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Examiners' report

A LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced GCE in Further Mathematics A

H245

For first teaching in 2017

Y543/01 Summer 2025 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y543 series overview

This proved to be a good standard of paper to test candidates' understanding of the course. At this level the higher scoring candidates confidently completed questions that involved resolving velocities and forces. Standard calculations were completed with ease and formal algebraic methods were used throughout. Questions 7 (c) and 8 (c) (i) and 8 (c) (ii) were less accessible to candidates, thoroughly testing problem solving and justification of conclusions. A number of candidates are showing a lack of understanding of the command words; missing necessary justification on 'show that' or 'determine' questions.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed clear labelled diagrams • set out work clearly and logically, using algebra confidently • considered their answers in the framework of the real-life context given.	• confused key formulae, for example in the context of elastic potential energy stored and Hooke's Law • missed out steps in their working in 'determine' and 'show that' questions • omitted multiple questions • were unable to calculate dot products.

Question 1 (a)

1 A particle P of mass 4 kg is moving in a straight line with a speed of $7\,\mathrm{m\,s^{-1}}$ on a smooth horizontal surface.

A constant force of magnitude 12 N acts on P for 5 seconds in the same direction as P's motion.

(a) Find the magnitude of the impulse exerted by the force on P. You must include the units. **[2]**

This question was answered well by the majority of candidates. Very few candidates were not credited full marks. Most commonly these students were uncertain about the units of impulse, misremembering these as $\mathrm{Ns^{-1}}$, even though they could correctly calculate its magnitude.

Question 1 (b)

(b) Hence determine the speed of P at the instant when the force stops acting. **[2]**

Many candidates were able to calculate speed through correct use of the impulse-momentum principle. A minority used kinematics as an alternative to calculate the speed, arriving at a correct answer via an alternative method to that specified in the question.

Question 1 (c)

(c) By considering the change in the kinetic energy of P, determine the work done by the force over the 5 seconds that it acts. **[2]**

The majority of candidates were able to effectively calculate the kinetic energy using their calculated speed from part c. As a determine question candidates were expected to show substitution of values into the correct formula before manipulating to arrive at their answer.

Question 1 (d)

(d) Hence find the distance travelled by P while the force is acting on it. **[2]**

Most of the candidates were able to answer this part fully correctly, making effective use of their calculated work done from part c. A small minority of candidates chose to approach this part via constant acceleration formulae, typically achieving a correct answer but losing a mark for not following the specified approach.

Question 2 (a)

2 Dynamic viscosity is a physical property of liquids and is denoted by η.

The units of η are $\mathrm{Ns/m^2}$.

(a) Determine the dimensions of η. **[3]**

The vast majority of candidates were credited these marks. Some candidates left [ ] around their dimensions which is not the correct notation and was not credited. A small minority chose to work with units rather than dimensions, which was only partially credited in this 'determine' question that expected clear justification using dimensional analysis.

Question 2 (b)

Kinematic viscosity is another physical property of liquids and is denoted by ν.

Kinematic viscosity and dynamic viscosity are related by the formula $\nu = \frac{\eta}{\rho}$ where ρ is the density (mass per unit volume) of the liquid.

(b) By finding the dimensions of ν, state a possible unit for ν. **[3]**

Candidates who had completed part (a) well, typically completed this part equally well. Most candidates could effectively state the dimensions of density. This was a 'find' question, however candidates benefitted from showing their working in order to avoid arithmetic errors when simplifying indices.

Question 3 (a)

3 **In this question you must show detailed reasoning.**

A particle P of mass 3.25 kg is moving on a smooth horizontal plane with velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j}$.

(a) Find the kinetic energy of P. **[2]**

Throughout this question, the **detailed reasoning** requirement meant candidates needed to show their working, ensuring clear explanations were shown. Candidates typically completed this efficiently, showing the result of their dot product embedded in their working. Successful alternative methods proceeded by squaring the magnitude of the velocity vector. A few candidates neglected to show their working and were only given partial credit for their answer.

Question 3 (b)

A system of horizontal forces starts to act on P. One of the forces is $\mathbf{F}$ N where $\mathbf{F} = \mathbf{i} - (3+t)\mathbf{j}$ and t is the time, in seconds, after the system of forces starts to act.

(b) Find the power developed by $\mathbf{F}$ at time $t = 0$. **[2]**

Candidates who answered the first part of the question generally were able to answer this question as well. Those who substituted t=0 into the force first, proceeded a little more efficiently. Of the small number of candidates who were given no marks, the most common error was to consider the scalar values of the velocity and the force at time t=0 before incorrectly multiplying these together.

