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Examiners' report

A LEVEL

FURTHER MATHEMATICS B (MEI)

OCR Level 3 Advanced GCE in Further Mathematics B (MEI)

H645

For first teaching in 2017

Y433/01 Summer 2025 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y433 series overview

This is a minor option paper for MEI Further Mathematics. It assesses Modelling with Algorithms under the three strands of algorithms, networks and linear programming.

Candidates made some attempt at all the questions, there is no evidence that suggest that candidates were unable to complete the paper.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• attempted all questions and gave responses appropriately for the number of marks available • worked neatly and explained their working where appropriately • considered the instructions within questions carefully and answered with the required accuracy • answered written responses precisely and unambiguously • checked all arithmetic.	• did not give written responses to questions asking for an explanation • worked in a muddled way and misread their own letters or numerical values • did not read the questions carefully enough • did not use all the information provided in the questions.

Assessment for learning

Centres should advise candidates that erased work can sometimes still show through on the scanned copy and may lead to ambiguities. This is especially prevalent on questions involving diagrams and drawn networks.

Question 1 (a)

1 Four workers, Ali (A), Beth (B), Casey (C) and Dev (D), are available to complete three tasks, P, Q and R. Each worker can only be assigned to at most one task, and each task must be done by at most one worker.

The table in **Fig. 1.1** shows the average time, in minutes, that each worker takes to complete each task.

Fig. 1.1

	P	Q	R
A	32	39	37
B	29	28	32
C	35	36	39
D	38	39	34

The four workers need to know, based on the times in the table, who should be allocated to each task so that the total time to complete all three tasks is minimised.

(a) Formulate this allocation problem as an LP. **[4]**

Most candidates were able to formulae the allocation problem stating an objective and two sets of constraints.

Candidates demonstrated a good understanding that the constraints for the workers were inequalities. The most common error occurred when candidates set up the constraints for tasks as inequalities rather than the equalities required.

Question 1 (b) (i)

The LP was run in an LP solver and the output is shown in the table in **Fig. 1.2**.

Fig. 1.2

Variable	Value
AP	1.000 000
AQ	0.000 000
AR	0.000 000
BP	0.000 000
BQ	1.000 000
BR	0.000 000
CP	0.000 000
CQ	0.000 000
CR	0.000 000
DP	0.000 000
DQ	0.000 000
DR	1.000 000

(b) (i) State which worker will **not** be assigned to a task according to the output given in **Fig. 1.2**. **[1]**

Candidates mostly answered this question correctly.

Question 1 (b) (ii)

(ii) Find the predicted total time for the **three** assigned workers to complete the **three** tasks, according to the output given in **Fig. 1.2**. **[1]**

Candidates mostly answered this question correctly.

Some candidates attempted to find the least time with three workers working at the same time rather than a total time.

Question 2 (a)

2 Consider the following algorithm.

Line 10	Let $A = 5, B = 6, C = 150$
Line 20	Calculate $D = (A + B) \div 2$
Line 30	Calculate $E = C - D^3$
Line 40	If $E^2 < 0.1$ go to Line 100
Line 50	If $E > 0$ go to Line 80
Line 60	Let $B = D$
Line 70	Go to Line 20
Line 80	Let $A = D$
Line 90	Go to Line 20
Line 100	Output D
Line 110	Stop

(a) Work through the algorithm, recording the values of A, B, D and E every time they change. You should record the exact values of A, B and D, and the value of E correct to **3** decimal places. Give the final output to **2** decimal places. **[4]**

Most candidates were able to complete algorithm to line 30. Some candidates replaced A and/or B at the wrong time during the algorithm. Also, not all candidates read the instructions carefully and were giving D to 3 decimal places as well as the final output to a different degree of accuracy.

Question 2 (b)

The algorithm gives the cube root of 150 correct to 2 decimal places.

A student adapts the algorithm to try to find the cube root of 1500 correct to 2 decimal places by changing only Line 10 to 'Let $A = 5, B = 6, C = 1500$'.

