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Examiners' report

A LEVEL

FURTHER MATHEMATICS B (MEI)

OCR Level 3 Advanced GCE in Further Mathematics B (MEI)

H645

For first teaching in 2017

Y421/01 Summer 2025 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y421 series overview

This is one of the two major examination components for the A Level examination for GCE Further Mathematics B (MEI). It is a two-hour fifteen-minute paper consisting of 120 marks. The paper consists of two sections, A and B. Section A will have between 25 and 35 marks and will comprise more straightforward questions. Section B will have between 85 and 95 marks and will comprise a mixture of more and less straightforward questions.

Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on all the associated mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word featured in Questions 1, 3 (b), 4 (c), 5 (d), 5 (e), 7 (a), 7 (d), 8 (c), 9 (a), 10 (b), 11 (b), 12 (b), 12 (c), 13 (b), 14 (b) and 15.

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 5 (c), 7 (b), 7 (c), 8 (b), 10 (a), 12 (a), 13 (a) and 14 (a).

While there is no specific level of working needed to justify answers to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation). Regardless of the final required accuracy, candidates should be careful of not rounding prematurely but also take care to avoid over specifying rounded answers where the context does not support that level of accuracy.

One general point with regards to the answering of certain mechanics questions should be made in this overview. This is that unless told otherwise (for example on this paper in Question 13) the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 9.81 or 10 (and this value is stated explicitly on the front cover of the examination paper).

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• used formal language and notation correctly • defined the letters used for speed, time, mass, etc. in unstructured questions • understood the level of response required for the command words used in the questions • made efficient use of their calculator • read questions carefully and provided the answers that were requested.	• made careless mistakes with algebraic manipulation • did not give sufficient evidence on 'Show that' and 'Determine' questions • used imprecise notation or language • did not extract all the relevant information when reading the question • did not give answers to the required degree of accuracy.

Section A overview

Section A in this paper covers questions 1 – 6 and was worth 33 marks. While designed to provide a gentle introduction to the exam session questions 4 and 6 did prove quite challenging for some candidates.

Question 1

1 A small box of mass 3 kg is pulled along a horizontal floor by a constant force of magnitude 35 N. The force acts at an angle of θ above the horizontal.

The total resistance to the motion of the box has magnitude 25 N.

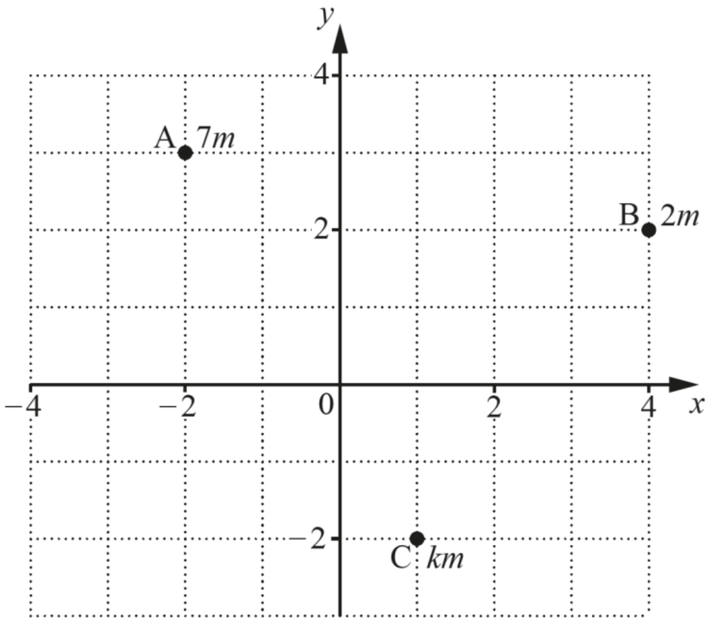
The box starts from rest at a point O, and passes through a point 4 m from O with speed $1.5\ \mathrm{m\,s^{-1}}$.

Use an energy method to determine the value of θ. **[3]**

Most candidates correctly applied the work-energy principle to find the required angle. When errors occurred, they were usually either sign errors or using $35\sin\theta \times 4$ rather than $35\cos\theta \times 4$ for the work done by the force of magnitude 35 N.

Question 2 (a)

2



The diagram shows a system of three particles of masses $7m$, $2m$ and km situated in the x-y plane at the points A$(-2, 3)$, B$(4, 2)$ and C$(1, -2)$ respectively.

The centre of mass of the three particles is at the point with coordinates $(0, d)$.

(a) Find the value of k. **[2]**

This part was answered extremely well with most candidates correctly taking moments about the *y*-axis to find the correct value of *k*. The most common errors were either using an incorrect distance or sign, and in some cases divided by 3 which examiners assumed was a misunderstanding that the three particles formed the vertices of a triangular lamina.

Question 2 (b)

(b) Find the value of d. **[2]**

Like part (a), this part was done equally as well with most candidates now correctly taking moments about the *x*-axis to find the value of *d*. Those responses that contained errors in part (a) were usually replicated in this part too.

Question 3 (a)

3 At time t seconds, the total force acting on a particle P of mass 4 kg is $\begin{pmatrix} 32\cos 2t \\ -8\sin t \\ 0 \end{pmatrix}$ N.

When $t = 0$ the velocity of P is $\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}$ m s^{-1}.

(a) Find, in column vector form, the acceleration of P at time t seconds. **[1]**

Almost all candidates correctly divided the given vector by 4 to obtain the correct acceleration vector.

