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Examiners' report

A LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced GCE in Further Mathematics A

H245

For first teaching in 2017

Y540/01 Summer 2025 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from [Teach Cambridge].

Paper Y540/01 series overview

This paper, along with Y541/01, assesses the compulsory core content of the A Level Further Mathematics qualification. Questions in each paper can assess any part of the core specification. This paper had several straightforward questions, and most candidates had a good attempt at these questions. However, there were some unfamiliar problem-solving aspects in some questions, and some candidates struggled to process the issues that the questions posed in order to approach their response in a productive way. Most candidates appeared to be able to complete the paper in the time available.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• showed a secure grasp of all standard techniques • communicated well using mathematical language correctly • applied the breadth of their mathematical knowledge to find ways to tackle problems unfamiliar to them • made minimal arithmetic and algebraic errors • organised their time well so that they had time to check (and possibly correct) their work	• demonstrated gaps in their knowledge of standard techniques • produced unclear or incomplete mathematical arguments, often resulting in algebraic and arithmetic errors • could not find routes into problems unfamiliar to them

Question 1 (a)

1 **(a)** A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by $\mathbf{M}$. **[2]**

Most candidates scored at least one mark in this part for correctly stating that the matrix represented an enlargement of scale factor 5 (or area scale factor 25). However, many did not give the complete description and often forgot to include the fact that the enlargement was centred at the origin.

Misconception

It was common for examiners to see the transformation stated as a 'stretch' rather than 'enlargement' and mathematically speaking these are two very different types of transformation.

Question 1 (b)

(b) Write down the 2×2 matrix that represents a rotation of 90° anticlockwise about the origin. **[1]**

Most candidates answered this part very well. The most common errors were to state a matrix which represented a 90 degree clockwise rotation or to leave their answer in terms of sine and cosine.

Question 1 (c)

(c) Write down the 3×3 matrix that represents a reflection in the x–z plane. **[1]**

The responses to this part were mixed with some candidates leaving this part blank or giving incorrect answers of $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

Question 2 (a)

2 **(a)** Given that $y = \cosh^{-1}\left(\frac{1}{3}x\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. **[1]**

This question was answered extremely well with most candidates able to write down the correct derivative with minimal working. The most common error seen was failing to apply the chain rule correctly, i.e. forgetting to multiply by $\frac{1}{3}$. A number of candidates, probably unfamiliar with the standard results given in the H245 formula booklet, tackled this problem by first re-writing as $\frac{1}{3}x = \cosh y$ and then differentiated implicitly; these attempts were not as successful as those who simply used the result that $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$.

Assessment for learning

Candidates need to be familiar with the results that appear in the H245 formula booklet and, unless a question says otherwise, they may quote these results without proof.

Question 2 (b)

(b) Determine an equation of the normal to the curve $y = \cosh^{-1}\left(\frac{1}{3}x\right)$ at the point where $x = 5$.

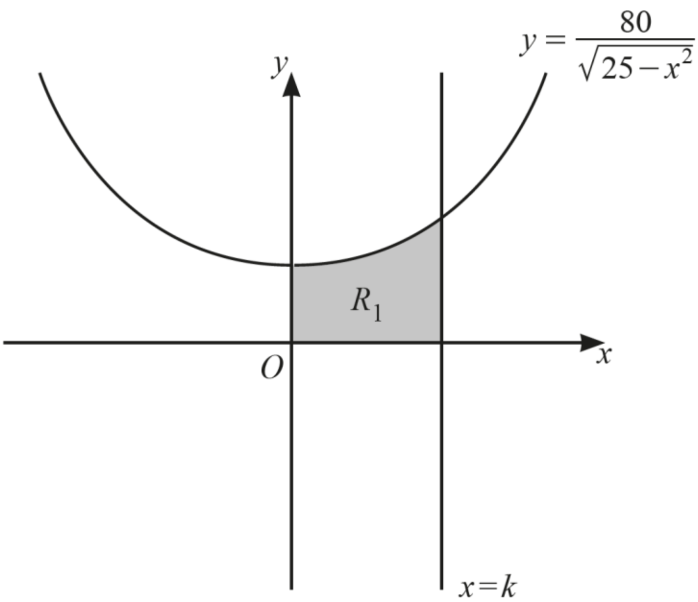
Give your answer in the form $ax + by = c + \ln d$, where a, b, c and d are integers. **[4]**

