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Examiners' report

A LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced GCE in Further Mathematics A

H245

For first teaching in 2017

Y542/01 Summer 2025 series

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y542 series overview

Many candidates were well prepared for this paper. Many of the solutions to significance tests were fully accurate. The paper was found to be largely accessible, although there were several details to stretch the more successful candidates. Candidates who scored very high marks on this paper certainly had a good knowledge and understanding of statistics.

Two issues that were identified in the previous series remained, although there was some improvement. Many candidates misunderstood the Central Limit Theorem (CLT). Many also thought that the statement 'If X has a Poisson distribution, then $E(X) = Var(X)$' applies to the *sample* means. Such candidates made statements such as '4.04 is not equal to 3.5184, so the distribution is not Poisson.' This second error (confusion between expected and observed values) appeared in a different context in chi-squared tests. The condition 'frequency < 5' applies to expected values and not to observed values.

There was frequent confusion between the different significance tests, particularly about when a result that was less than the critical value means 'reject H_0' and when it means 'do not reject H_0'. There was also confusion between the assumptions and hypotheses involved in the two Wilcoxon tests.

When a null hypothesis is not rejected, most candidates have learned that the conclusion usually has to be stated as a double negative ('there is *insufficient* evidence that the population mean length is *not* 13.2'). Some, however, overdid this, with conclusions such as 'there is insufficient evidence that there is not no association'.

In several areas, there was confusion between why you can do something and why you need to do something. In Question 3 (b) (i), the comment 'we do not know if the parent distribution is normal' does not explain why the CLT can be used, only why it might need to be used. In Question 6 (a), the answer 'you can't combine rows until you have found the expected frequencies' refers to why you might need to combine rows, but it does not tell you why you can't combine them.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• had a genuine understanding of the ideas, assumptions and conditions • had a clear grasp of the differences between the various hypothesis tests • understood that $E(X) = Var(X)$ is an exact relationship for population parameters but applies only approximately to sample values • understood that the Central Limit Theorem (CLT) is a statement about the distribution of a particular variable, namely the sample *mean* • had a clear grasp of the difference between a reason why something can be done and why it needed to be done • had a clear understanding of what a random variable is.	• relied largely on repeating learned phrases without fully understanding what they meant • confused the assumptions or hypotheses or rejection criteria for the various tests • said that as the sample mean was not exactly equal to the sample variance, the distribution could not be Poisson • thought that the CLT allows you to assume that the population, or a sample drawn from it, has a normal distribution • confused the reason why something can be done and why it needed to be done • had only a vague understanding of what a random variable is, often using it as simply an abbreviation for words.

Question 1

1 A set of 16 observations of the bivariate data (X, Y) are summarised as follows.

$n = 16 \quad \sum x = 136 \quad \sum y = 352 \quad \sum x^2 = 1496 \quad \sum y^2 = 9104 \quad \sum xy = 3642$

Determine an estimate of the value of y corresponding to $x = 8.5$. **[4]**

Most candidates answered this question correctly. As 8.5 is the mean of the *x*-values, the corresponding estimate for *y* is equal to the mean of the *y*-values and could be written down without calculating the equation of the line. Some candidates realised this.

Question 2 (a)

2 The number of trees of a particular species found in $1\,\text{km}^2$ of a forest can be modelled by the distribution $\text{Po}(\lambda)$.

The forest consists of two regions, A and B.

In the region A, $\lambda = 4$. The number of trees of this species in a randomly chosen area of $3\,\text{km}^2$ in region A is denoted by X.

(a) Find $\text{P}(16 < X < 20)$. **[3]**

Candidates either answered this correctly or only made a mistake in dealing with the < signs. Errors that were seen several times were P(≤ 19) – (1 – P(≤ 16)) or even P(≤ 19) × (1 – P(≤ 16)).

Question 2 (b)

In region B, $\lambda = 8$. The number of trees of this species in a randomly chosen area of $3\,\text{km}^2$ in region B is denoted by Y.

The random variable Z is defined by $Z = X + Y$. It may be assumed that X and Y are independent.

