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# Examiners' report

## A LEVEL

# FURTHER MATHEMATICS B (MEI)

**OCR Level 3 Advanced GCE in Further Mathematics B (MEI)**

**H645**

For first teaching in 2017

## Y420/01 Summer 2025 series

Version 1

ocr.org.uk/maths

# Contents

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

# Paper Y420/01 series overview

The Y420 paper is relatively long and presents a challenging test to candidates. Nevertheless, most candidates found all parts of the paper to be accessible and many scored very highly. Many candidates produced sustained, high-quality work across the whole paper, providing detailed, clearly structured solutions. On most question parts the most commonly given score was full marks.

| Candidates who did well on this paper generally: | Candidates who did less well on this paper generally: |
|---|---|
| • showed all their working and each method they used<br>• set out their working clearly and logically<br>• understood the requirements of the defined command words<br>• gave complete explanations for results they were asked to 'show' or 'prove', supported by precise language and terminology. | • did not apply standard A Level Mathematics techniques when they were expected to, opting to use less efficient and often less successful methods, instead<br>• did not provide the required level of rigour when asked to prove a result<br>• did not provide complete arguments, instead offering more piecemeal working which was often insufficient<br>• did not use all the information given in the question, did not consider every part of it, or did not give their final answer in the form explicitly required by the question<br>• did not use brackets when necessary<br>• were not aware of which formulae are provided in the formula booklet and which are not and quoted formulae incorrectly or used less efficient methods. |

**OCR support**

Section 2b of the specification provides guidance for the 'command word' instructions used in examination questions. Students can download a useful summary sheet from the qualification webpages.

# Section A overview

Most candidates scored very well on Section A and high marks were seen across each question.

Many questions were answered very well. Question 4 (a) was the least well answered.

## Question 1

**1** The complex number $z$ satisfies the equation $z + 2iz^* + 1 - 4i = 0$.

You are given that $z = x + iy$, where $x$ and $y$ are real numbers.

Determine the values of $x$ and $y$. **[4]**

This question was very well answered. A small number of candidates made an arithmetic slip or misread the equation, typically missing the $i$ in the $2iz^*$ term.

## Question 2

**2** **In this question you must show detailed reasoning.**

Find the acute angle between the planes $2x - y + 2z = 5$ and $x + 2y + z = 8$. **[4]**

Most candidates were able to get full marks on this question. Most used the main method given in the mark scheme which was the most efficient. Those candidates who did not get a fully correct answer typically either made a slip in finding the modulus of one of their vectors or, more commonly, spoilt their answer by subtracting it from $90°$ at the end.

## Question 3

**3** Using standard summation formulae, show that, for integers $n \geqslant 1$,

$$1 \times 3 + 2 \times 4 + ... + n \times (n+2) = \frac{1}{6}n(n+1)(an+b),$$

where $a$ and $b$ are integers to be determined. **[5]**

Most candidates answered this question correctly. Errors typically appeared when manipulating the algebra after successfully writing their sum using the standard formulae. Most candidates used the more efficient approach of factorising as soon as possible, although a notable minority expanded and combined their terms before factorising afterwards.

Although errors with sigma notation were not penalised, examiners noted that these appeared frequently. Most commonly, $r$ and $n$ were used incorrectly.

## Question 4 (a)

**4** **(a)** You are given that **M** and **N** are non-singular $2 \times 2$ matrices.

Write down the product rule for the inverse matrices of **M**, **N** and **MN**. **[1]**

Approximately two thirds of candidates answered this question correctly. Of those who did not there was a relatively even split between those who quoted the product rule incorrectly, those who did not recognise it at all and those who gave the rule for the inverse of $\mathbf{NM}$ rather than $\mathbf{MN}$. Given the importance of commutativity in matrix multiplication, or rather the lack of it, examiners were unable to credit this as a misread.

## Question 4 (b)

**(b)** Verify this rule for the matrices **M** and **N**, where

$\mathbf{M} = \begin{pmatrix} a & 1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & b \end{pmatrix}$ and $a$ and $b$ are non-zero constants. **[6]**

Those candidates who answered part (a) correctly were typically able to achieve a fully correct answer in part (b). Those who gave the correct rule for the inverse of $\mathbf{NM}$ were also typically able to verify their rule and were able to receive full credit for doing so. Some candidates necessarily did not gain marks by omitting brackets in this part, in particular struggling to distinguish between $(\mathbf{MN})^{-1}$ and $\mathbf{MN}^{-1}$, or by not clearly labelling the different matrices.