Although this part was generally answered well, a number of candidates did not correctly write $\cosh^{-1}\frac{5}{3}$ in logarithmic form with many incorrectly finding $\cosh^{-1}5$. Of those that did find the correct gradient and expression for $\cosh^{-1}\frac{5}{3}$ most then went on to give the equation of the normal in the required form.

Question 3 (a)

3 The region R_1 is bounded by the curve $y = \dfrac{80}{\sqrt{25 - x^2}}$, the line $x = k$ and the coordinate axes, as shown in **Fig. 1**.

Fig. 1



(a) In this question you must show detailed reasoning.

Given that the area of R_1 is $\dfrac{40}{3}\pi$, determine the value of k. **[3]**

This part was answered extremely well with most candidates correctly integrating $\frac{80}{\sqrt{25-x^2}}$ between the limits of 0 and k, obtaining $80\sin^{-1}\left(\frac{1}{5}k\right) = \frac{40}{3}\pi$ and then solving this equation correctly to obtain a value of 2.5 for k.

Assessment for learning

Many of the questions in this paper asked candidates to either show detailed reasoning, were 'show that' questions, or used the command word 'determine'. In these questions, candidates are required to demonstrate understanding of the topic being assessed and this requires them to show all the steps in their working and/or give sufficient reasoning for their answers. Candidates would benefit from understanding these command words.

Further information about these command words, including the differences between determine, show that and detailed reasoning can be found in section 2b of the specification. OCR have produced a posters and blogs that can be used to support candidates understanding of these.

https://www.ocr.org.uk/Images/533967-a-level-maths-command-words-poster-a4-size.pdf

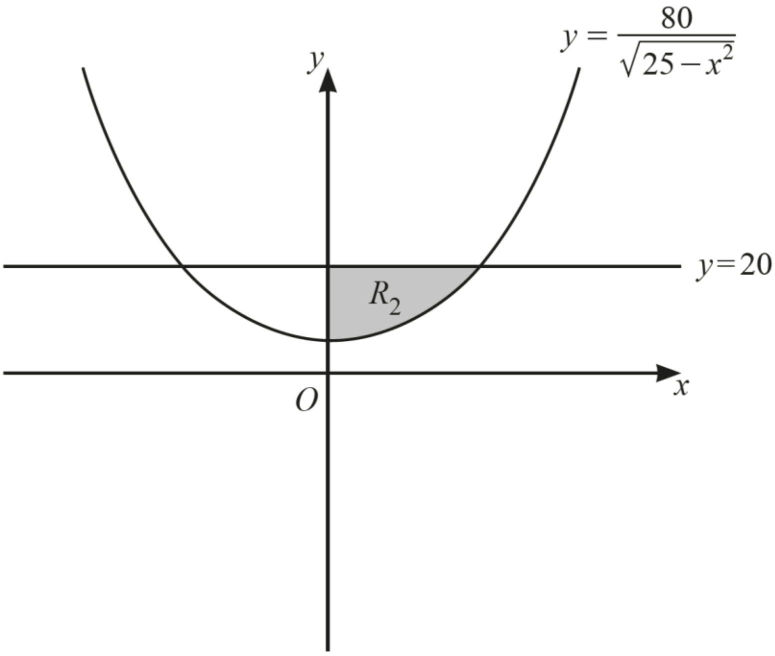
https://www.ocr.org.uk/blog/a-level-maths-three-things-to-consider-for-detailed-reasoning/

https://www.ocr.org.uk/blog/alevel-maths-a-deep-dive-into-the-command-word-determine/

Question 3 (b)

The region R_2 is bounded by the curve $y = \dfrac{80}{\sqrt{25 - x^2}}$, the line $y = 20$ and the y-axis as shown in **Fig. 2**.