(b) Explain why this change will **not** give the output of the cube root of 1500 correct to 2 decimal places. **[1]**

The candidates that understood the algorithm's purpose and correctly worked through it in part (a) were more able to understand the significance of A and B. Most candidates were able to state that C needed to be between A and B. Candidates that answered this incorrectly mostly attempted to multiply A and B by the same value (10) as D.

Question 3 (a)

3 The table lists the duration (in hours) and immediate predecessors for each activity in a project.

Activity	Duration (hours)	Immediate predecessors
A	3	–
B	5	A
C	6	–
D	6	B
E	4	B, C
F	2	B
G	5	B
H	4	C
I	7	C
J	4	E, H
K	2	G, I, J
L	4	I

(a) Draw an activity network, using activity on arc, to represent the project. **[3]**

Candidates were not all able to use dummys correctly. The two dummys required to activity E were not always present with candidates attempting to use a single dummy for activity E so the IPAs were wrong for some activities. Many candidates missed a dummy at the end of D or E so both activities started and ended at the same event.

Question 3 (b) (i)

(b) (i) Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of your network. **[3]**

Most candidates that correctly drew the network in part (a) were able to complete both forward and backward passes correctly.

Candidates were mostly able to attempt a forward and backward pass although errors and missing values were seen for candidates regardless of their marks achieved in part (a).

Question 3 (b) (ii)

(ii) State the minimum project completion time. **[1]**

There were many correct responses to this 1-mark question.

Question 3 (b) (iii)

(iii) List the critical activities. **[1]**

The candidates that correctly completed the network in part (a) were able to correctly list the critical activities.

Some candidates were able to list the critical activities despite having an incorrect network drawn.

Question 3 (c)

Each activity requires one worker. When an activity is started it must be completed without interruption.

(c) Use the diagram in the Printed Answer Booklet to show how **four** workers can complete the project in the minimum time found in part **(b)(ii)**. Each column in the diagram represents 1 hour. For each worker, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. **[3]**

Most candidates attempted this question and were able to schedule activities based on their drawn network in part (a). The candidates that had a correct network in part (a) were mostly able to correctly complete the scheduling.

Candidates did not gain marks due to errors in length of activities, preceding activities as well as some candidates scheduling activities to be completed by more than one worker.

Question 3 (d)

(d) Explain why it is not possible for only **three** people to complete the project in the minimum time found in part **(b)(ii)**. **[1]**

This part of the question, requiring candidates to articulate why it was not possible for the project to complete with three workers, was the most challenging item on the paper.

The most successful candidates were able to consider the total time required for activities as well as the time available with three workers. These candidates looked at the unused time at the start of the project due to the initial activities A and C and that all other activities require at least one of their completions.

Most candidates that attempted to list the four activities that must be completed at the same time did so incorrectly.

Question 4 (a)

4 The following LP problem in x, y and z is as follows.

Maximise $P = 2x + y + 4z$

Subject to

$$\begin{aligned} x - 3y + z &\leqslant 40 \\ 3x + y + z &\leqslant 56 \\ -2x + 4y + z &\leqslant k \\ x \geqslant 0,\ y &\geqslant 0,\ z \geqslant 0 \end{aligned}$$

where k is a non-negative constant.

(a) Explain why k must be non-negative if the simplex method is to be used to solve this problem. **[1]**

This was well answered by most candidates with the majority referring to the origin needing to be in the feasible region.

Incorrect answers mainly focused on the requirements of the constraints and the direction of the inequalities.

Question 4 (b)

(b) Complete the initial tableau in the Printed Answer Booklet so that the simplex method may be used to solve this problem. **[2]**

Candidates were mostly able to complete the initial tableau from the given LP problem.

Question 4 (c)

If $k = 0$, the maximum value of P is 112. When the value of k is increased then the maximum value of P also increases until k reaches some value k_1. After reaching this value of k, the maximum value of P increases no further.