## Question 5

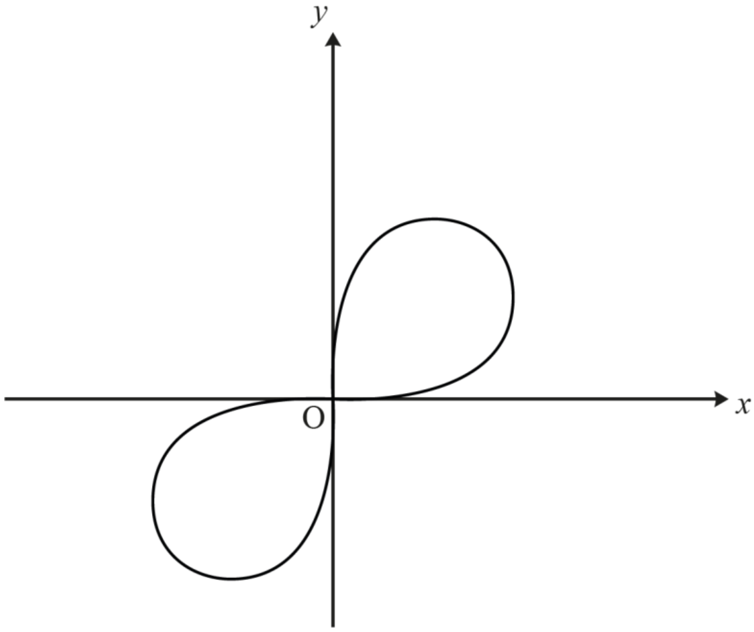
**5** The cubic equation $2x^3 - 3x + 4 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

Determine a cubic equation with integer coefficients whose roots are $\frac{1}{2}(\alpha + 1)$, $\frac{1}{2}(\beta + 1)$ and $\frac{1}{2}(\gamma + 1)$. **[4]**

Most candidates received full marks. Those who used a substitution method most often did so correctly, with only algebraic slips when expanding the cubic preventing full credit from being awarded. Those candidates who considered the sums of roots were generally less successful, with sign errors appearing relatively frequently.

## Question 6 (a)

**6** The figure below shows the curve with cartesian equation $(x^2+y^2)^2 = xy$.




**(a)** Show that the polar equation of the curve is $r^2 = a\sin b\theta$, where $a$ and $b$ are positive constants to be determined. **[3]**

This was correctly answered by most of the candidates.

## Question 6 (b)

**(b)** Determine the exact maximum value of $r$. **[2]**

Most candidates were able to identify the maximum value $r^2 = \frac{1}{2}$ correctly. A common error, however, was to then find $r = \frac{1}{4}$.

## Question 6 (c)

**(c)** Determine the area enclosed by **one** of the loops. **[4]**

This question was answered well, although some candidates did not realise they already had an expression for $r^2$ and need not square again. In these instances, no credit was given.

# Section B overview

Section B contained longer, less structured and less straightforward questions and candidate performance was more variable as a result, although many were able to score very highly. Most candidates demonstrated at least a good understanding of the majority of topics tested and found every question part accessible.

The questions that were answered best were Questions 7, 12 and 15. Questions 8, 14 and 16 were the least well answered overall, along with Question 9 (a) (iii).

## Question 7

7 **In this question you must show detailed reasoning.**

By first expressing $\frac{1}{x^2-4}$ in partial fractions, show that $\int_3^\infty \frac{1}{x^2-4}\mathrm{d}x = \frac{1}{m}\ln n$ , where $m$ and $n$ are integers to be determined. **[8]**

Almost all candidates were able to correctly express the integrand as partial fractions and integrate correctly and a large proportion were able to give the correct final value for the integral. However, only approximately half of candidates were able to provide a complete limit argument. There were a number of errors seen among those who did not, such as struggling to combine their logarithm terms and attempting to argue that two infinitely large terms cancel each other out, confusing $\rightarrow$ and $=$, and using $x$ as a limit.

Some candidates left their answer as $-\frac{1}{4}\ln\frac{1}{5}$ and did not note the form of the final answer required by the question.

## Question 8 (a)

**8** The function $\mathrm{f}(x)$ is defined as $\mathrm{f}(x) = \ln(1+x)$, for $x > -1$.

**(a)** Prove by mathematical induction that the $n$th derivative of $\mathrm{f}(x)$, $\mathrm{f}^{(n)}(x)$, for all $n \geqslant 1$, is given by $\mathrm{f}^{(n)}(x) = \dfrac{(-1)^{n+1}(n-1)!}{(1+x)^n}$. **[4]**

Many candidates struggled with the level of rigour required by a proof by induction question and only approximately one quarter of candidates received full marks as a result.