Question 3 (b)

(b) Determine the speed of P when $t = \frac{1}{3}\pi$. **[5]**

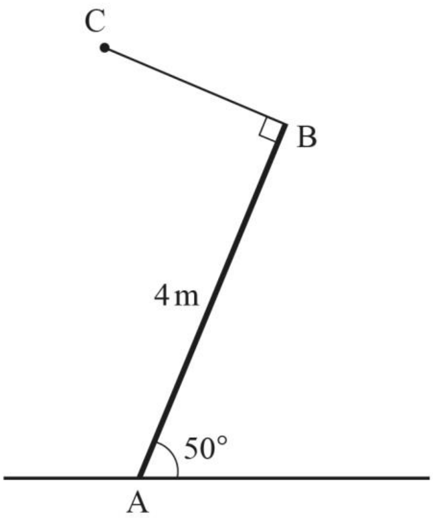
While most candidates correctly integrated their expression from **(a)** to obtain the corresponding velocity vector some differentiated, some ignored the need for a vector constant of integration, and some assumed that it would be a zero. Some candidates who did correctly obtain the velocity at $t = \frac{1}{3}\pi$ as $\mathbf{v} = \begin{pmatrix} 4\sin\left(\frac{2}{3}\pi\right) \\ 2\cos\left(\frac{1}{3}\pi\right) + 3 \\ 6 \end{pmatrix}$ either did not find the magnitude (even though the question clearly asked for the speed of P at this time) or gave an answer that indicated that their calculator was set in degrees mode rather than radians.

Assessment for learning

When a question involves the integration or differentiation of trigonometric functions candidates are reminded that the angle will be measured in radians and not degrees (and this will often not be explicitly mentioned in the question).

Question 4 (a)

4



A uniform rod AB of mass 3 kg and length 4 m rests with one end A on rough horizontal ground. A light inextensible string has one end attached to the rod at B and the other end attached to a fixed point C.

The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 50° to the ground, as shown in the diagram.

(a) Complete the diagram in the **Printed Answer Booklet** to show all the forces acting on the rod. **[1]**

While examiners commented that most candidates correctly completed the given diagram by adding the four required forces it was surprising how many candidates missed one of the two forces at A (either forgetting the normal contact and/or the frictional contact force) or had the frictional force at A acting in the wrong direction.

Question 4 (b)

(b) By taking moments about A, find the tension in the string. **[2]**

This part was answered extremely well with most candidates correctly taking moments about A to find the tension in the string. The most common errors were either did not include a distance when taking moments either clockwise or anticlockwise, or using $\sin 50$ instead of $\cos 50$ when taking the moment of the weight.

Question 4 (c)

(c) Determine the magnitude of the total contact force between the rod and the ground. **[4]**

While many fully correct responses were seen in this part it was surprising how many candidates thought the 'total contact force between the rod and the ground' was the normal contact force at A only. If anything, the number of marks in this part should have given those candidates a bit of a hint that there was more to it than just a need to resolve vertically. While most candidates, in their attempt to find the normal and frictional contact forces, favoured resolving both horizontally and vertically for the rod, some resorted instead to taking moments about a second point; these candidates were usually nowhere near as successful.

Misconception

Candidates are reminded that in situations like this when the contact between the rod and the ground (at A) is rough then the total contact force is given by $\sqrt{F_{\mathrm{A}}^2 + R_{\mathrm{A}}^2}$ where F_{A} and R_{A} are the frictional contact and normal contact forces at A respectively, and not just R_{A}.

Question 5 (a)

5 In this question you may assume that if r and s are any physical quantities then $\left[\frac{\mathrm{d}r}{\mathrm{d}s}\right] = \left[\frac{r}{s}\right]$.

You may also assume that all given numerical constants are dimensionless.

(a) Find the dimensions of power. **[1]**

While most candidates used a correct method to find the dimensions of power (with the most common being to consider $P = F \times v$) it was disappointing how many used incorrect notation. When a question asks for the dimension of a quantity then only an expression in terms of M, L and T is acceptable.

Question 5 (b)

A car of mass m moves horizontally in a straight line. When the car is a distance x from a point O, it is moving away from O with speed v.

The power developed by the car is P. The maximum possible speed of the car during its motion is U.

The differential equation for the motion of the car is given by

$$m^{\alpha} v^{\beta} \frac{\mathrm{d}v}{\mathrm{d}x} = P^{\gamma}\left(1 - \frac{v^{\delta}}{U^3}\right).$$

(b) Explain why $\delta = 3$. **[1]**

The responses to this part were extremely pleasing with most candidates correctly explaining that for the equation to be dimensionally consistent then the dimensions of *v* and *U* had to be the same and as these are both speeds, $\delta = 3$.

Question 5 (c)

(c) Show that the dimensions of $\frac{\mathrm{d}v}{\mathrm{d}x}$ are T^{-1}. **[1]**

This part was also answered well with most candidates either using $\frac{\mathrm{LT}^{-1}}{\mathrm{L}}$ or $\frac{\mathrm{LT}^{-2}}{\mathrm{LT}^{-1}}$ (from $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}v}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$) to show the given result.

Question 5 (d)

(d) Use dimensional analysis to determine the values of α, β and γ. **[3]**

Apart from the occasional incorrect dimensions or sign slip, most candidates correctly found the correct values of α, β and γ from the equation $\mathrm{M}^{\alpha}(\mathrm{LT}^{-1})^{\beta}\mathrm{T}^{-1} = (\mathrm{ML}^2\mathrm{T}^{-3})^{\gamma}$.

Question 5 (e)

Once the car attains a speed of $\frac{1}{2}U$, the power developed by the car is immediately reduced to zero.