(c) By first using an entry in the z column as the pivot element, use the simplex method to determine the value of k_1 and the corresponding optimal values of P, x, y and z. **[7]**

The candidates that correctly chose the second row as their pivot row mostly completed the question losing at most 2 marks for errors.

Some candidates incorrectly chose their initial pivot which did not allow them to complete the question and find the required values.

Exemplar 1

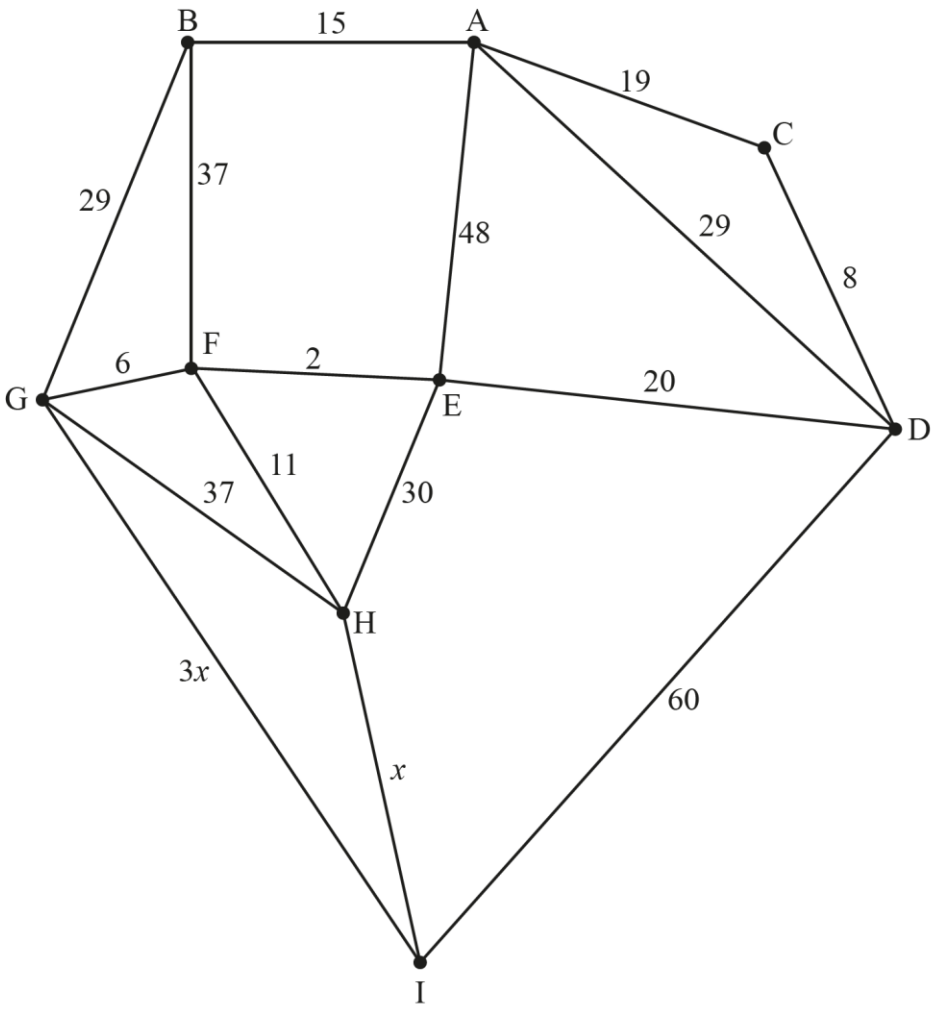
P	x	y	z	s_1	s_2	s_3	RHS
1	-10	15	0	0	0	4	4k
0	3	-7	0	1	0	-1	40-k
0	5	-3	0	0	1	-1	56-k
0	-2	4	1	0	0	1	k

P	x	y	z	s_1	s_2	s_3	RHS
1	0	9	0	0	2	2	112+2k
0	0	-5.2	0	1	-0.6	-0.4	6.4-0.4k
0	1	-0.6	0	0	0.2	-0.2	11.2-0.2k
0	0	2.8	1	0	0.4	0.6	22.4 +0.6k

Some candidates (such as this exemplar) struggled to correctly decode the information in the question and incorrectly pivoted on the row involving k. Candidates where this occurred were unable to proceed beyond the first iteration. Their second iteration was based on an unknown value of k and did not lead to correct values for k, P, x, y or z.