When differentiating it was not uncommon for candidates to have difficulty in differentiating an expression in terms of $k$ and $x$, with some believing $k$ to be a variable. Others omitted brackets which led to both accuracy marks being lost.

Although a slight majority of candidates were able to reach a result for $n = k + 1$, most did not give their result in terms of $k + 1$ as required, with the most common approach being to reach a target expression which had been previously stated and only given simplified in terms of $k$ (see exemplar). As a result, those candidates who used a target expression approach were typically less successful.

A number of candidates did not provide a clear or correct conclusion, with the most common error typically being having a conclusion which was not suitably conditional. Poor use of 'if… then…' and a lack of clarity as to what had been definitively proven meant that some of those candidates who had provided a complete argument up to that point could not receive full credit.

## Exemplar 1

8(a)

① $n=1$

$$f'(x) = \frac{1}{1+x}$$

$$f'(x) = \frac{(-1)^{1+1}(0)!}{(1+x)^1} = \frac{1}{1+x}$$

$\therefore$ true for $n=1$.

② assume true for $n=k$,

$$f^{(k)}(x) = \frac{(-1)^{k+1}(k-1)!}{(1+x)^k}$$

③ target expression for $n=k+1$,

$$\boxed{f^{(k+1)}(x) = \frac{(-1)^{k+2}(k)!}{(1+x)^{k+1}}}$$

④ proof of target expression

$$f^{(k+1)}(x) = \frac{d}{dx} f^{(k)}(x)$$

$$= \frac{d}{dx}\left(\frac{(-1)^{k+1}(k-1)!}{(1+x)^k}\right)$$

$$= \frac{(-k)(-1)^{k+1}(k-1)!}{(1+x)^{k+1}} \quad \text{(power rule)}$$

$$= \frac{(-1)^{k+2}(k)!}{(1+x)^{k+1}} \quad \text{QED}$$

⑤ conclusion.

if it is true for $n=k$, then it is true for $n=k+1$. since it is true for $n=1$, it is true for all positive integer $n$ such that $n \geq 1$.

In this response the candidate scores B1 for showing that the result holds when $n = 1$ and then M1 for differentiating correctly in the third line of what they have titled 'proof of target expression'. The next line was condoned for the A1 but since an expression in terms of $k + 1$ was not seen anywhere in the response the final A1 mark could not be scored, even though the conclusion given here is otherwise satisfactory.

## Question 8 (b)

**(b)** Hence prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{(-1)^{n+1}x^n}{n} + ...$ for $-1 < x \leqslant 1$.

[You are not required to show this series for $\ln(1+x)$ converges for $-1 < x \leqslant 1$.] **[3]**

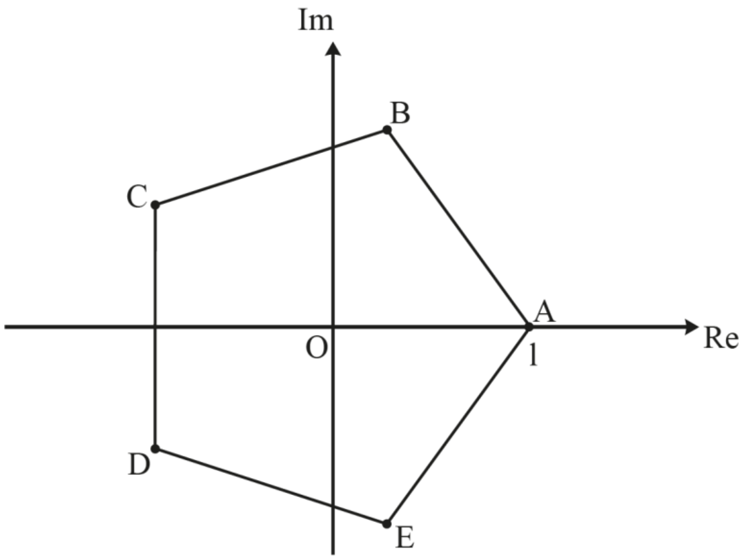
This question proved to be the most challenging on the whole paper, with less than 10% of the cohort scoring full marks here. Most candidates did not heed the instruction 'hence prove that'. Many attempted to derive the result from the general Maclaurin series formula by considering individual terms without referring to their work from part (a); most of those who did refer to the result from part (a) did not recognise the need for generality, instead working with individual terms in the series rather than explicitly considering their sum. Some candidates identified the simplification of the factorial terms and could receive some credit for it. Many did not indicate that the sum is of infinitely many terms, instead giving the series' $n^{th}$ term as its final one.