Question 3 (c)

(c) Given that the velocity of P does not change at any time, find the **total** power developed by the forces other than $\mathbf{F}$ at time $t = 6$. **[2]**

This part proved more challenging, with few candidates completing it correctly. The most common wrong answer was 58W, with candidates discarding the negative sign produced, or calculating the total power developed by the original force. A number of incorrect units were seen but not penalised in this instance. Some candidates attempted to integrate from 0 to 6s rather than consider an instantaneous power at t=6. Some candidates calculated the dot product correctly and then added another incorrect step such as subtracting the power at t=0.

Misconception

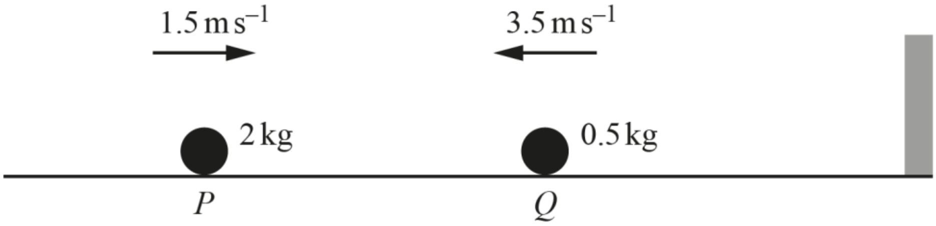
?

Confusion about the validity (and inherent meaning) of the negative sign produced when calculating the power developed by forces via a dot product. Candidates tended to discard the sign, deciding that a scalar quantity should not include it, rather than understanding that it was a necessary part of the answer.

Question 4 (a)

4 Two particles, P of mass 2 kg and Q of mass 0.5 kg, are moving on a smooth horizontal surface.

Particle P is moving with speed $1.5\,\mathrm{m\,s^{-1}}$ directly towards a vertical wall. Particle Q is between P and the wall and is moving directly towards P with speed $3.5\,\mathrm{m\,s^{-1}}$ (see diagram).



The particles collide directly. After the collision, P moves away from the wall with speed $0.3\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between P and Q is denoted by e.

(a) Determine the value of e. **[4]**

Candidates generally completed this part well. Clear diagrams were typically included. If errors were seen, they tended to be either in using the correct signs when combining the speeds of the two particles to find the coefficient of restitution or accidentally using a mass of one of the particles in place of a velocity.

Question 4 (b)

After this collision, Q goes on to collide directly with the wall. The coefficient of restitution between Q and the wall is $\frac{1}{10}e$.

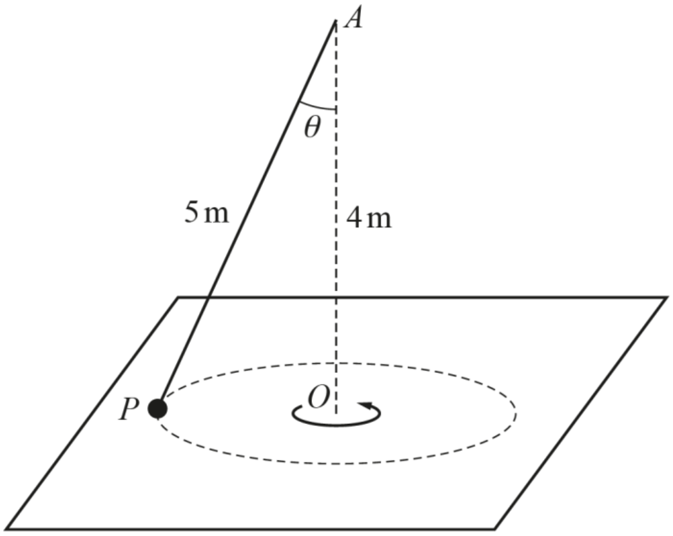
(b) Determine whether there will be a subsequent collision between P and Q. **[2]**

This part was also completed competently in general. As expected, many candidates wrote a formal conclusion for this 'determine' question, showing clear comparison of the velocities of P and Q, before concluding that there would be no subsequent collision. A common error here involved use of 1/10 instead of e/10 for the coefficient of restitution at the wall, resulting in the wrong conclusion as to whether there would be a further collision between P and Q.

Question 5 (a)

5 One end of a light inextensible string of length 5 m is attached to a fixed point A. The other end is attached to a particle P of mass m kg which lies on a smooth horizontal plane.