Fig. 2



(b) Find, in an exact form, the volume of the solid formed when R_2 is rotated by 2π radians about the ***y*-axis**. **[3]**

Most candidates realised that they needed to re-arrange the given equation to make x^2 the subject and then use that result when rotating about the *y*-axis; the volume is given by the integral $\pi \int_{16}^{20} x^2 \, \mathrm{d}y$. Some incorrectly rotated about the *x*-axis and a number forgot the factor of π, therefore only gaining 1 of the 3 marks available.

Assessment for learning

The command word used in this part was to 'find' the volume of the solid formed. Many candidates spent time solving the integral $\pi \int_{16}^{20} \left(-\frac{6400}{y^2} + 25\right) \mathrm{d}y$ by hand instead of efficiently using their calculators to obtain the correct value of 20π.

Question 4 (a)

4 The equation $5x^3 - 4x^2 + 10 = 0$ has roots α, β and γ.

(a) Write down the values of $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha\beta\gamma$. **[2]**

This was question was answered very well, with many candidates scoring all 9 marks. In this part the most common error was to state that $\alpha + \beta + \gamma = -\frac{4}{5}$ and/or $\alpha\beta\gamma = 2$. A few candidates did not simplify $-\frac{10}{2}$ as -5, which meant they were only awarded 1 out of 2 marks, which is a reminder to future candidates that all final answers that can be given as integers must be).

Question 4 (b)

(b) By expanding $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$, determine the value of $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$. **[3]**

Most candidates correctly expanded the given expression and obtained the required value with the most common errors being either due to sign errors or forgetting the 2's when expanding (so instead implying 6 rather than 9 terms).

Question 4 (c)

(c) By expanding a suitable expression, find the value of $\alpha^2+\beta^2+\gamma^2$. **[2]**

Again, this part was answered extremely well with most candidates scoring both marks.

Question 4 (d)

(d) Hence find a cubic equation with integer coefficients that has roots α^2, β^2, and γ^2. **[2]**

While most candidates did use the preceding results to write down a cubic equation with the desired roots, many did not give their final answer with integer coefficients, as requested. Some did not give their answer as an equation, they did not include an '= 0', as the question demand asked for.

Question 5 (a)

5 A vector equation of the plane Π_1 is $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(a) **Verify** that a cartesian equation of Π_1 is $x - y + 2z = 3$. **[1]**

The responses to this part were mixed with few candidates using the most obvious method of taking the parametric form of the vector equation and substituting into the given cartesian form (e.g. $(1 + 3\lambda + \mu) - (4 + \lambda + \mu) + 2(3 - \lambda) = 1 + 3\lambda + \mu - 4 - \lambda - \mu + 6 - 2\lambda = 3$) to verify this equation.

Assessment for learning

The command word used in this part was to 'verify' and while candidates were not penalised for 'showing' that this was the cartesian equation of the given plane the command word and the number of marks available should have been a hint to candidates regarding the amount of work that was required here.

Misconception

A vey common misconception in this part was to assume that substituting one point (most candidates used (1, 4, 3)) into the given cartesian equation was sufficient to verify that this was indeed the correct equation. Candidates are reminded that at least three points are required to uniquely define a 3D-plane.