**Assessment for learning**

When a question uses the word 'hence', it is an indication that the next step should be based on what has gone before. Not doing so could result in no credit being given even if the method used is valid.

## Question 9 (a) (i)

**9** The figure below shows an Argand diagram with a **regular** pentagon ABCDE. The point A represents the real number 1. The point B represents the complex number $w$.




**(a) (i)** Write down, in terms of $w$, the complex numbers represented by the points C, D and E. **[1]**

This was answered correctly by most of the candidates, although only a minority of those gave their answers strictly as powers of $w$; most gave answers such as $we^{\frac{2\pi}{5}\mathrm{i}}$ which were condoned.

## Question 9 (a) (ii)

**(ii)** Write down an equation whose roots are the complex numbers represented by the points A, B, C, D and E. **[1]**

Most candidates answered this question correctly.

## Question 9 (a) (iii)

**(iii)** Show that the sum of these roots is zero. **[2]**

Only approximately one third of candidates gave a fully creditworthy response to this question. A notable proportion did not recognise the geometric series of terms and attempted to write the roots in modulus-argument form and use compound angle formulae and symmetry arguments to reach zero. This approach made no progress towards the answer and so received no credit. As in part (a) (i), it was noted that most responses gave an answer in terms of exponentials, rather than working with $w$ alone, which increased the scope for slips.

## Question 9 (b) (i)

**(b) (i)** Find $w$. Give your answer in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $\theta = k\pi$, where $k$ is a positive constant to be found. **[1]**

This was correctly answered by most of the candidates.

## Question 9 (b) (ii)

**(ii)** By considering the line segment AB, show that the length of each side of the pentagon is $2\sin\frac{\pi}{5}$. **[5]**

Roughly equal numbers of candidates approached this question by strictly considering the modulus of $w - 1$ and by using the cosine rule with triangle AOB. Typically, they were successful in achieving an expression for the length of $AB$ or its square. Many responses lacked sufficient detail when working towards the final answer from this point, however, and many candidates could not be credited with the correct use of the double angle formula as their answers lacked detail and did not include the necessary intermediate steps for the method to be clear.

A slightly less popular method – considering the right-angled triangle which is half of AOB leading to a value for $\frac{1}{2}AB$ – was the most efficient and was generally done very well by those who attempted it. Very few successful responses used the sine rule as candidates were generally unable to use the fact that $\sin\theta = \cos(\frac{\pi}{2} - \theta)$.

## Question 10

**10 In this question you must show detailed reasoning.**

Evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{2}{x^2 - x + 1}\,\mathrm{d}x$. Give your answer in exact form. **[4]**

Almost all candidates heeded the request to show detailed reasoning and explicitly completed the square, integrated and substituted their limits before reaching a final answer. Roughly four fifths received full marks. The most common errors came when integrating, especially when candidates used a substitution to do so. Although many did so successfully, some did not change their variable when integrating and used an implied substitution $x = x - \frac{1}{2}$ which received no credit. Others did not change their limits after substituting. It is encouraged that candidates do so as standard when integrating by substitution so that they do not use incorrect limits.

## Exemplar 2

**10**

$$x^2 - x + 1 = (x - \tfrac{1}{2})^2 - \tfrac{1}{4} + 1 = (x - \tfrac{1}{2})^2 + \tfrac{3}{4}$$

$$\hookrightarrow 2\int_0^{1/2} \frac{1}{(x-\frac{1}{2})^2 + \frac{3}{4}}\,dx = 2\left[\frac{1}{(\frac{\sqrt{3}}{2})}\arctan\left(\frac{x}{(\frac{\sqrt{3}}{2})}\right)\right]_0^{1/2}$$

$$= 2\left[\frac{2\sqrt{3}}{3}\arctan\left(\frac{2\sqrt{3}}{3}x\right)\right]_0^{1/2}$$

$$\tan^{-1}(0) = 0 \longrightarrow \hookrightarrow 2\left(\frac{2\sqrt{3}}{3}\arctan\left[\left(\frac{2\sqrt{3}}{3}\right)\left(\frac{1}{2}\right)\right]\right)$$

$$\hookrightarrow 2\left(\frac{2\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\right)\right)$$

$$\hookrightarrow = 2\left(\frac{2\sqrt{3}}{3}\left[\frac{1}{6}\pi\right]\right) = 2\left(\frac{2\sqrt{3}}{18}\pi\right)$$

$$\hookrightarrow = \underline{\frac{2\sqrt{3}}{9}\pi}$$

In this response the candidate has correctly completed the square for the first M1 mark but has not integrated correctly so they get A0. Given their integral now has $x$ rather than $x - \frac{1}{2}$ the correct limits would now be $0$ and $-\frac{1}{2}$ and so the method mark for substituting in correct limits correctly is withheld.