Question 5 (a)

5 The diagram shows a network. The weights on the arcs are distances.



The weight of arc HI is x and the weight of arc GI is $3x$, where x is an integer such that $6 \leqslant x \leqslant 59$.

(a) • Apply Dijkstra's algorithm to the network to find the **three** possible shortest paths from A to I.

• State the length of each path, leaving your answer in terms of x where necessary. **[7]**

Candidates answered this question well.

Some candidates did not complete all order of labelling or missed working values at different nodes. Even under these conditions most candidates were able to list the three possible paths in terms of x.

Question 5 (b) (i)

(b) (i) Apply Prim's algorithm, **starting at A**, to find the minimum spanning tree for the network in the diagram. You must state the order in which the arcs were included in the tree. **[2]**

Most candidates were able to apply Prim's algorithm and listed arcs in the correct order.

The most common error occurring was the inclusion of the arc BG although it was not the third arc to be added (CD was required). In these cases, candidates did not make the criteria to achieve any marks.

Question 5 (b) (ii)

(ii) State, in terms of x, the weight of the minimum spanning tree. **[1]**

All candidates that had an answer for part (i) were able to state a weight.

Those that answered part (i) correctly mostly stated this weight correctly.

Question 5 (c)

For the network in the diagram, you are given that the shortest path from A to I passes through F. Furthermore, the weight of the minimum spanning tree is greater than the weight of the shortest path from A to I that passes through G.

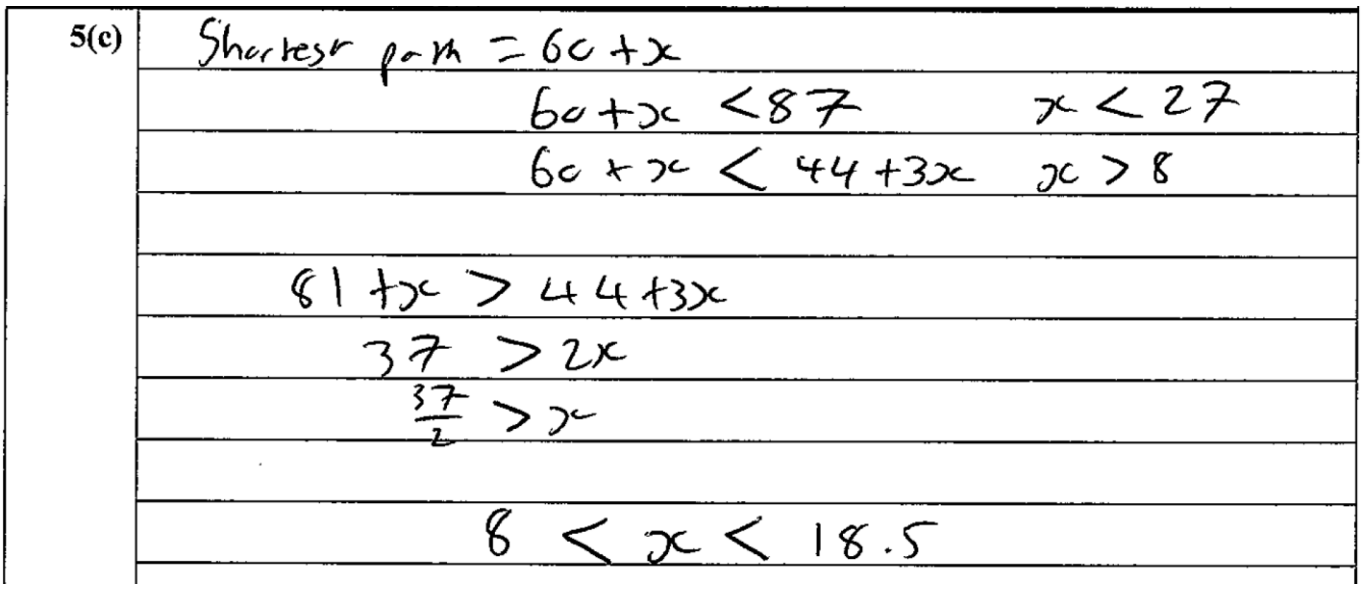
(c) Determine the range of possible values of x. **[3]**