Question 5 (b)

For some real constant a, cartesian equations of planes Π_2 and Π_3 are

$$\begin{aligned} \Pi_2&: \quad x \phantom{{}-y} - 3z = 1 \\ \Pi_3&: \quad ax - y - z = 4 \end{aligned}$$

(b) By considering a suitable matrix, show that Π_1, Π_2 and Π_3 intersect at a single point for all values of a except $a = 2$. **[3]**

Most candidates scored at least the first 2 marks in this part for correctly calculating the determinant of a suitable matrix (although some incorrectly set up the matrix with the planes represented down the columns instead of across the rows). However, many did not articulate that there would be a single point of intersection of the three planes when the determinant of the relevant matrix was non-zero.

Question 5 (c)

(c) Use the matrix from part **(b)** to find the coordinates of the point of intersection of Π_1, Π_2 and Π_3 in the case where $a = 3$. **[2]**

This part was answered well by most candidates correctly using the inverse of the matrix from part (b) to find the required point of intersection. Some candidates simply solved the three equations without using the required matrix and therefore did not gain the first mark.

Question 5 (d)

(d) In the case where $a = 2$, determine the geometrical arrangement of Π_1, Π_2 and Π_3. **[2]**

While most candidates realised that the three equations were consistent and therefore geometrically the planes meet in a straight line many candidates showed insufficient working to justify this consistency. Future candidates are advised to label the three equations as say [1], [2] and [3] and then explicitly show the linear combination, e.g. in this case [1] + [2] = [3] that gives evidence to their argument of consistency.

Question 6 (a)

6 In this question you must show detailed reasoning.

The series S_n is given by $S_n = \left(\frac{1}{5} \times \frac{1}{15}\right) + \left(\frac{1}{15} \times \frac{1}{25}\right) + \cdots + \left(\frac{1}{10n-5} \times \frac{1}{10n+5}\right)$ for $n \in \mathbb{Z}^+$.

(a) Use the method of differences to show that, for all $n \in \mathbb{Z}^+$, $S_n < \frac{1}{50}$. **[5]**

Most candidates realised that before the method of differences could be applied to the given sequence, the expression $\frac{1}{(10r-5)(10r+5)}$ had to be re-written in partial fractions and most who recognised this were ultimately successful in doing so. The fact that this question required 'detailed reasoning' meant that sufficient terms of the method of differences had to be shown to convince examiners that $S_n = \frac{1}{50}\left(1 - \frac{1}{2n+1}\right)$ had been found legitimately, and furthermore, some correct justification had to be given for why this expression implied that $S_n < \frac{1}{50}$. Many candidates either did not attempt to justify this result or instead simply stated that as $n \to \infty, \frac{1}{2n+1} \to 0$, so they did not deal with the direction of the inequality.

Question 6 (b)

Let $S_\infty = \lim_{n\to\infty} S_n$.

(b) Given that, for some value of k, $S_\infty = \frac{1}{2450} + S_k$, find the value of k. **[3]**

Of those that correctly found the required expression for S_n in part (a) most correctly went on to find the correct value of k in this part.

Question 7 (a)

7 A 3-D coordinate system, whose units are metres, is set up to model a street containing telephone cables T_1 and T_2.

The cables are modelled as straight lines with vector equations

$$T_1\colon\ \mathbf{r} = \begin{pmatrix}3\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}5\\4\\1\end{pmatrix} \text{ and } T_2\colon\ \mathbf{r} = \begin{pmatrix}8\\2\\4\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\1\end{pmatrix}.$$

(a) Show that the cables do not intersect. **[3]**

This part was answered extremely well with most candidates scoring at least 2 marks for setting up a correct pair of simultaneous equations and then solving them to find a correct value of either λ and/or μ. Most realised that to show that the cables did not intersect they then had to show that their value(s) (of the scalar parameters) were inconsistent with the third equation. As always with this type of question the most common errors were due to either sign issues or miscopying a value from the question. While other methods were seen (for example, attempting to show that the shortest distance between the two skew lines was not zero) they were not always as successful.

Question 7 (b)

To access the cables for maintenance, a ladder can be used. The base of the ladder is placed at a fixed point on the ground.