## Question 11 (a)

**11** The lines $l_1$ and $l_2$ have equations

$$l_1: \frac{x-3}{a} = \frac{y+1}{b} = \frac{z-2}{1} \qquad l_2: \mathbf{r} = 2\mathbf{i}+\mathbf{j}+3\mathbf{k}+\lambda(-\mathbf{i}+c\mathbf{j}+2\mathbf{k})$$

where $a$, $b$ and $c$ are constants.

**(a)** In the case where $c = 0$ and $l_1$ and $l_2$ intersect at right angles, determine the coordinates of the point of intersection of the two lines. **[6]**

The point of intersection was found correctly by most candidates. Most successfully used the scalar product to find $a$ and then correctly solved linear simultaneous equations for $\lambda$ and $\mu$. Examiners were impressed that most candidates did so efficiently, ignoring $b$ which was not required for the final answer.

It was noted, however, that some candidates felt uncomfortable with the second line's equation containing $\lambda$, rather than the first line, and so switched $\lambda$ for $\mu$ in the second equation. Although this was not incorrect, it did lead to some accuracy issues when substituting values back in to find the point of intersection as some did so into the wrong equation.

A few candidates did not gain the final accuracy mark for giving their answer as a position vector rather than coordinates as requested in the question.

## Question 11 (b) (i)

**(b)** Now consider the case where $c = 4$ and lines $l_1$ and $l_2$ are parallel.

**(i)** Find the values of $a$ and $b$. **[3]**

Most candidates successfully found both values. Those who did not typically attempted to use the scalar product or the vector product, often setting them equal to 1.

## Question 11 (b) (ii)

**(ii)** Determine the distance between the two lines. **[5]**

Candidates used a variety of methods to determine the distance between the lines and were often successful in doing so. The most common approach was to find the vector product of a vector between one point on each line and a multiple of the direction vector. This vector product had to be seen for full credit to be given.

The main errors when following this method were arithmetic and sign slips in finding the vector product and the moduli of vectors. Some candidates confused where they should use position vectors and direction vectors.

**Assessment for learning**

When a question asks candidates to determine a result, they should give justification for any results found, including working where appropriate.

## Question 12

**12 In this question you must show detailed reasoning.**

The roots of the equation $z^4 - z^3 + cz^2 + dz + 18 = 0$ are $\alpha$, $\frac{2}{\alpha}$, $\beta$ and $-\beta$.

Determine, in any order, the exact values of the following.

- The **four** roots of the equation
- The value of $c$
- The value of $d$ **[8]**

This question was very well answered and most candidates scored at least 5 marks. For the last 3 marks, awarded for finding $c$ and $d$, some candidates made sign errors, either in manipulation of their products or, more fundamentally, in the sign of $d$.

A number of successful alternative approaches were used, with the most successful being substituting a known root into the equation leading to equations for the real and imaginary components. This minimised the scope for slips.

## Question 13 (a)

**13** The gradient of a curve $y = \mathrm{f}(x)$ satisfies the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 2 + x^2$.

**(a)** Show that the integrating factor for this differential equation is $x^{-2}$. **[3]**

This question was the single best-answered on the paper. A small number of candidates did not show division by $x$ in their initial step.

## Question 13 (b)

**(b)** You are given that the curve passes through the point $(1, 0)$.

By solving this differential equation, determine the exact $x$-coordinate of the stationary point on the curve $y = \mathrm{f}(x)$. **[8]**

Successful responses here required the accurate application of correct methods across multiple steps and almost half of responses were fully correct. Most candidates identified each of the steps correctly and began each one, although accuracy issues were costly for some. These were most common when dividing through by $x^{-2}$, resulting in either incorrect index terms appearing or the constant of integration term inexplicably disappearing despite having been found successfully in the preceding step.

After differentiating and equating to 0, some candidates did not know how to solve with an equation including both $x$ and $\ln x$. Others did so successfully but did not reject $x = 0$ as a solution, as was required.

Few candidates stopped after finding an equation for $y$, seemingly expecting that this was all that was required.