Most candidates attempted to use their weights found in parts (a) and (b).

Some candidates misinterpreted the question and used non-strict inequalities, and some had their inequalities the wrong way around.

Many candidates did not gain the A mark as they did not consider the correct range based on x being an integer value.

Exemplar 2



This candidate correctly set up two inequalities to find out the limits to the range of values for x.

However, they left their range including 18.5 rather than changing the top limit to be 18.

Question 6 (a)

6 An incomplete initial tableau for a two-stage simplex solution for a maximisation linear programming problem, in x, y and z where $x, y, z \geqslant 0$, is shown below.

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
0	1	−2	−1	−3	0	0	0	0	0	0	0
0	0	2	−3	4	1	0	0	0	0	0	55
0	0	3	1	1	0	−1	0	0	1	0	23
0	0	2	4	1	0	0	−1	0	0	1	23
0	0	2	4	1	0	0	0	1	0	0	23

(a) Write down the equality constraint for the LP problem. **[1]**

This question was well answered by most candidates. The candidates that incorrectly answered this stated the objective instead of the equality constraint.

Question 6 (b)

(b) In the **Printed Answer Booklet**, complete the top row of the initial tableau for the LP problem. **[2]**

Most candidates were able to find Q by adding the two rows with artificial variables together.

When candidates didn't attempt to work with the 4th and 5th rows the question was left without a response.

Question 6 (c)

A 2-D graphical method is to be used to solve the LP problem.

(c) Explain why P is maximised when $4x + 11y$ is minimised. **[3]**

Although very few candidates achieved full marks on this question, most were able to eliminate z in the objective function by substitution for two of the three available marks.

Question 6 (d)

(d) Use a 2-D graphical method, with axes representing x and y, to determine the following.

- The maximum value of P
- The corresponding values of x, y and z

[8]

Very few candidates secured full marks, with most struggling to obtain or draw all the constraints in the LP problem.

Nearly all candidates attempted to substitute in $z = 23 - 2x - 4y$ into at least one constraint. Most candidates successfully found two out of three required constraints.

Some candidates were able to draw three lines but most only drew one or two correctly. Very few candidates drew all four constraints that contained the feasible region.