The ladder is modelled as a straight-line segment. The base of the ladder is modelled as being located at the point (4, 5, 0).

(b) Determine the minimum length of the ladder required so that it reaches cable T_1. Give your answer in centimetres to the nearest centimetre. **[4]**

The responses to this part were mixed with many candidates either leaving this part blank or applying an incorrect method. Examiners commented on the variety of correct methods seen, for example, finding an expression for the distance between (4, 5, 0) and the cable and then using calculus, or methods involving the vector product. The most common correct method seen was to set a relevant scalar product equal to zero - so using the fact that the minimum length would be when the cable and the line from the point to the cable are perpendicular - and then using Pythagoras to find the required distance. The most common errors seen from this approach were once again sign errors or finding the distance from the origin to the cable rather than from the point (4, 5, 0). Finally, many candidates did not give the answer in the form requested (in centimetres to the nearest centimetre).

Question 7 (c)

(c) Identify a modelling assumption used in part **(a)** that is unrealistic, **and** which could affect your answer to this part. **[1]**

This part was mixed with some candidates identifying a correct modelling assumption and saying how it would affect their answer. The most common answer was to mention the fact that the cables were modelled as straight lines but in reality, are likely to bend.

Question 8 (a)

8 In this question you must show detailed reasoning.

In this question, arguments of complex numbers are in the interval $[0, 2\pi)$.

(a) Given that $z = -5 + 5\text{i}$, express z in exact exponential form. **[2]**

While many candidates could correctly write *z* in exact exponential form few showed sufficient detailed reasoning. Many simply stated either the modulus or argument (or both) without any indication of where these values came from.

Question 8 (b)

(b) Given that $w = \dfrac{36\cos\left(\frac{1}{7}\pi\right) - 36\mathrm{i}\sin\left(\frac{1}{7}\pi\right)}{27\sin\left(\frac{3}{7}\pi\right) + 27\mathrm{i}\cos\left(\frac{3}{7}\pi\right)}$, express w in exact exponential form. **[5]**

Candidates found this a demanding question part, with many making little progress apart from recognising that $|w| = \frac{4}{3}$ and/or that the numerator could be re-written as $36\mathrm{e}^{-\mathrm{i}\frac{1}{7}\pi}$. Many candidates assumed that the denominator was $27\cos\left(\frac{3}{7}\pi\right) + \mathrm{i}\sin\left(\frac{3}{7}\pi\right)$ which over-simplified the problem. Some candidates, without showing any mathematical justification, simply stated (possibly from their calculators) that the argument of the denominator was $\frac{1}{14}\pi$, and as the question required detailed reasoning, this was not acceptable. Finally, some candidates correctly showed that $w = \frac{4}{3}\mathrm{e}^{-\mathrm{i}\frac{3}{14}\pi}$ but did not give the argument in the form required, so they did not gain the final accuracy mark.

Exemplar 1

8(b)

$\cancel{|w| = 36e^{-\frac{\pi}{7}}}$ $36\cos\frac{\pi}{7} - 36i\sin\frac{\pi}{7} = 36e^{-\frac{i\pi}{7}}$

$27\sin\left(\frac{3\pi}{7}\right) + 27i\cos\left(\frac{3\pi}{7}\right) = i \times \left(27\cos\frac{3\pi}{7} - i\sin\frac{3\pi}{7}\right)$