Some candidates used incorrect notation, using $\frac{\mathrm{d}y}{\mathrm{d}x}$ for $\frac{\mathrm{d}}{\mathrm{d}x}$, which was not condoned by examiners.

**Read the question**

Candidates should make sure they are clear on exactly what the question is asking them to do. Questions may require additional results on top of what candidates might assume is the final answer.

## Question 14 (a)

**14 (a)** By using the definition of $\cosh x$ and $\sinh x$ in terms of $e^x$ and $e^{-x}$, show that $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$. **[2]**

Although most candidates seemed to know the exponential definitions of $\cosh x$ and $\sinh x$ and were able to manipulate these correctly, many did not provide a complete argument, omitting either $\cosh^2 x + \sinh^2 x$ or $\cosh 2x$ from their answer.

Examiners noted that many candidates did not structure their answers clearly or sequentially, with steps instead scattered around the answer space.

### Exemplar 3

| | |
|---|---|
| **14(a)** | $\frac{1}{4}(e^{x} + e^{-x})^2 + \frac{1}{4}(e^{x} - e^{-x})^2$ |
| | $= \frac{1}{4}(e^{2x} + 2 + e^{-2x}) + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$ |
| | $= \frac{1}{4}(2e^{x} + 2e^{-x})$ |
| | $= \frac{1}{2}(e^{2x} + e^{-2x})$ |
| | $= \cosh 2x$ |

In this response the candidate has worked with the correct exponential definitions and provided an almost complete argument finishing with $\cosh 2x$. However, they have not stated that they are working with $\cosh^2 x + \sinh^2 x$ and so their argument is incomplete and M1 A0 is awarded.

## Question 14 (b)

**(b)** The transformation T of the plane has associated matrix **M**, where $\mathbf{M} = \begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}$ and $x > 0$.

Show that T transforms the unit square with coordinates (0, 0), (1, 0), (0, 1) and (1, 1) to a rhombus of unit area. **[6]**

Notably few candidates provided a complete answer to this question, although most were nevertheless able to achieve a significant number of marks. Candidates were required to consider three parts: the images of the points, the area of the image and the proof of the rhombus.

The images of the points were found very successfully. The most common mistake was to give the image of $(1,1)$ as the product of $\sinh x$ and $\cosh x$, rather than their sum.

A very large majority of candidates found the determinant of the transformation matrix and stated this was equal to 1 but many equated this to the area itself without first justifying this as the scale factor of enlargement. Some evidence of this was required for the accuracy mark.

Most commonly candidates provided insufficient detail in their work to show that the resulting image was a rhombus. Although almost all candidates who attempted this part of the question knew some necessary conditions for a shape to be a rhombus, many did not know which were sufficient. The most efficient approach was to show that all four sides had the same length, but candidates almost always stated that they were the same without work to derive the lengths of all four sides. Since the result that the image is a rhombus is given, this fact is already known, and the second A1 mark was only rarely awarded.

Some candidates did not recognise the need to consider all three aspects and instead only considered two.

**Assessment for learning**

The command words 'show that' indicate that a given result must be achieved from the starting information. Because the result has been given, the explanation has to be sufficiently detailed to cover every step of the working.

## Question 14 (c)

**(c)** You are given that the length of each side of the rhombus is 2 units.

Determine the exact value of $x$. Give your answer in logarithmic form. **[2]**

Roughly half of candidates received full marks for this question. The most common error was to equate the square of the length of each side with 2 rather than the length of each side itself.

After writing a correct initial equation some candidates used the exponential definition of $\cosh 2x$ and worked with exponentials, rather than using the arcosh function immediately. Besides being an inefficient method, those who did so commonly made slips and so did not find the correct final answer.

**Formula booklet**

Candidates should be aware of which formulae are given to them in the formula booklet, and which are not. They should be encouraged to make efficient use of the formulae provided, particularly when dealing with hyperbolic functions.

## Question 15 (a)

**15 (a)** Show that $(3-\mathrm{e}^{4\mathrm{i}\theta})(3-\mathrm{e}^{-4\mathrm{i}\theta}) = a + b\cos 4\theta$, where $a$ and $b$ are integers to be determined. **[2]**

Most candidates provided a fully correct response. When they did not errors were typically accuracy-related, such as writing $4\theta$ as $\theta$ or $\sin 4\theta$ instead of $\mathrm{i}\sin 4\theta$ when writing their terms in modulus-argument form.