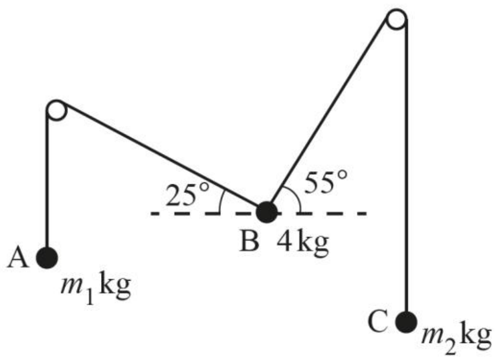
A student calculates that the time taken, *t*, for the speed of the car to reduce from $\frac{1}{2}U$ to $\frac{1}{4}U$ is given by $t = \frac{2mU^2}{P}$.

(e) Determine whether this expression for *t* is dimensionally consistent. **[2]**

This part, like the other parts of this question, was answered extremely well with most candidates correctly showing that the given expression for *t* was indeed dimensionally consistent by showing that the expression $\frac{\mathrm{M}\times(\mathrm{LT}^{-1})^2}{\mathrm{ML}^2\mathrm{T}^{-3}}$ was equivalent to T.

Question 6 (a)

6



One end of a light inextensible string is attached to a particle A of mass $m_1$kg. The other end of the string is attached to a second particle B of mass 4kg.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass $m_2$kg. Both strings are taut and pass over small smooth fixed pulleys.

The diagram shows the three particles A, B and C, and the two strings. The diagram also shows the angles that the non-vertical portions of the strings make with the horizontal.

The system is in equilibrium with all three particles at rest in the same vertical plane.

(a) Draw a closed figure to represent the three forces acting on B. **[1]**

It was surprising the number of candidates who drew an open figure of the three forces acting on B instead of the requested closed figure. Of those that did draw a closed figure most correctly drew a triangle of forces and labelled the sides accordingly (with either relevant masses or tensions) and including at least two correct/relevant angles.

Question 6 (b)

(b) Hence, or otherwise, find the following.

- The value of m_1
- The value of m_2 **[4]**

Most candidates in this part opted to ignore the closed figure from part (a) and instead resolved vertically and horizontally at B to form simultaneous equations in m_1 and m_2. Although the majority were successful in their endeavours it was much more efficient to use the closed figure from part (a) and apply the sine rule to find the required masses.

Section B overview

Section B in this paper covers questions 7 – 15 and was worth 87 marks. While designed to incorporate more problem solving in less routine scenarios, most candidates appeared to be able to demonstrate their knowledge on each of the questions and gain some credit.

Question 7 (a)

7 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 4 kg, and the mass of B is 6 kg.

The spheres are travelling in the same direction in a straight line on a smooth horizontal surface, A with speed $6\,\mathrm{m\,s^{-1}}$ and B with speed $3\,\mathrm{m\,s^{-1}}$.

Sphere A collides directly with B and, immediately after the collision, the speed of B is $5\,\mathrm{m\,s^{-1}}$.

(a) Determine the exact value of the coefficient of restitution between A and B. **[3]**

This question was answered extremely well with most candidates in this first part correctly applying the conservation of linear momentum (CLM) and Newton's experimental law (NEL) for the collision between A and B to find the required coefficient of restitution.

Assessment for learning

Unless a question explicitly mentions the direction of motion of the bodies immediately after direct impact, candidates are advised to assume that both bodies move in whichever direction is being taken as the positive direction of motion. Therefore, candidates are better off assuming that both bodies are moving in the same direction of motion after impact, this then eliminates most sign errors which are all too common in this type of question.

Question 7 (b)

Sphere B subsequently collides directly with a uniform smooth stationary sphere C. The mass of C is 2 kg, and C has the same radius as A and B. The coefficient of restitution between B and C is 0.6.

(b) Show that there will be **no** further collisions between any of the three spheres. **[4]**

While many candidates correctly realised that they needed to find the speed of B after the collision between B and C it was surprising how many candidates did not explain clearly why there would be no further collisions. In situations like this, candidates would be best placed to give an answer that explicitly states that the speeds of A and B are both equal after this second collision (and therefore there cannot be any further collisions). Some candidates commented as well on the speed of C after the collision, but this was not necessary (as it is a given in such scenarios that B and C cannot collide again).

Question 7 (c)

(c) Show that the impulse of B on C is equal in magnitude to the impulse of A on B. **[2]**

This part was answered extremely well too with most candidates correctly showing that the magnitude of both impulses was 12 N s.

Misconception

Candidates are reminded that when a question in mechanics asks for the magnitude of a particular quantity the corresponding answer must always be positive.

Question 7 (d)

(d) Determine the total kinetic energy lost due to the **two** collisions. **[3]**

Although not as well done as the other three parts, most candidates did find the total kinetic energy lost due to the two collisions correctly as 18 J. The most common (and successful) method was to subtract the total KE of the system after the second collision from the total KE of the system before the first collision. Those that tried to consider the total KE of the system between the two collisions were not normally as successful.

Question 8 (a)

8 A box of mass m kg rests on a horizontal platform. The platform moves vertically with simple harmonic motion of period T seconds and amplitude a m. At time t seconds, the upward displacement of the platform from its mean position is x m.

(a) Express $\ddot{x}$ in terms of and π, T and x. **[2]**

Erratum notice

Turn to **page 6** of the **question paper** and look at **question 8(a)**.

Cross out the word 'and' after the word 'of'.

The question should now read:

Express $\ddot{x}$ in terms of π, T and x.

This part was answered extremely well with most candidates scoring at least 1 mark for correctly stating either that $\ddot{x} + \omega^2 x = 0$ or $T = \frac{2\pi}{\omega}$, and while most correctly found that $\ddot{x} = -\frac{4\pi^2}{T^2}x$ occasionally one of the terms was squared incorrectly or the negative sign was absent.