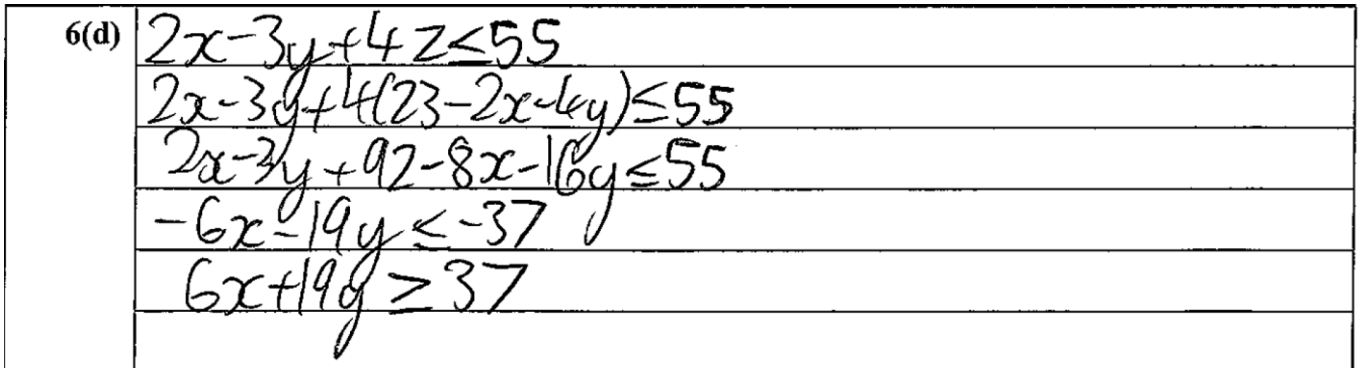
Candidates were able to find values for P, x, y and z from $6x + 19y \geqslant 37$ and $x \geqslant 3y$ without completely finding the feasible region The candidates that only worked with two constraints did not score above 4 marks.

Assessment for learning

When completing a graphical method candidates should make sure that the feasible region is completely enclosed.

All constraints need to be drawn including x, y, $z \geq 0$.

Exemplar 3



6(d)

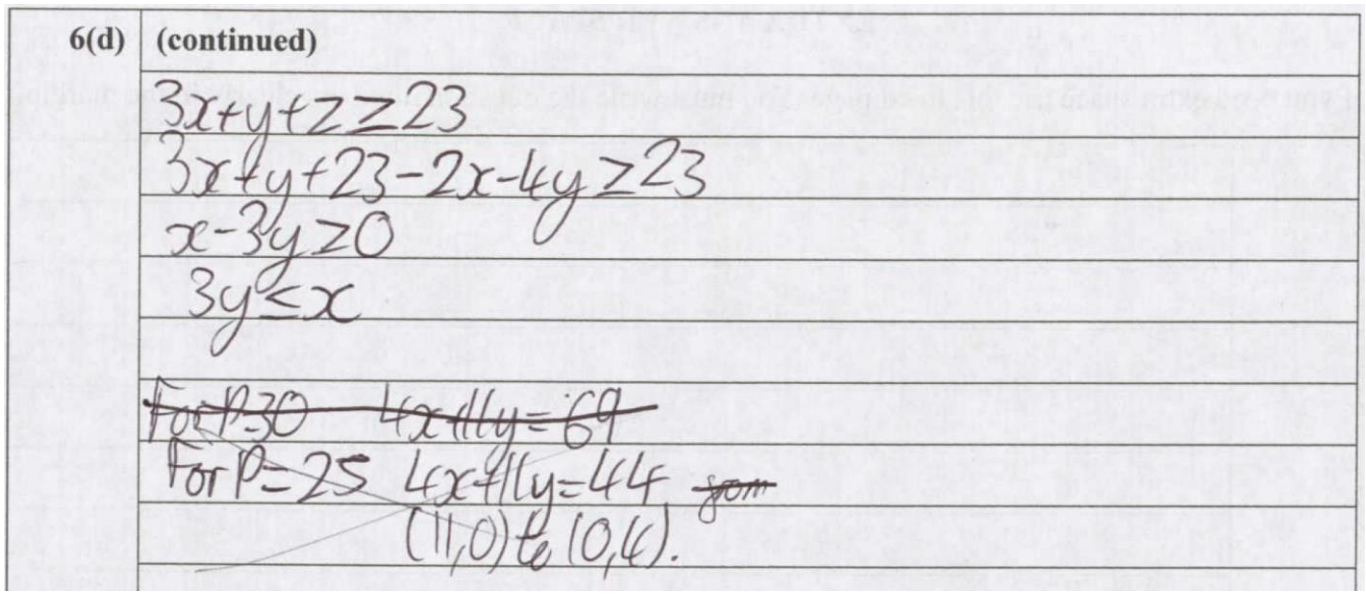
$2x-3y+4z\leq 55$

$2x-3y+4(23-2x-4y)\leq 55$

$2x-3y+92-8x-16y\leq 55$

$-6x-19y\leq -37$

$6x+19y\geq 37$



6(d) (continued)