With the string taut, P moves on the plane with constant speed $v\,\mathrm{m\,s^{-1}}$ in a circular path. The centre of the circular path is the point O which is 4 m vertically below A. The string makes an angle of θ with the downward vertical through A (see diagram).



(a) Given that the magnitude of the contact force between P and the surface is equal to the tension in the string, determine the value of v. **[4]**

This was an accessible question for candidates, with many able to effectively resolve forces horizontally and vertically for P, resulting in an accurate speed. Common errors seen included resolving tension as $T = mgcos(\theta)$, calculating the radius for P as 5 rather than 5sinθ, or confusing sinθ and cosθ when resolving. A few candidates neglected the contact force in their equations, limiting their ability to effectively tackle the question.

Question 5 (b)

(b) Given instead that P is about to lose contact with the surface, determine the angular velocity of P. **[3]**

Typically, candidates who had tackled the first part effectively, were able to adjust their equations by setting the contact force to be equal to zero. Some candidates stopped before finding angular speed, not able to relate this to their calculated *v*.

Question 6 (a)

6 One end of a light inextensible string of length r m is attached to a fixed point O. The other end is attached to a particle P of mass m kg.

Particle P hangs in equilibrium vertically below O with the string taut. An initial horizontal impulse is applied to P so that it instantaneously starts moving horizontally with a speed of $u\,\mathrm{m\,s^{-1}}$.

Particle P subsequently moves in complete circles in a vertical plane containing O.

When the line OP makes an angle of θ with the downward vertical through O, the speed of P is $v\,\mathrm{m\,s^{-1}}$ and the tension in the string is T N.

(a) Show that $T = \frac{m}{r}\left(u^2 + \alpha gr + \beta gr\cos\theta\right)$ where α and β are constants to be determined. **[6]**

Many candidates made a strong start to this question, giving a thorough consideration of conservation of energy to derive an expression for v^2. In the main algebraic steps were shown clearly, with some candidates making sign errors as they attempted to expand a bracket and collect terms in one step. Most commonly students who found the second part of the derivation more challenging had neglected the component of weight in their construction of Newton's 2nd Law.

Question 6 (b)

(b) Determine, in terms of m, g and r, the minimum initial impulse required to ensure that P describes complete vertical circles around O with the string remaining taut. **[3]**

This question was completed more variably. Just under half of the candidates were able to complete this question successfully, giving a clear calculation of minimum initial impulse. Many candidates had the correct instinct that tension should be greater than or equal to zero but then attempted to combine this with the incorrect statement that the velocity should be zero. Some candidates chose to restart completely in this part, using Newton's 2nd Law to find the minimum velocity needed at the top of the vertical circle and then weaving this into energy considerations to arrive at the correct minimum initial impulse.

Question 7 (a)

7 A particle P of mass 36 kg moves in a straight line on a smooth horizontal plane. At time t seconds the displacement of P from a fixed point O is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. Initially, P is at rest at O.

At time $t = 0$, P is acted on by a horizontal force, directed along the line. The magnitude of the force is proportional to $\sqrt{9-v^2}$. When $v = 2.4$, the magnitude of the force is 32.4 N. When the velocity of P reaches $3\,\mathrm{m\,s^{-1}}$, the force ceases to act.

(a) Show that while the force is acting the motion of P is modelled by the differential equation $\frac{1}{\sqrt{9-v^2}}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}$. **[3]**

The majority of the candidates answered this question comprehensively showing all steps needed to gain 3 marks. In this 'show that' questions, candidates needed to show substitution for the mass, force and acceleration in Newton's 2nd Law before writing the final expression.

Question 7 (b)

(b) Hence find v in terms of t, showing that this expression for v is valid for $0 \leqslant t \leqslant \pi$. **[5]**

Many candidates were confident in stating the result of integration for both sides of the differential equation. Algebraic solutions were typically well set out. Candidates should be encouraged to show clear substitution when finding integration constants. Only a small number of candidates were able to fully justify the validity of their expression for *v.* While many candidates could state the values of *v* that related to $t = 0$ and $t = \pi$, few referred to the key idea that this was the first time that this function reaches a velocity of 3, at which point the force ceased to act. Some candidates effectively considered the graph of the function to aid them in their analysis.

Question 7 (c)

(c) Determine the total distance travelled by P in the interval $0 \leqslant t \leqslant 10$. Give your answer in an exact form. **[4]**

A significant minority of candidates answered this part effectively, confidently considering the two-stage journey; using integration with correct limits to find the length of journey for the first part and combining this with the second stage travelling at constant velocity. Many candidates integrated the function found in part (b) correctly and then assumed that this held for the full 10 seconds and were unable to gain any further marks from this point.