Question 8 (b)

(b) Show that the box will maintain contact with the platform throughout its motion if $T > 2\pi\sqrt{\frac{a}{g}}$. **[4]**

This part, however, was not as well done with most candidates showing insufficient working to indicate that a correct method had been applied; far too many candidates started with an inconsistent/incorrect statement about the contact force between the box and platform and/or inequalities that were only a step away from the given answer. It was expected that candidates would use their answer to part (a) to set up an equation for the normal contact force between the box and the platform, e.g. $R = mg - \frac{4\pi^2}{T^2}x$, set this expression for *R* greater to zero and replace *x* with *a* (which if done correctly would lead to the given answer).

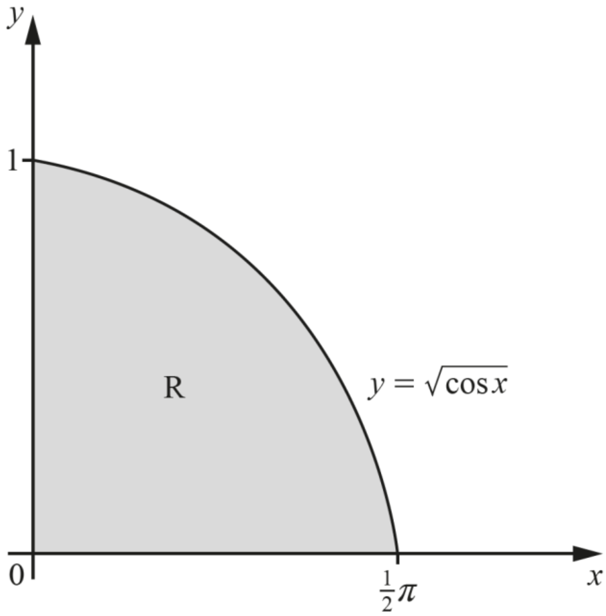
Question 8 (c)

(c) Given that $T = 4\pi\sqrt{\frac{a}{g}}$, determine the ratio of the magnitudes of the greatest and least forces exerted by the platform on the box. Give your answer in the form $p:q$ where p and q are integers to be found. **[2]**

While noticeably better answered than part (b), it was surprising how many candidates would correctly show (via a correct method) that the least force was $\frac{3}{4}mg$ but then would decide not to use of a similar method to find the greatest force (as $\frac{5}{4}mg$ and hence give a ratio of 5 : 3) but instead would state incorrectly that the greatest force would be equal to the weight of the box.

Question 9 (a)

9



The diagram shows the shaded region R bounded by the curve $y = \sqrt{\cos x}$, the x-axis and the y-axis.

The region R is rotated through 2π radians about the x-axis to form a uniform solid of revolution S.

(a) Determine the exact x-coordinate of the centre of mass of S. **[6]**

While full marks were very common in this part it was surprising how many candidates forgot a factor of π from one of the two integrals or introduced sign errors when integrating $x \cos x$ by parts. Even though the question explicitly asked for an exact *x*-coordinate some gave their answer to 2 or 3 significant figures. Sadly, some misread the question and found the centre of mass of a uniform lamina that occupied the shaded region.

Question 9 (b)

The uniform solid S is placed with its plane face on an inclined plane. The inclined plane makes an angle θ with the horizontal.

(b) Given that the inclined plane is sufficiently rough to prevent S from sliding and that S is on the point of toppling when $\theta = \alpha$, find the angle α. **[2]**

Most candidates recognised that $\tan\alpha = \frac{1}{\bar{x}}$, however, some found the complement angle or instead considered $\tan\alpha = \frac{\frac{1}{2}\pi}{\bar{x}}$.

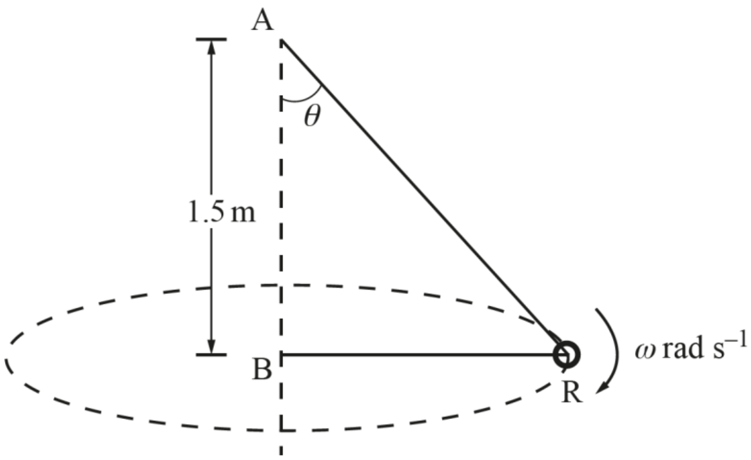
Question 9 (c)

(c) Given instead that S is on the point of sliding down the plane when $\theta = \beta$ and the coefficient of friction between S and the plane is 0.4, find the angle β. **[2]**

This part was answered extremely well with most candidates correctly stating that on the point of sliding $\tan\beta = 0.4$. Some candidates, however, did spend a considerable amount of time deriving this equation even though this is a result directly from the specification (Md4) that candidates can state without proof in a 'find' question.

Question 10 (a)

10



One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where AB = 1.5 m.

A small smooth ring R of mass 2 kg is threaded on the string. The ring R moves in a horizontal circle with centre B.

The upper part of the string makes an angle θ with the downward vertical and R moves with constant angular speed ω rad s^{-1} (see diagram).