## Question 15 (b)

The infinite series $C$ and $S$ are defined as follows.

$$C = \cos\theta + \frac{1}{3}\cos 5\theta + \frac{1}{9}\cos 9\theta + \frac{1}{27}\cos 13\theta + \ldots$$

$$S = \sin\theta + \frac{1}{3}\sin 5\theta + \frac{1}{9}\sin 9\theta + \frac{1}{27}\sin 13\theta + \ldots$$

**(b)** Show that $C + \mathrm{i}S = \dfrac{3\mathrm{e}^{\mathrm{i}\theta}}{3-\mathrm{e}^{4\mathrm{i}\theta}}$. **[4]**

This question was very well answered. Most candidates recognised the geometric series and correctly used and simplified the sum to infinity formula.

## Question 15 (c)

**(c)** Hence show that $C = \dfrac{9\cos\theta - 3\cos 3\theta}{10 - 6\cos 4\theta}$. **[3]**

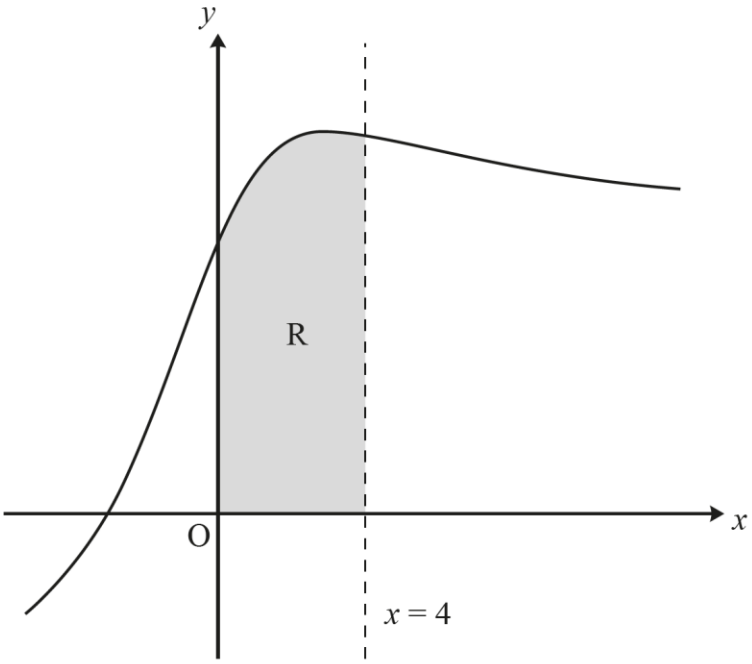
This question was well-attempted but only a little over half of candidates received full marks. Most correctly multiplied through, using their results from parts (a) and (b), but many then neglected to use modulus-argument form to establish the result, instead immediately jumping to a result that $C = \frac{9\cos\theta - 3\cos(-3\theta)}{10 - 6\cos 4\theta}$. With the command 'show that' this was not sufficient for full credit.

Bracketing errors – typically omitting them entirely – was a common error which generally led to the final 2 marks being withheld. Some candidates provided an almost fully correct answer but did not include $C =$ anywhere in their answer, either leaving the left-hand side of the result to be assumed or leaving it only as $\mathrm{Re}(C + \mathrm{i}S)$; these were both insufficient.

## Question 16 (a)

**16 In this question you must show detailed reasoning.**

The diagram shows the curve with equation $y = \frac{x+3}{\sqrt{x^2+9}}$.




The region R, shown shaded in the diagram, is bounded by the curve, the $x$-axis, the $y$-axis, and the line $x = 4$.

**(a)** Determine the area of R. Give your answer in the form $p + \ln q$ where $p$ and $q$ are integers to be determined. **[6]**

Many candidates found this question to be particularly challenging and fewer than half scored full marks on it. Those who split the fraction and then recognised they could use standard results typically made good progress and scored at least 4 marks. However, many did not recognise they could integrate immediately using results learned in A Level Mathematics and many different substitutions were attempted. These were inefficient and the extra work they required increased the scope for accuracy errors and slips. Generally, candidates were much less likely to reach a correct final answer after substituting. Some candidates attempted to integrate by parts twice but rarely made substantial progress.

It was noted that many candidates persevered with inefficient methods over a number of additional pages without reaching an answer rather than considering trying a different method instead.

Some candidates seemingly did not read the question sufficiently closely and gave a final answer of $2 + 3\ln 3$ which meant the final accuracy mark was withheld.

**Misconception**

The content of Y420 Core Pure builds on top of the pure content of A Level Mathematics and the mathematics it covers is not entirely distinct from that seen in A Level Mathematics. Candidates should expect to use A Level Mathematics techniques, including techniques for integration, in their work on Core Pure.