(b) Write down a formula for $\text{P}(Z = z)$, in terms of z. **[2]**

This question was usually answered correctly. Some candidates did not calculate the parameter correctly.

Question 2 (c)

(c) The average numbers of trees of this species in one square kilometre are not the same in regions A and B.

Explain why the associated modelling assumption for the distribution of Z is nevertheless valid. **[1]**

The crucial piece of information in this question is that X and Y are independent of each other. This is not the same as the basic assumption for a Poisson distribution: trees within a single region are independent of one another. Many candidates tried to argue about constant average rates, which are irrelevant here. The point of the question is that if candidates think of A and B as a single region combined, the average rate throughout this single region is *not* constant, it is greater in region B.

Misconception

The following response shows the problems associated with explaining modelling assumptions:

> 'X and Y can both be modelled by a Poisson distribution. Therefore, X is independent, and so is Y, so their sum can be modelled by a Poisson distribution'.

The statement 'X is independent' is meaningless. X is a random variable, taking numerical values. It is *the events that trees occur* that are independent. The reason that the numerical variables X and Y can be added is that the occurrences of trees in region A do not affect the occurrences of trees in region B, and vice versa.

Question 2 (d)

(d) Show that $Y-X$ does **not** have a Poisson distribution. **[1]**

This question showed that candidates had several misunderstandings about Poisson distributions. Several candidates wrote, for example, '24 – 12 = X so the distribution of $Y – X$ is the same as the distribution of X'. Some calculated E($Y – X$) wrongly and said '8 – 12 < 0 and a Poisson distribution cannot have negative λ.'

There were two ways to answer the question. One was to say that the value of $Y – X$ (not its *expected* value) could be negative, which is not possible for a Poisson distribution. The other was to calculate E($Y – X$) and Var($Y – X$) and to state that these are not the same. Here, we are dealing with *population parameters* (expected values), and if these are not *exactly* equal, the distribution is not Poisson. This contrasts with the *approximate* relationship of *sample* mean and variance, which is explored in Question 9 (a).

Question 3 (a)

3 The length, in cm, of sand lizards when fully grown is a random variable which is known to have the distribution $N(13.2, 3.4^2)$.

Some scientists discover a colony of what they believe to be sand lizards in a remote location. The scientists measure the lengths of a random sample of 50 fully grown lizards from this remote location. The mean of this random sample is 14.02 cm. You may assume that the variance of the length of lizards on the remote island is $3.4^2\,\text{cm}^2$.

(a) Test, at the 5% significance level, whether the fully grown lizards in this location have a mean length which is different from 13.2 cm. **[7]**

This question was largely well-answered. Most candidates knew that they had to define any parameter they might use, such as μ, and that they had to say that this was the population mean length. Hypotheses and conclusions should always be given in context, so some mention of lengths was necessary.

Almost all candidates used a divisor of 50 for the variance, although some attempted to multiply it by $\frac{50}{49}$. This is incorrect here, as 3.4^2 is the population variance, not the sample variance.

Question 3 (b) (i)

(b) Now suppose it is **not** known that the lengths are normally distributed.

(i) Explain why the Central Limit Theorem can be used in this context. **[1]**

The correct explanation is 'the sample is big enough'. The question is not asking 'do we *need* to use the Central Limit Theorem (CLT)?', so it is wrong to add 'we do not know that the parent population is normally distributed'.

Question 3 (b) (ii)

(ii) State where in your test the Central Limit Theorem would be used. **[1]**

The only correct answer to this is that 'the distribution of the sample mean is normal'. 'The sampling distribution of means is normal' is another way of saying the same thing.

Misconception

As has been explained in numerous recent reports, the CLT does *not* say that you can assume that the *population*, or the *sample*, is normally distributed. There are two random variables here: the length of a single lizard, say X, and the mean of the lengths of a sample of 50 lizards, $\bar{X}$. These two random variables have different distributions.

The CLT says that, regardless of the distribution of X, the distribution of $\bar{X}$ is approximately normal for large enough samples. Candidates should always think of two different random variables with two different distributions.

