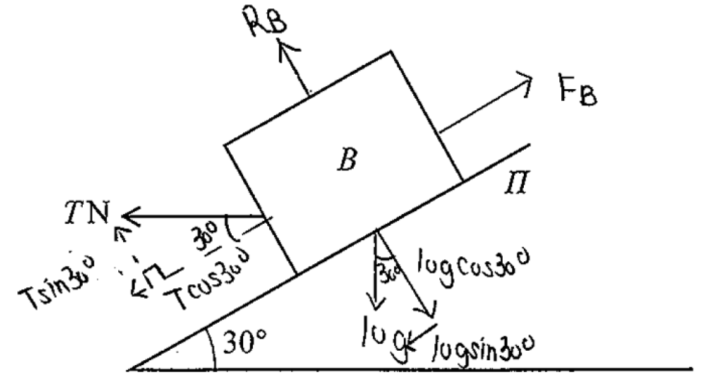
Furthermore, as this part was a 'show that' style question, examiners needed to see either the correct answer to at least 4 significant figures or a correct expression for *T* before stating the given answer of 14.9.

Assessment for learning

When resolving in two orthogonal directions candidates are reminded that if a component of a force appears in one term, then it must appear in the other term with the 'opposite' trig ratio. For example, in this question when resolving parallel to the plane the expression for the frictional force contains a component of the *T* N force ($T\cos 30$) and a component of the weight ($10g\sin 30$). Therefore, when resolving perpendicular to the plane the equivalent expression for the normal contact force must also contain a component of the *T* N force ($T\sin 30$) and a component of the weight ($10g\cos 30$) too.

Exemplar 3

12(a)



$F = \mu R \quad \mu = 0.8$

$F = T\cos 30° + 10g\sin 30°$

$\cancel{R}\ R + T\sin 30° = 10g\cos 30°$

$R = 10g\cos 30° - T\sin 30°$

$T\cos 30° + 10g\sin 30° = 0.8(10g\cos 30° - T\sin 30°)$

$T\cos 30° + 10g\sin 30° = 8g\cos 30° - 0.8T\sin 30°$

$T\cos 30° + 0.8T\sin 30° = 8g\cos 30° - 10g\sin 30°$

$T(\cos 30° + 0.8\sin 30°) = 8g\cos 30° - 10g\sin 30°$

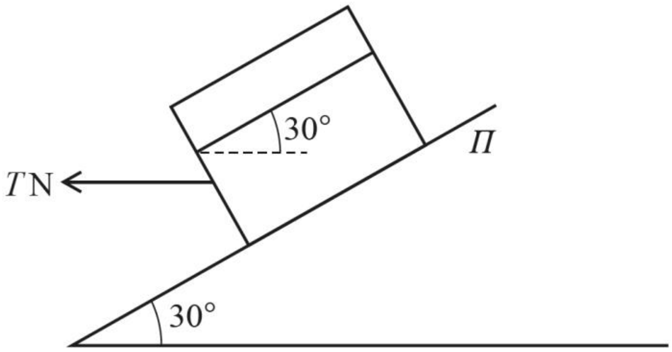
$T = 14.9$ (3 sf)

The candidate has correctly resolved parallel and perpendicular to the plane and correctly applied $F = \mu R$. However, to be awarded full marks, as this part was a 'show that', examiners needed to see either the correct answer to at least 4 significant figures or a correct expression for T before stating the given answer of 14.9.

Question 12 (b)

For the remainder of the question, you should you use this value of *T*.

Fig. 2



Block B is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg, and the lower block has a mass of 6 kg. The two blocks are held at rest with the lower block on Π. The horizontal force of magnitude T N is now applied to the lower block (see **Fig. 2**).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration 3.5 m s^{-2}.

(b) Determine the coefficient of friction between the two blocks. **[4]**

This part was answered relatively well with many candidates correctly applying Newton's second law for the upper block and obtaining an equation for the coefficient of friction between the two blocks. The most common errors were to include *T* in their equation (even though this force was being applied to the lower block only) or to include other forces that were not acting on the upper block.

Question 12 (c)

(c) Show that the lower block does **not** move. **[3]**

This part was extremely demanding with only the most able candidates making any real progress. Some managed to find a correct expression for the maximum frictional force between the lower block and the plane but only the most able could then find a correct expression for the resultant force acting down the plane for the lower block (with most attempts missing at least one required term). Those that did find both correct expressions usually argued correctly that as $F_{max} >$ Resultant force down the plane, this implied that the lower block would not move.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don't have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students' results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

We've made it easier for Exams Officers to download copies of your candidates' completed papers or 'scripts'. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular professional development courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is the exam paper builder platform for a range of our qualifications. Find out more.

Free for all OCR centres with an Interchange account, it allows unlimited users per centre. We require an Interchange username to verify your centre's users for ExamBuilder.

If you do not have an Interchange account, please contact your centre administrator (usually the Exams Officer) to request a username.

Once you have an Interchange username, use the form on this page to sign up for ExamBuilder, or ask an existing user at your centre to create an ExamBuilder account for you.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals (examined units only).

Find out more.

Tell us what you think

Your feedback plays an important role in how we develop, market, support and resource qualifications now and into the future. We want you and your students to enjoy and get the best out of our qualifications and resources, but to do that we need your honest opinions to tell us whether we're on the right track or not.

You can email your thoughts to resources.feedback@ocr.org.uk or visit our feedback page to learn more about how you can help us improve our resources.

Designing and testing in collaboration with you and your students

Helping young people develop an ethical view of the world

Equality, diversity, inclusion and belonging (EDIB) are part of everything we do

Contact the team at:

01223 553998

ocr.org.uk

To stay up to date with all the relevant news about our qualifications, register for email updates at ocr.org.uk/updates

Visit our Online Support Centre at support.ocr.org.uk