Exemplar 1

7(c)

$v = 3\sin\left(\frac{1}{2}t\right) \qquad \int v\,dt = x \qquad v = \frac{dx}{dt}$

$$\int_0^{\pi} 3\sin\left(\tfrac{1}{2}t\right)\,dt = \left[-3(2)\cos\left(\tfrac{1}{2}t\right)\right]_0^{\pi}$$

$$= \left(-6\cos\left(\tfrac{1}{2}\pi\right) + 6\cos 0\right) = 6$$

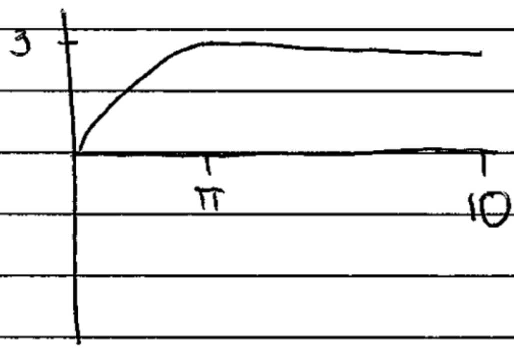
$10-\pi$ to 10 no force

$3\cos^{-1}$

$36g$

$3 \times (10-\pi) = (30 - 3\pi)\,m$

$30 - 3\pi + 6 = 36 - 3\pi$



This candidate has shown clear consideration of both stages of the journey, arriving at a well explained calculated distance. Although the sketch of velocity is not needed it has provided them with a useful scaffold on which to base their explanation.

Question 8 (a) (i)

8 In this question you must show detailed reasoning.

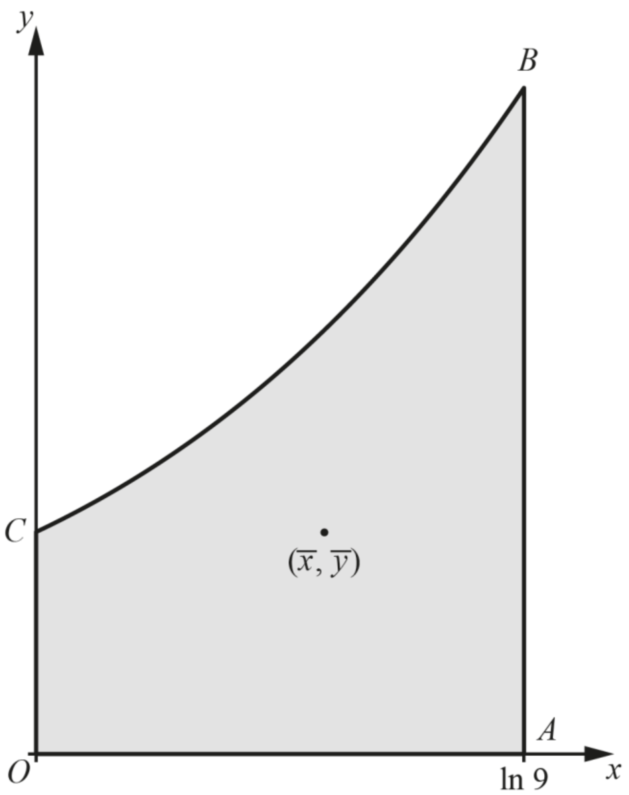
The units of the coordinate system in this question are metres.

A designer is constructing a sign $OABC$ which is to be made from a flat board.

The designer creates two possible models for the sign, Model 1 and Model 2. In both models the sign is modelled by a uniform lamina.

In Model 1, the lamina occupies the region bounded by the x-axis, the lines $x = 0$ and $x = \ln 9$ and the curve with equation $y = \mathrm{e}^{0.5x}$.

The vertices of the sign are the points $O(0, 0)$, $A(\ln 9, 0)$, $B(\ln 9, 3)$ and $C(0, 1)$.The centre of mass of the lamina in Model 1 is at the point with coordinates $(\overline{x}, \overline{y})$ (see diagram).



(a) (i) Show that $\overline{x} = 3\ln 3 - 2$. **[4]**

This question, as a 'show that' question needed to involve clear reasoning with algebraic integration shown. Almost all candidates understood the underlying concept here but needed to show clear substitution of limits to arrive at the given answer in order to gain full marks.