Misconception

Many candidates think that in going from a population variance of 3.4^2 to a sample variance of $\frac{3.4^2}{50}$ they are using the CLT, but this is not so. The CLT is about the normal distribution, not the parameters.

Perhaps some are confused by statements seen in some textbooks such as: 'the CLT says that $\bar{X} \sim N(\mu, \sigma^2 / n)$'. The statement that $\mathrm{Var}(\bar{X}) = \sigma^2/n$ is always true for any distribution and any sample size (provided the members of the sample are independent of one another).

Exemplar 1

3(b)(ii) In modelling the length of the island lizards by a normal distribution

Assuming that the length of a lizard (that is, the length of a single observation X) can be normally distributed is a common misunderstanding. It is only the sample mean $\bar{X}$ that can be taken to be approximately normally distributed. The population distribution is not changed by taking a sample, however big. This misunderstanding was also seen in some answers to Question 9 (b).

Question 4 (a)

4 In this question you must show the parameters of any distributions you use.

The continuous random variable V has the distribution $N(42, 3^2)$.

The continuous random variable W has the distribution $N(48, 4^2)$.

The random variable X is the sum of 5 independent observations of V.

The random variable Y is the sum of 7 independent observations of W.

(a) Determine $P(X+Y > 560)$. **[3]**

This question was often answered well. The most common mistake was to confuse the rule for adding independent variables with the rule for multiplying one variable by a constant. Many took the variance to be $5^2 \times 3^2 + 7^2 \times 4^2$. This would be the variance of $5V + 7W$, but that is not the same as adding five independent observations of V and seven independent observations of W. The correct variance is $5 \times 3^2 + 7 \times 4^2$.

Question 4 (b)

(b) Determine the probability that X is less than 60% of Y. **[4]**

Some candidates did not know how to approach this question. Some tried to find $P(X/Y < 0.6)$, but this cannot be done as there is no simple way of finding the distribution of X/Y. Others tried to find $P(X < 0.6E(Y))$. Some of those who correctly used $X - 0.6Y$ found the wrong variance, but there were many correct answers.

Question 5 (a)

5 For many years a coach company has used a make of tyre that has an average driving lifespan of 1.52×10^5 km. The coach company decides to fit a new make of tyre to 12 of its coaches, and records the lifespan of one of the tyres, randomly chosen, for each of these 12 coaches. The distances, in multiples of 10^5 km, are given below.

1.35	1.46	1.48	1.53	1.54	1.57
1.62	1.74	1.76	1.82	1.88	1.91

The coach company wishes to test whether the new make of tyres has improved the driving lifespan.

(a) Explain why a hypothesis test based on a normal distribution may not be valid. **[1]**

Most candidates gave one or both correct reasons, which are that the sample size is too small for the CLT to be applied and that the parent distribution is not known to be normally distributed. A few candidates repeated the mistake discussed in Question 3 (b) (i), by saying 'the sample is too small to assume that the population is normal'.

Assessment for learning

It would be worthwhile giving candidates examples of several different hypothesis tests for the sample mean, with different parent distributions, and asking:

- For which of these do you *need* to use the Central Limit Theorem?
- For which of these *can* you use the Central Limit Theorem?

Question 5 (b)

(b) Carry out a suitable Wilcoxon test at the 5% significance level. **[7]**

Although many good answers were seen to this question, many candidates were not fully prepared for the topic. Some did not find the differences from 1.52 but simply ranked the data values in increasing order. As the first three data values were less than 1.52, their test statistic was given as 1 + 2 + 3. A few ranked the differences incorrectly from 1 = greatest to 12 = least (this is permissible in the rank-sum test). The rankings were often wrongly calculated, perhaps suggesting that candidates were not very confident about the method.

It is necessary to state both ΣP and ΣQ, or at least to show that both have been considered.

Hypotheses should refer to the population median, and not just say things like 'H_0: the lifespan has not improved'. Some said that the distributions had to be identical apart from the median, but this is the condition for the rank-sum test, not the signed-rank test.