$3x+y+z\geq 23$

$3x+y+23-2x-4y\geq 23$

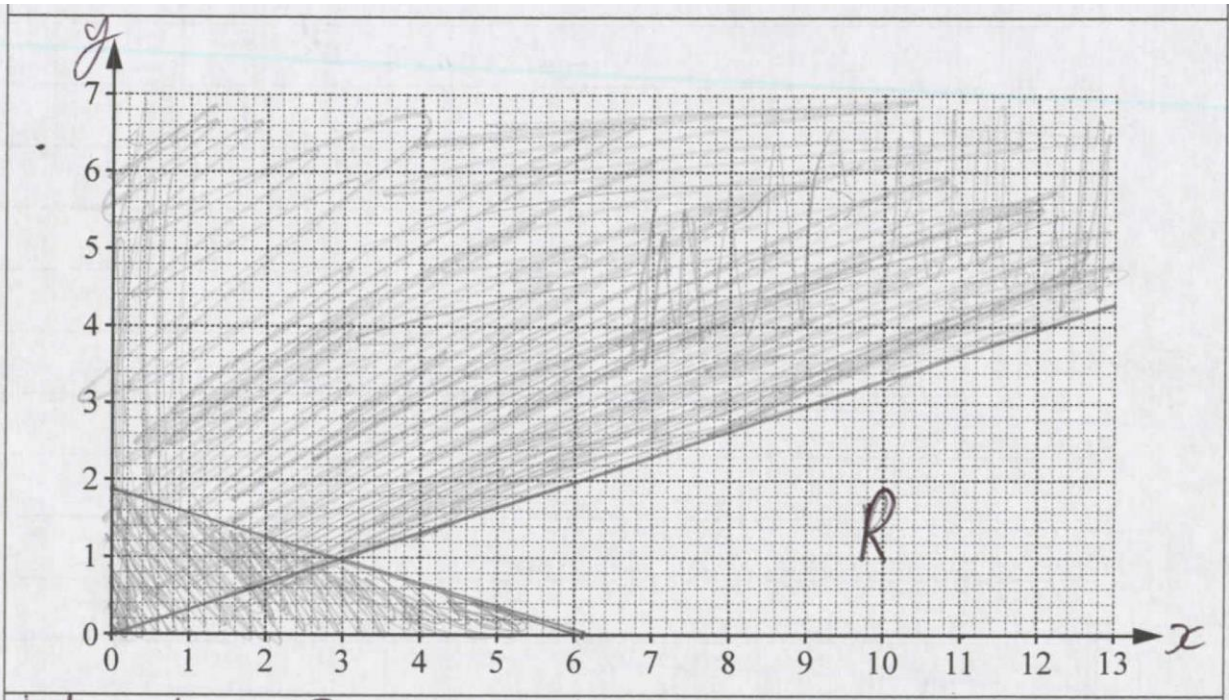
$x-3y\geq 0$

$3y\leq x$

~~For P=30 $4x+11y=69$~~

For P=25 $4x+11y=44$ ~~gom~~

(11,0) to (0,4).



Intersection of $3y = x$ and $6x + 19y = 37$

$18y + 19y = 37$

$37y = 37$

$y = 1$

$x = 3(1) = 3$

$z = 23 - 6 - 4$

$z = 26$

$P = 69 - 12 - 11 = 46$

Maximum value of P = 46

x = 3 y = 1 z = 26

Candidate was able to correctly find $P = 46$ and $x = 3$, $y = 1$, $z = 13$ but did not create an enclosed feasible region to find their values. The question required a 2-D graphical method however this candidate did not find all of the constraints in terms of x and y nor draw the four required lines that made up the feasible region.

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