$= i \times 27e^{-\frac{3i\pi}{7}}$, $i = e^{\frac{i\pi}{2}}$

$w = 27e^{\frac{i\pi}{14}}$

$= 36e^{-\frac{i\pi}{7}} \times \frac{1}{27}e^{-\frac{i\pi}{14}}$

$= \frac{36}{27}e^{-\frac{3i\pi}{14}} = \frac{4}{3}e^{-\frac{3i\pi}{14}}$

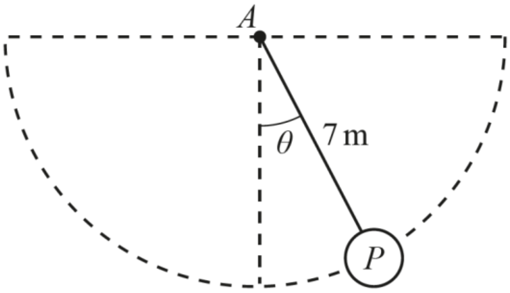
This exemplar highlights the common error discussed above of not giving the argument in the form required. This response scored all but the final accuracy mark, so scored 4 out of the 5 marks available.

Question 9 (a)

9 A pendulum comprises an object P of mass m kg and a string of length 7 m. One end of the string is attached to P and the other end is attached to a fixed point A.

At time t seconds, $t \geqslant 0$, the string forms an angle of θ radians, measured anti-clockwise, from the downward vertical through A, as shown in the diagram.

When $t = 0$, $\theta = \theta_0 > 0$ and P is released from rest. You may assume that in the subsequent motion P moves along the arc of a circle, centre A and radius 7 m, and that $|\theta| \leqslant \theta_0$ for all $t \geqslant 0$.



The motion of P is modelled by the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\sin\theta = 0 \, (*)$$

In some situations, it is appropriate to approximate $(*)$ with the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\theta = 0 \, (**)$$

(a) Explain why it would be appropriate to model the motion of P with the differential equation $(**)$ when $\theta_0 = \frac{1}{15}\pi$ but not when $\theta_0 = \frac{1}{3}\pi$. **[1]**

Many candidates realised that it was only appropriate to use one of the given angles due to the small angle approximation of $\sin\theta \approx \theta$ being valid in only one of the two cases. To be awarded this mark, candidates had to state mathematically that in this situation $\sin\theta \approx \theta$ and mention when this result was valid (e.g. when the angle is small).

Question 9 (b)

You are now given that $\theta_0 = \frac{1}{15}\pi$.

(b) By finding the particular solution to the differential equation (∗∗), determine the total **distance** travelled by P in the first 6 seconds of the motion according to (∗∗). **[6]**

The responses to this part were very mixed and can be broadly placed into the following three categories:

Those who could correctly find the general solution of the given differential equation, so scored 1 mark, but then did not make any significant progress in finding the particular solution. This lack of progress was due to not being able to find the values of the arbitrary constants, usually due to not appreciating that when $t = 0, \theta = \frac{1}{15}\pi$ and $\dot{\theta} = 0$.

Those that could correctly find the particular solution of the differential equation and therefore scored at least the first 3 marks. Most of these candidates made some further progress and correctly calculated that when $t = 6, \theta = 0.1434$ ... which scored in total 4 marks.

However, those that recognised that the distance travelled was given by 7θ and that at this time (of 6 seconds) the pendulum had already completed one complete oscillation and therefore the required distance was given by $7\left(\frac{4}{15}\pi + \frac{1}{15}\pi - 0.1434 \ldots\right)$ were generally able to succeed. A common incorrect response of those that fell into this final category was to assume that the angle was being measured from the horizontal and not the vertical and instead found the distance to be $7\left(\frac{4}{15}\pi + 0.1434 \ldots\right)$ - this incorrect approach scored 5 of the 6 marks available.

Exemplar 2

9(b)

$$\lambda^2 + \frac{7}{5} = 0$$

$$\lambda = \pm \frac{\sqrt{35}}{5} i$$

$$\therefore \quad \theta = A\sin\left(\frac{\sqrt{35}}{5}t\right) + B\cos\left(\frac{\sqrt{35}}{5}t\right)$$

$$t = 0, \quad \theta = \frac{\pi}{15}. \qquad t = 0, \quad \frac{d\theta}{dt} = 0$$