Question 5 (c)

(c) Explain why a Wilcoxon test, if it is valid, is preferable to a sign test in this context. **[1]**

Most candidates could state that the Wilcoxon test uses more information as it takes into account the magnitudes of the differences, although a variety of incorrect responses were also seen.

Some candidates said 'It is not known whether the distribution is symmetrical'. The Wilcoxon test requires the distribution to be symmetrical, so this would be a reason why the Wilcoxon test should *not* be used.

Question 6 (a)

6 The editor of a mathematical journal investigated whether there was association between the types of articles published in the journal and the academic backgrounds of the authors.

The results for a random sample of 64 articles are shown in the table.

	Pure maths	Applied maths	Other
Schools and colleges	6	4	5
Universities	30	12	7

The editor carries out a test at the 10% significance level.

(a) Explain why it is not possible to combine rows of the table before carrying out the test. **[1]**

This question was often answered well. Successful answers included 'there would be 0 degrees of freedom' or 'there would be nothing to test, as the difference between academic backgrounds has been lost'. It was not enough to just say 'there would be only one row'.

Some candidates overlooked the word 'possible' and said 'you would first need to find whether any of the expected frequencies are less than 5'. That is an answer to a different question: 'why might you *need* to combine rows?' This is another example of the difference between necessary and sufficient conditions.

Question 6 (b)

(b) Show that it is necessary to combine columns before carrying out the test. **[2]**

Most candidates correctly calculated one or both of the expected frequencies that turned out to be less than 5. Some said that one of the observed frequencies is less than 5, which is the wrong condition.

Exemplar 2

6(b) ~~association~~

Oi for schools and colleges buying other < 5

therefore must combine last two columns.

In this exemplar, the error is very clear. It is not the observed frequency O_i that has to be greater than 5 but the expected frequency E_i.

Question 6 (c)

(c) Carry out the test. **[6]**

There were many answers that were either completely correct or which merely omitted Yates's correction. The hypotheses were usually well stated, and the critical value was usually correct.

Many used Yates's correction wrongly. Candidates who showed no working risked not getting several marks if their answer was wrong, as no method marks could be given without evidence of a correct method being used. It is advisable to show at least one term of the calculation, for instance $(|8.4375 - 6| - 0.5)^2 / 8.4375$.

Some candidates combined the first two columns instead of the last two columns. This is wrong because the expected value in the top-right-hand cell is less than 5. A few combined only the two individual cells with expected frequencies less than 5, rather than combining the complete columns.

Question 7 (a)

7 (a) A large number of competitors take part in two races. A random sample of 6 competitors who take part in both races is obtained. The value of Spearman's rank correlation coefficient for the finishing positions of these 6 competitors is 0.771, correct to 3 significant figures.

Test, at the 5% significance level, whether there is agreement between the finishing positions in the two races. **[4]**

This question was largely well-answered. The main error was that 'agreement' means 'positive association' rather than 'any association', which means that a 1-tailed test is required.

Question 7 (b)

(b) Show that, when only 3 competitors are selected at random, the value of Spearman's rank correlation coefficient can take only 4 different values. You should state what these values are. **[5]**

Many candidates correctly looked at different possible permutations of the rankings. However, many used only four different permutations, rather than all six. The command words are 'show that', so simply finding four different values of r_s did not get full marks. It was necessary to show that the six different permutations lead to only four different values of r_s.

Some went straight to values of d_i, which was again inadequate and usually led to wrong answers. Some misunderstood the question and tried to consider six competitors rather than three.

Question 8 (a)

8 The proportion X of charge in a phone battery is measured on a scale from 0 (uncharged) to 1 (fully charged).

For a certain model of phone, X can be modelled as a continuous random variable with the following probability density function.

$$\mathrm{f}(x) = \begin{cases} (k+1)x^k & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $\mathrm{f}(x)$ is a valid probability density function for all positive values of k. **[3]**

Most candidates were able to show that the total area under the curve was 1 for any positive value of k, although some did not use limits and found the cumulative distribution function. This would need further justification to answer the question; it is not enough merely to show that $\mathrm{F}(1) = 1$.