## Question 16 (b)

The region R is rotated through $2\pi$ radians about the $x$-axis.

**(b)** Determine the volume of the solid of revolution formed. Give your answer in the form $\pi\left(a+b\ln\left(\frac{c}{d}\right)\right)$ where $a$, $b$, $c$ and $d$ are integers to be determined. **[6]**

This part of the question was marginally better answered than part (a) overall. Again, those who wrote their integral as the sum of proper fractions and integrated straight away were typically able to get at least 4 marks. Few attempts at integrating by substitution were successful; the most commonly attempted was to let $x = 3\tan u$.

Across both parts of Question 16 it was clear that some of those candidates who attempted substitutions were unclear on whether to use a circular or hyperbolic one. Confusion between $\tan u$ and $\tanh u$ was especially common. Additionally, examiners noted that some candidates attempted to write $\arctan x$ using logarithms.

## Question 17 (a) (i)

**17** A researcher is modelling the height of a particular type of tree over its lifetime.

Data suggests that the maximum possible height of this type of tree over its lifetime is double the height of the tree 5 years after planting.

It is given that, $t$ years after planting a seed for this type of tree, the corresponding height of the tree is $h$ m, and that $h = 0$ when $t = 0$.

**(a)** The researcher first models the height of the tree by assuming that the rate of increase of $h$ is proportional to $(20-h)$, with constant of proportionality 0.2.

**(i)** Write down the first order differential equation for this model. **[1]**

Most responses were correct.

## Question 17 (a) (ii)

**(ii)** Show that this model predicts that the maximum possible height of the tree is 20 m. **[1]**

This part was also very well answered with most responses correctly finding $h = 20$ after setting $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$.

## Question 17 (a) (iii)

**(iii)** Show by integration that $h = 20(1 - e^{-0.2t})$. **[4]**

Candidates were relatively evenly split between separating variables and using an integrating factor to obtain the required result. Regardless of method most did so successfully.

## Question 17 (a) (iv)

**(iv)** Determine whether this model's prediction for the height of the tree 5 years after planting is consistent with the maximum possible height of the tree being 20 m. **[2]**

Most candidates were able to secure at least 1 mark by substituting $t = 5$ into the expression for $h$. Explicit comparison with the relevant value was then required for full credit to be awarded: the resulting value had to be compared with 10 or doubled in order to be compared with 20. Some candidates did not do this, typically comparing 12.64 with 20. An explicit comment of the model's inconsistency was also required which some candidates did not provide, stating only that 12.64 is not equal to 10.

It was noted by examiners that many candidates either wrote or implied that consistent results required the values to be equal, not simply close.

## Question 17 (b) (i)

**(b)** The researcher refines the model for the height of the tree using the following second order differential equation.

$$\frac{d^2h}{dt^2} + 0.3\frac{dh}{dt} + 0.02h = 0.4$$

**(i)** Determine the general solution of this second order differential equation. **[4]**

This was well answered by most of the candidates. Almost all were able to get at least 2 marks for finding the correct complementary function. Those few who did not go on to get a fully correct answer either used $x$ and $y$ in place of $t$ and $h$ or did not realise they needed to find a particular integral.

Slips were more common when candidates chose a linear or quadratic form for their particular integral, both of which worked provided they had a constant term, but which increased the scope for slips.

## Question 17 (b) (ii)

**(ii)** Show that the refined model also predicts that the maximum possible height of the tree is 20 m. **[1]**

Although no question on the paper was often left unanswered, this one was the most commonly left blank, with few candidates making no attempt to answer it. A little under half of candidates scored the mark for this question.

A clear argument that both exponential terms, or their sum, would tend to zero as $t$ tended to infinity was required. Some candidates lacked precision in their answers, for instance arguing that as $t$ grew the exponential terms became negligible. This was not sufficient. Others stated only that as $t \to \infty$, $h \to 20$ without showing why.

## Question 17 (b) (iii)

Further research determines that the initial rate of growth of this type of tree is 2.9 metres per year.

**(iii)** By applying the initial conditions to find the particular solution of this differential equation, determine whether the refined model's prediction for the height of the tree 5 years after planting is consistent with the maximum possible height of the tree being 20 m. **[5]**

Over half of candidates received full credit for this question. Errors tended to result from arithmetic slips when differentiating or solving for the constants. As in Question 17 (a) (iv) some candidates again did not compare appropriate values or did not make a comment about the consistency of the model.

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