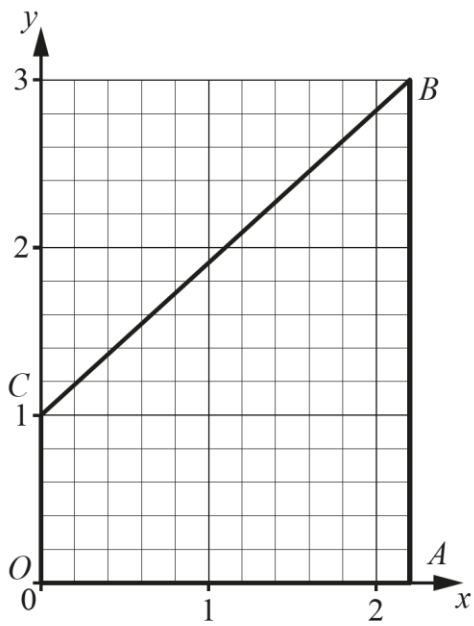
Question 8 (a) (ii)

(ii) Find the value of $\overline{y}$. **[2]**

The candidates who achieved full marks for part (a) (i) generally were credited full marks for this part too. A minority had not recalled the formula correctly, neglecting the 0.5 needed in the numerator. As in the first part, candidates needed to show detailed reasoning to gain both marks.

Question 8 (b)

Model 2 is a simplified model for the sign. In this model, the lamina is in the shape of a trapezium with vertices at the same points O, A, B and C as in Model 1 (see diagram).



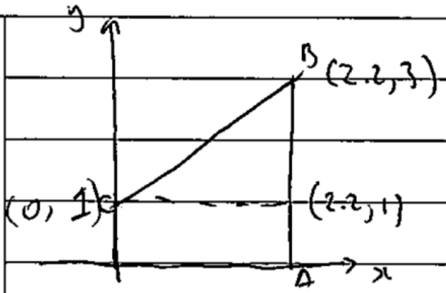
The centre of mass of the lamina in Model 2 is at the point with coordinates $(\overline{X}, \overline{Y})$.

(b) Find the value of $\overline{X}$ and the value of $\overline{Y}$. **[3]**

Many candidates were able to answer this question effectively and efficiently, supported by a clear split of the given trapezium into a rectangle and triangle. Some candidates struggled to find the centre of mass of the triangle or chose to split the shape into more than two pieces, which was a more demanding approach. Some misapplied the co-ordinate average method which works for triangles to the trapezium, for which it does not work. Those candidates who used a table or vectors to consider distances generally had good success, helped in part by the equal masses of the rectangle and triangle. Some candidates attempted to integrate to find the centre of mass, experiencing initial challenges in finding a sufficiently accurate gradient for their line.

Exemplar 2

8(b)



Shape	mass	com $\bar{x}$	com $\bar{y}$
triangle	2.2	$\frac{22}{15}$	$\frac{5}{3}$
rectangle	2.2	1.1	0.5
whole	4.4	$\bar{x}$	$\bar{y}$

$$\frac{2.2+2.2+0}{3} = \frac{22}{15}$$

$$\frac{3+1+1}{3} = \frac{5}{3}$$

$$4.4\bar{X} = 2.2\left(\frac{22}{15}\right) + 1.1(2.2)$$

$$4.4\bar{X} = \frac{847}{150}$$

$$\bar{X} = \frac{77}{60} = 1.28$$

$$4.4\bar{Y} = 2.2(0.5) + 2.2\left(\frac{5}{3}\right)$$

$$4.4\bar{Y} = \frac{143}{30}$$

$$\bar{Y} = \frac{13}{12} = 1.08$$

$\bar{X} = 1.28$ $\bar{Y} = 1.08$

This candidate has made concise and effective use of a diagram and table to consider distances and masses before combining these to accurately find the coordinates of the centre of mass.

Question 8 (c) (i)

The sign is placed on a rough plane inclined at 50° to the horizontal. The line OA rests on the plane with O below A. The line OA lies along a line of greatest slope of the plane.

You may assume that all forces acting on the sign lie in the vertical plane containing OA and the plane is sufficiently rough to prevent sliding.

(c) **(i)** You are given that $\bar{x}$ and $\bar{y}$ from **Model 1** are both within 0.04 m of the actual coordinates of the centre of mass of the sign.