(a) By finding an expression for ω^2 in terms of g and $\sin\theta$, show that $\omega > 2\sqrt{\frac{1}{3}g}$. **[6]**

The responses to this part were mixed; while many candidates correctly resolved vertically for *R* and obtained the correct equation $T\cos\theta = 2g$ several candidates struggled to apply N2L correctly in the horizontal direction and stated that $T = mr\omega^2$ or $T\sin\theta = mr\omega^2$ while others did not appreciate that the radius of the horizontal circle was $1.5\tan\theta$. Of those that correctly obtained the equation $T\sin\theta + T = 2(1.5\tan\theta)\omega^2$ most eliminated *T* correctly but even though the question explicitly required an expression for ω^2 many did not do so. Only the most able could argue correctly why $\omega^2 = \frac{2g}{3}\left(1 + \frac{1}{\sin\theta}\right)$ implied the given 'show that' answer.

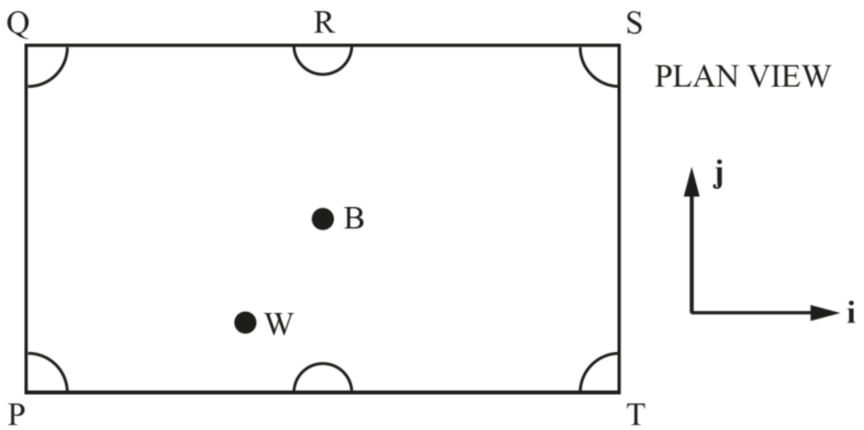
Question 10 (b)

(b) Given that $\omega = \sqrt{2g}$, determine, in terms of g, the exact tension in the string. **[3]**

Most candidates that had a correct expression for ω^2 in part (a) were successful in finding the tension in the string in this part (by first finding the corresponding angle of $\frac{1}{6}\pi$ and then substituting this into one of the two equations found earlier involving *T*). However, once again some did not read the question carefully and gave a non-exact decimal approximation of the answer.

Question 11 (a)

11



A student is playing snooker. The snooker table, PQST, is rectangular and the unit vectors, **i** and **j**, are parallel to PT and PQ respectively (see diagram).

The surface of the table is horizontal and smooth. The balls on the table are all uniform smooth spheres with the same radius and mass. The coefficient of restitution between the balls is 0.6. The blue ball, B, lies at rest at the centre of the table.

The student strikes the white ball, W, so that it collides obliquely with B. Immediately before the collision with B, W is travelling with velocity $(6\mathbf{i} + 8\mathbf{j})\,\mathrm{m\,s^{-1}}$. As a result of the collision, B travels directly towards R, where R is the mid-point of QS.

(a) Explain why, at the moment of the collision, the line of centres of B and W must be parallel to **j**. **[1]**

It was surprising how many candidates in this part did not explain that as B was at rest before the collision then the line of centres must be parallel to the direction of motion of B after collision and as B travels directly to R then the line of centres must be parallel to **j**.

Question 11 (b)

(b) Determine, correct to the nearest degree, the angle through which the direction of motion of W is deflected by the collision. **[7]**

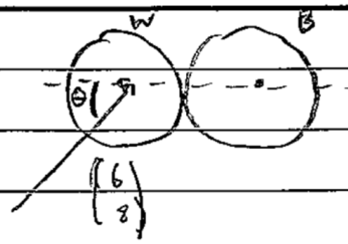
This part was answered extremely well with many candidates correctly applying CLM and NEL parallel to the line of centres to find the velocity of sphere W after impact. When errors occurred, they were usually sign errors in applying NEL, applying one or both principles perpendicular to the line of centres, calculating an incorrect angle, or being unable to find the required angle of deflection from two correctly found relevant angles (e.g. 53.1, 14.9 or 36.9, 75.1).

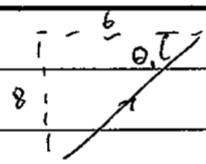
Assessment for learning

When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question it is the responsibility of the candidate to define the variables they use. In this question some candidates did not make it explicitly clear which letters were being used for the speeds of the spheres at different stages of their motion. In this type of question candidates may have benefitted from drawing diagrams in the Printed Answer Booklet with all speeds clearly labelled to assist both theirs (and the examiners) understanding.

Exemplar 1

11(b)





$\tan\theta = \frac{8}{6}$

$\theta_1 = 53.13°$

only momentum in j changes.

$m8 + 0 = mV_{Bj} + mV_{wj}$ $\qquad \frac{V_B - V_w}{8} = 0.6$

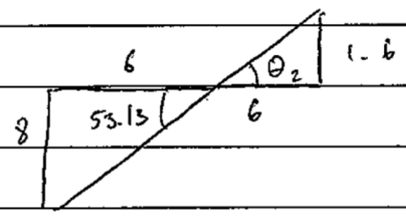
$V_B = 4.8 + V_w$

$8 = V_{Bj} + V_{wj}$

$8 = 4.8 + 2V_{wj}$

$3.2 = 2V_{wj}$

$1.6 = V_{wj}$ $\qquad \Rightarrow |V_w| = \begin{pmatrix} 6 \\ 1.6 \end{pmatrix}$



$\theta_2 = \arctan \frac{1.6}{6}$

$= 14.93°$

angle turned through $= 38.2°$

$= 38°$ (nearest degree)

This response to Question 11 (b) scored full marks. The candidate has made good use of several diagrams throughout their response and has set their work out in a logical manner to make it very clear to the examiner which principle of mechanics is being applied at each step.