Most candidates forgot the other condition: $\mathrm{f}(x) \geq 0$ for all x. This needed to be stated.

Question 8 (b)

(b) Suppose that $k = 3$.

Determine $\mathrm{Var}(X)$. **[5]**

Most candidates were often completely correct, although a few candidates used the wrong powers of x. Some did not use limits, obtaining answers such as $\frac{2}{3}x^6 - \frac{16}{25}x^{10}$.

Question 8 (c)

(c) Suppose instead it is known that X is less than 0.8203 for 50% of this model of phone.

Determine the 70th percentile of X. **[5]**

Again, this was often answered correctly. The most common mistake was to interchange 0.8203 and 0.5, writing $0.5^{k+1} = 0.8203$ rather than $0.8203^{k+1} = 0.5$. A few candidates went from $k + 1 = 3.5$ to $k = 4.5$. Most were able to solve the exponential equations correctly.

Question 9 (a)

9 A set of 50 independent observations of a discrete random variable X, with an unknown distribution, is found.

(a) The mean and variance of the 50 observations are 4.04 and 3.5184 respectively.

Determine whether these results are consistent with the following distributions.

- A geometric distribution
- A Poisson distribution **[4]**

For the geometric distribution, most candidates found two different estimates of p based on the mean and on the variance or found the variance corresponding to $p = 0.2475$. Candidates then made statements such as '0.2475 is not equal to 0.4096'. Likewise, for the Poisson distribution some said '4.04 is not equal to 3.5184'. Both statements show that candidates do not appreciate the difference between expected mean/variance (E(X) and Var(X)) and sample mean/variance (in this case 4.04 and 3.5184). The issue is not whether the sample values are equal, but whether they are close together. A full statement might be 'If X has a Poisson distribution, E(X) must equal Var(X), so the sample mean, and the sample variance should be close together.'

The symbols for 'not approximately equal', $\not\approx$, and 'much greater than', >>, are useful here.

Assessment for learning

It would be worth showing candidates some pairs of values of the sample mean and the sample variance and asking which pairs suggest that a Poisson model might be appropriate.

It might also be worth showing candidates some pairs of values of E(X) and Var(X) and asking which pairs showed that a Poisson distribution was *not* appropriate.

You cannot ever infer from either E(X) and Var(X) or from a sample mean and variance that a Poisson distribution is definitely appropriate.

Question 9 (b)

(b) The values of the observations are given in the table.

x	0	1	2	3	4	5	6	7	8	> 8
Observed frequency	0	4	7	10	11	7	5	3	3	0

Without carrying out a goodness of fit test, determine whether these frequencies are consistent with the following distributions.

- A geometric distribution
- A Poisson distribution **[4]**

This question allowed a range of possible approaches, but there was no point in trying to use the sample mean and variance as this had been done in part (a). Here are two successful approaches:

- Use Po(4.04) and Geo(0.2475) to calculate several expected frequencies and compare them with the observed frequencies.
- Use two or three different observed frequencies to calculate two different estimates of p and compare these two estimates. Then do the same for λ.

Many correctly said that the observed frequencies for a geometric distribution should decrease, and as these didn't, they were inconsistent with a geometric distribution. Some said that for a Poisson they would increase then decrease (and some said that the mode would be close to the mean), although that was insufficient, and some incorrectly said that a Poisson distribution would be symmetrical. Some answers seemed to be saying, wrongly, that the CLT made the population normally distributed.

Most were able to draw inferences about the results, such as 'they are not consistent with a geometric distribution but reasonably consistent with a Poisson distribution.'

It is not possible to draw definite deductions about distributions from samples such as this, so conclusions should not be too definite. 'Inconsistent with geometric' is a good answer whereas 'It is not geometric' is not.

Some candidates wrote, for example, 'Poisson is consistent for some values but not others'. This is not sufficient. A single overall assessment for each distribution is required.

Assessment for learning

Several candidates calculated the expected frequency for 0 for the geometric distribution as 16.44 (= pq^{-1}) when it should be 0. Centres might like to check whether calculators can be misleading here.

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