Determine whether the sign will topple. Justify your answer. **[3]**

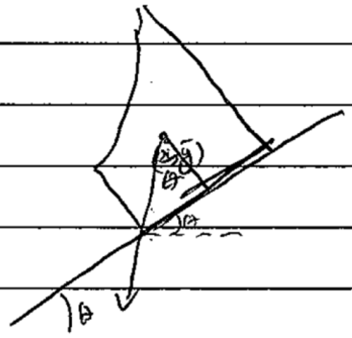
A minority of candidates managed this completely, with a small number calculating relevant angles and quoting an angle between OA and the vertical rather than the horizontal. Some were able to find an initial angle without considering the 0.04m errors correctly to arrive at a 'worst case' for toppling. In some cases, they had unfortunately considered the incorrect model. Sophisticated solutions were seen using a diverse choice of methods, including moments, comparison of distances parallel or perpendicular to the plane or comparison of angles. A good diagram often helped candidates to consider which way the error should be applied to give the 'most likely to topple' situation. In some cases, candidates had produced accurate calculations but then linked these to the incorrect conclusion.

Assessment for learning

Candidates will find it useful to produce clear diagrams in the space on their printed answer booklet, annotating these with all information from the question to support them in accurately answering toppling questions.

Exemplar 3

8(c)(i)



$\bar{x} = 1.296 \pm 0.04$

$\bar{y} = 1 \pm 0.04$

$\tan^{-1}\left(\dfrac{1.296 \pm 0.04}{1 \pm 0.04}\right)$

Min angle $= \tan^{-1}\left(\dfrac{1.296 - 0.04}{1 + 0.04}\right) = 50.4°$ (3s.f)

$50.4° > 50°$ so the sign will topple

This response begins with a clear diagram, suggesting a strong understanding of the relationship between the angle of incline and the angle between the centre of mass and the vertical. Although it moves to an initially ambiguous consideration of the tangent of the angle, it continues with a correct minimisation to arrive at the minimum angle for toppling. This is followed by a comparison to the correct angle, with an incorrect conclusion seed. 2 out of 3 marks awarded.

Question 8 (c) (ii)

(ii) Use your answer to part **(b)** to explain whether **Model 2** would have predicted the same result as in part **(c)(i)**. **[1]**

Only a few candidates completed this correctly. Common errors included: considering Model 1 rather than Model 2, calculating the correct angle and then stating the incorrect conclusion. Candidates should be encouraged to support assertions with calculations in order to make sure they have fully explored the given situation.

Question 9

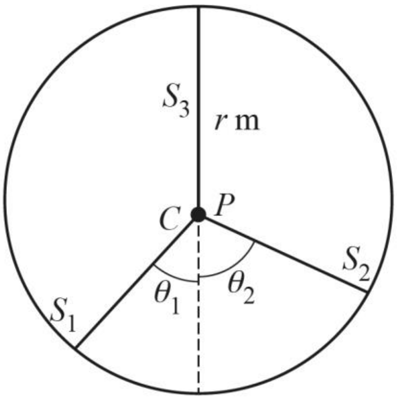
9 In this question you must show detailed reasoning.

A rigid circular frame of centre C and radius r m is fixed in a vertical plane. A particle P of mass 1.25 kg is attached to one end of each of three elastic strings, S_1, S_2 and S_3.

The natural length of each string is $\frac{2}{3}r$. The modulus of elasticity of S_1 is 20 N. The other ends of the strings are attached to the circular frame in such a way that P remains in equilibrium at C.

The string S_3 is attached to the frame at the point vertically above C while S_1 and S_2 are attached at points such that S_1 makes an angle of θ_1, and S_2 an angle of θ_2, with the downward vertical through C, one on each side of this vertical (see diagram).

You are given that $\sin\theta_1 = \frac{3}{5}$ and $\sin\theta_2 = \frac{12}{13}$.



Find, in terms of r, the total elastic potential energy stored in all three strings. **[8]**

Over a quarter of candidates were completely successful, with good efforts at resolving forces seen by many. The most common error seen was missing the weight out of their vertical application of Newton's 2nd Law. Another common error was to assign the given modulus of elasticity of 20N to string 3 instead of string 1. A few candidates calculated the extension incorrectly or were uncertain how to calculate it, making it challenging to make further progress at this point. Some students incorrectly put three weights at the attachment points on the circle. Almost all candidates showed good understanding of the differences between Hooke's Law and the elastic potential energy formula.

Assessment for learning

Candidates should be encouraged to work systematically through equilibrium problems, resolving in two orthogonal directions, in addition to giving clear statement and use of both Hooke's Law and elastic potential energy calculations.

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