Question 12 (a)

12 One end of a light elastic string of natural length a m and modulus of elasticity $4mg$ N is attached to a fixed point O. The other end of the string is attached to a particle P of mass m kg. The particle is projected vertically downwards from O with speed $\sqrt{4ga}\,\mathrm{m\,s^{-1}}$.

(a) When P is at a distance of h m below O, where $h > a$, it has speed, $v\,\mathrm{m\,s^{-1}}$.

By using the principle of conservation of energy, show that $v = \sqrt{10gh - \dfrac{4gh^2}{a}}$. **[4]**

This part was answered extremely well with most candidates correctly applying the conservation of energy to derive the given result. When errors occurred, they were usually sign errors in the conservation of energy equation or an error in the elastic potential energy term (e.g. only dividing by *a* and not 2*a* or not realising that the extension was given by *h* – *a*).

Question 12 (b)

(b) Determine, in terms of a, the greatest distance below O attained by P. **[2]**

Almost all candidates correctly set *v* = 0 and obtain the correct distance as 2.5*a*.

Question 12 (c)

(c) Determine, in terms of g and a, the maximum speed of P during its downward motion. **[4]**

This part was not answered as well as the first two parts with the most common incorrect answer being $\sqrt{6ga}$ from incorrectly assuming that the maximum speed of P was when *h* = *a*. Those candidates who realised that the maximum speed occurred when the acceleration was zero and hence $mg - \frac{4mg}{a}(h-a) = 0$ (from N2L and Hooke's law) were usually successful. Those that attempted to differentiate the expression for *v* (rather than differentiating the expression for v^2) from part (a) were not usually as successful due to errors in applying the chain rule.

Question 12 (d)

(d) Find, in terms of a, the greatest height above O attained by P. **[2]**

This part discriminated well with very few candidates realising that all that was required in this part was to take their un-simplified equation from part (a), e.g. $\frac{1}{2}m \times 4ga = -mgh + \frac{1}{2}mv^2 + \frac{4mg(h-a)^2}{2a}$ and set $v = 0$ and replace h with $-h$. Many candidates in this part incorrectly assumed that the greatest height above O would be when the string was slack and so incorrectly had and equation with only KE and GPE terms.

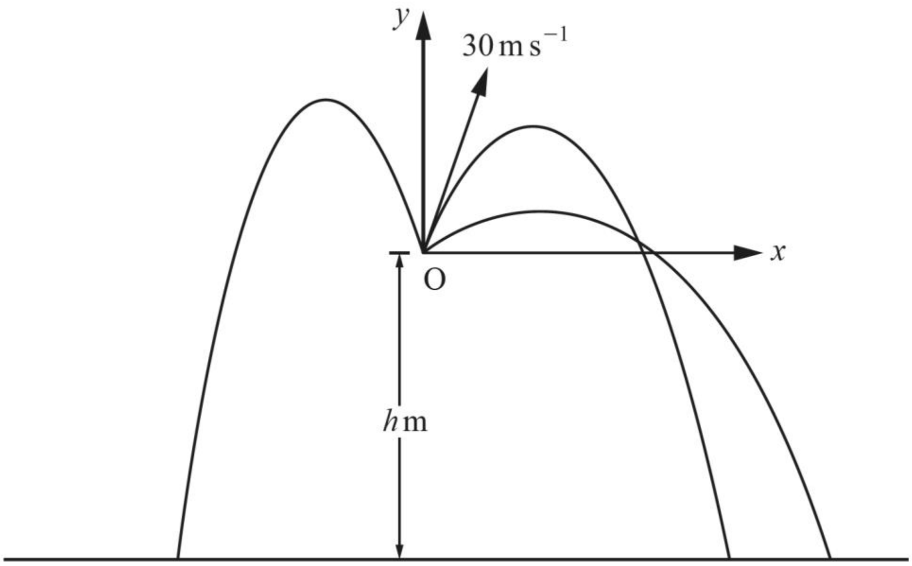
Question 12 (e)

(e) Explain why the greatest distance below O attained by P may **not** in practice be given by your answer to part **(b)**. **[1]**

Most candidates in this part either gave an answer based on the elastic limit of Hooke's law or that energy could be lost to other sources. The one common incorrect answer seen was the mention of friction.

Question 13 (a)

13



In this question you should take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.

A machine can launch balls with speed $30\,\mathrm{m\,s^{-1}}$ from a height h m above horizontal ground. The point of projection O is the origin for a horizontal x-axis and a vertical y-axis which define the vertical plane in which the balls move (see diagram). You should assume that the balls, as they leave the machine, can be modelled as particles moving freely under gravity.

At time $t = 0$, a ball B is projected at an angle θ above the horizontal in the x-y plane. Before B hits the ground, the horizontal and vertically upwards displacements of B from O at time t seconds are denoted by x m and y m respectively.

You are given that B hits the ground at the point $(X, -h)$.

(a) By expressing x and y in terms of t, show that

$$X^2 \tan^2\theta - 180X\tan\theta + X^2 - 180h = 0.$$ **[5]**

This part was answered extremely well with most candidates correctly deriving the required result by first using the *suvat* equation $s = ut + \frac{1}{2}at^2$ to obtain both $x = 30\cos\theta \times t$ and $y = 30\sin\theta \times t - \frac{1}{2} \times 10 \times t^2$. Most candidates then eliminated *t* correctly to form an equation in terms of *y*, *x* and θ only, and then finally most were able to successfully complete the 'show that' by using $y = -h, x = X$ and $1 + \tan^2\theta = \sec^2\theta$. The most common errors were either due to not using 10 for g, sign errors (or missing terms) or surprisingly not following the request to start by expressing both *x* and *y* in terms of *t*.

Question 13 (b)

(b) Determine an expression, as $\tan\theta$ varies, for the maximum value of X^2 at the instant when B hits the ground. Give your answer in the form $X^2 = p + qh$, where p and q are integers to be determined. **[3]**

Although by far the most common (and simplest) way of attempting this part was to set the discriminant of the quadratic in tan from part (a) equal to zero, many opted for either completing the square or differentiating with respect to either $\tan\theta$ or θ; these methods were usually not as successful. It was also clear that candidates struggled with applying this part of the specification to solve this question due to the large number of blank responses seen in both this part and the next.

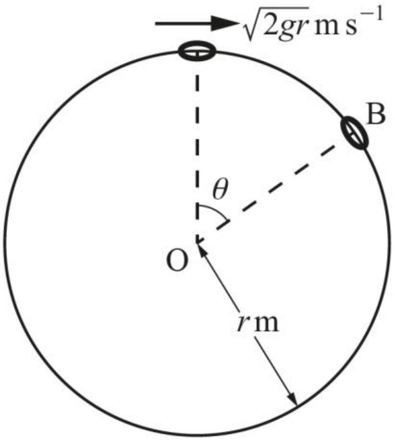
Question 13 (c)

(c) Hence find the equation of the curve that bounds the region in the x-y plane through which B can pass. Give your answer in the form $y = k_1x^2 + k_2$, where k_1 and k_2 are constants to be determined. **[2]**

This part appeared to be very challenging, with very few candidates correctly taking their answer to part (b) and setting $X = x$ and $h = -y$ to obtain the equation of the bounded parabola.

Question 14 (a)

14



A smooth thin wire is formed into a circle of radius r m and centre O and is fixed in a vertical plane. A small bead B of mass m kg is threaded on the wire.

The bead is initially at the highest point of the wire and is set in motion along the wire with speed $\sqrt{2gr}\,\mathrm{m\,s^{-1}}$ (see diagram).

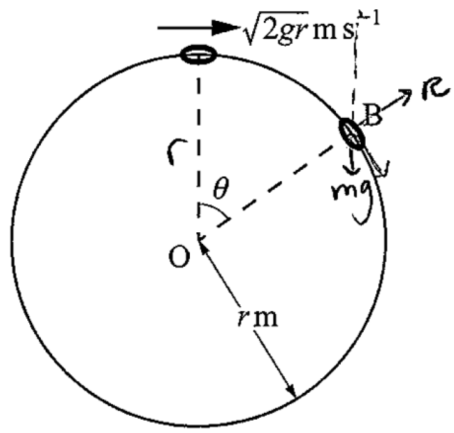
(a) Show that, when the radius from O to B is inclined at an angle θ to the upward vertical, the downward vertical component of the acceleration of B is

$(1+4\cos\theta-3\cos^2\theta)g$. **[5]**

While most candidates correctly used the conservation of energy to find an expression for v^2 at θ (so scoring the first 2 marks). Only the most able realised that the downward vertical component of acceleration could be found by either considering $\frac{v^2}{r}\cos\theta+\sin\theta\frac{\mathrm{d}v}{\mathrm{d}t}$ or by applying N2L both vertically and radially, e.g. $mg+R\cos\theta=ma$ and $R+mg\cos\theta=\frac{mv^2}{r}$ and then eliminating *R*.

Exemplar 2

14(a)



Energy conservation:

$\frac{1}{2}\cancel{m}\times 2gr + \cancel{m}g\times 2r = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g\times(r + r\cos\theta)$

$gr + 2gr = \frac{v^2}{2} + gr + gr\cos\theta$

$\frac{v^2}{2} = 2gr - gr\cos\theta$

$v^2 = 4gr - 2gr\cos\theta$

$= 2gr(2-\cos\theta)$

$F=ma$ (radial)

$mg\cos\theta - R = m\frac{v^2}{r} = \frac{2mgr(2-\cos\theta)}{r}$

$R = mg\cos\theta - 2mg(2-\cos\theta)$

$= mg(\cos\theta - 4 + 2\cos\theta)$

$R = mg(3\cos\theta - 4)$

$F=ma$ (vertical)

$mg - R\cos\theta = ma$

$\cancel{m}g - \cancel{m}g\cos\theta(3\cos\theta - 4) = \cancel{m}a$

$a = g - 3g\cos^2\theta + 4g\cos\theta$

$a = (1 + 4\cos\theta - 3\cos^2\theta)$ as required.

In this exemplar the candidate has correctly applied the conservation of energy to find an expression for v^2 at θ. They then correctly applied N2L both vertically and radially and eliminated the contact force to find a correct expression for the required acceleration. Sadly, when writing the expression in the required (given) form the candidate has forgotten the factor of g and so lost the final accuracy mark.

Assessment for learning

This exemplar highlights, in this instance, the importance of checking carefully an answer against the given answer on the question paper. In situations in which the answer is given it is vital that the candidates replicate this answer exactly and ensure that if an error is spotted then each step of working needs to be checked for consistency.

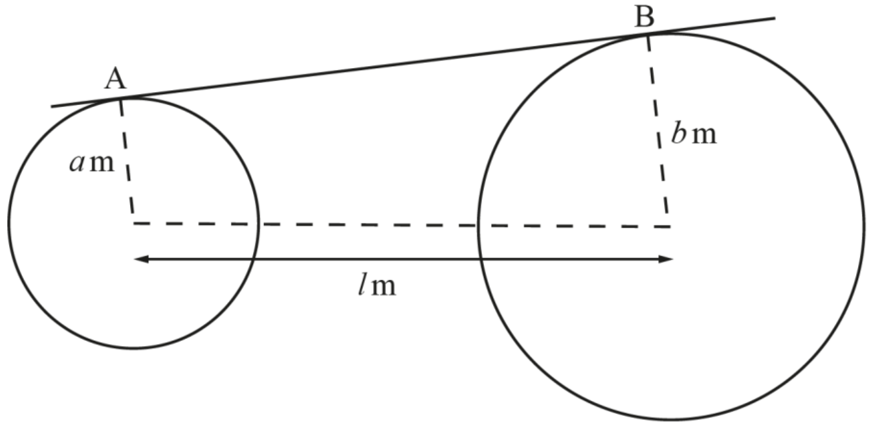
Question 14 (b)

(b) Determine, in terms of m and g, the magnitude of the force of the bead on the wire when the downward vertical component of the acceleration is a maximum. **[6]**

It was surprising how many candidates in this part incorrectly believed that the maximum acceleration would occur when $\cos\theta = 1$. Of those that differentiated the given expression from part (a) (or applied a similar correct method) most correctly obtained the result that $\cos\theta = \frac{2}{3}$. Surprisingly, many then struggled to apply N2L correctly in the radial direction with many incorrectly believing that $R = \frac{mv^2}{r}$.

Question 15

15



The diagram shows two fixed rough circular cylinders of radii a m and b m, where $b > a$. The axes of the cylinders are parallel and in the same horizontal plane at a distance l m apart. A heavy non-uniform straight rod rests in limiting equilibrium on the cylinders at A and B. The centre of mass of the rod lies between A and B but is not at the mid-point of AB. The coefficient of friction between the rod and each cylinder is μ.

Determine, in terms of a, b and l, an expression for μ. **[6]**

As expected (from the last question on the paper) this question provided stretch and challenge for those candidates that had done well on this paper, with only a small proportion score full marks with the correct expression for the coefficient of friction between the rod and each cylinder. Some candidates correctly managed to derive the result that $\mu = \tan\theta$ (where θ was the angle between the horizontal and the rod), but few realised that $\tan\theta = \frac{b-a}{\sqrt{l^2-(b-a)^2}}$.

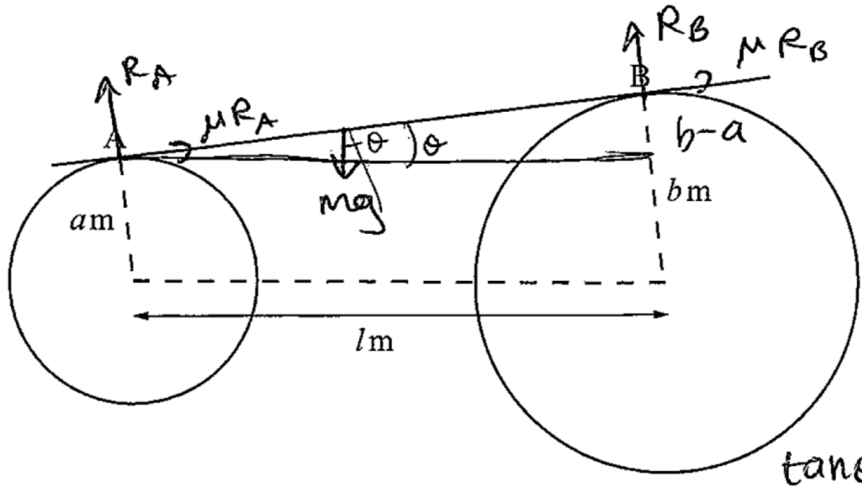
Many candidates incorrectly (or inefficiently) attempted to take moments about either A or B even though the question explicitly made it clear that the centre of mass of the rod was not known. Those that realised that because of this resolving either parallel and perpendicular to the rod (or just horizontally) would be the more sensible course of action were often the most successful.

Assessment for learning

When a question involves multiple stages of working that include the need to introduce a certain number of variables that have not been defined in the question it is the responsibility of the candidate to define the variables they use. In this question some candidates did not make it explicitly clear which letters were being used for the forces at different positions on the rod. In this type of question candidates may have benefitted from labelling these forces on the diagram provided in the Printed Answer Booklet.

Exemplar 3

15



$\tan\theta = \frac{b-a}{l}$

~~for horiz~~ for perp,

~~$\mu R_A \cos\theta =$~~

$R_A + R_B = mg\cos\theta$ (1)

par: $\mu R_A + \mu R_B = mg\sin\theta$ (2)

(2) ÷ (1):

$$\frac{\mu R_A + \mu R_B}{R_A + R_B} = \tan\theta$$

$$\frac{\mu\cancel{(R_A + R_B)}}{\cancel{R_A + R_B}} = \frac{b-a}{l}$$

$$\boxed{\mu = \frac{b-a}{l}}$$

This exemplar highlights the all-too-common error seen in this question of a candidate being able to correctly apply the required mechanical principles to solve the problem (and therefore scoring most of the marks available) but incorrectly assuming that the length AB rather than length *l* was the hypotenuse of the triangle shown in their diagram.

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