$$\frac{\pi}{15} = 0 + B$$

$$B = \frac{\pi}{15}$$

$$\frac{d\theta}{dt} = \frac{\sqrt{35}}{5}A\cos\left(\frac{\sqrt{35}}{5}t\right) - \frac{\sqrt{35}}{5}B\sin\left(\frac{\sqrt{35}}{5}t\right)$$

$$t = 0 \quad \frac{d\theta}{dt} = 0:$$

$$0 = \frac{\sqrt{35}}{5}A \quad \therefore \quad A = 0$$

$$\theta = \frac{\pi}{15}\cos\frac{\sqrt{35}}{5}t$$

Time for one oscillation $= 2\pi \times \frac{5}{\sqrt{35}} = 5.31\text{ s}$

9(b)	(continued)
	$6 - 5.31 = 0.6897\text{s}$
	$t = 0.6897\text{s}, \quad \theta = 0.1435$
	$\therefore$ total distance $= 2\left(7 \times \frac{2\pi}{15}\right)$ $+ 7 \times 0.1435$
	distance $= \underline{6.01\text{ m}}$

This exemplar highlights a response that falls into the third category mentioned above. The candidate has correctly found the particular solution (so scoring the first 3 marks), then calculates the angle at time t = 6 (so scoring the fourth mark). Finally, they calculate $7\left(\frac{4}{15}\pi + 0.1434 \ldots\right)$ (so treating the angle from the horizontal and not the vertical) and so scores 5 of the 6 marks available.

Question 9 (c)

An additional force now acts on P. It can be shown that it is now appropriate to model the motion of P with the differential equation $\frac{d^2\theta}{dt^2} + \frac{k}{m}\frac{d\theta}{dt} + \frac{7}{5}\theta = 0$ where $k > 0$.

(c) Find the range of values of m, in terms of k, for which the motion of the pendulum is overdamped. **[2]**

While many candidates scored at least 1 mark for considering the correct discriminant of the auxiliary equation $\lambda^2 + \frac{k}{m}\lambda + \frac{7}{5} = 0$ only some recognised that due to the physical context that the correct range of values of m would be $0 < m < \frac{\sqrt{35}}{14}k$ and not $-\frac{\sqrt{35}}{14}k < m < \frac{\sqrt{35}}{14}k$.

Question 10 (a)

10 In this question you must show detailed reasoning.

(a) Use de Moivre's Theorem to show that, if $\cos 5\theta \neq 0$,

$$\tan 5\theta \equiv \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}.$$ **[4]**

The responses to this first part were positive, with many candidates scoring at least the first 3 marks for correctly obtaining an expression for $\tan 5\theta$ in terms of powers of sine and cosine using de Moivre's Theorem. However, very few candidates scored the final mark as most did not show sufficient detailed reasoning in their attempts to derive the given answer. Many resorted to simply stating the need to 'divide by $\cos^5 x$' or did not show explicitly each term in both the numerator and denominator being divided by $\cos^5 x$.

Exemplar 3

10(a)

$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

$= c^5 + 5c^4 s i - 10c^3 s^2 - 10c^2 s^3 i + 5c s^4 + s^5 i$

$\sin 5\theta = \text{Im} = 5c^4 s - 10c^2 s^3 + s^5$

$\cos 5\theta = \text{Re} = c^5 - 10c^3 s^2 + 5c s^4$

$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta} = \dfrac{5c^4 s - 10c^2 s^3 + s^5}{c^5 - 10c^3 s^2 + 5c s^4}$

$\div\, c^{\cancel{4}5}$ $\quad \tan 5\theta = \cancel{\tan\ t - t^3 + t^5}$

$= \dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$

$= \dfrac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^3\theta + 1}$

This exemplar highlights the point made above – the candidate was awarded the first 3 marks for correctly applying de Moivre's and obtaining a correct expression for $\tan 5\theta$ in terms of powers of sine and cosine. However, the final accuracy mark could not be given due to the lack of detailed reasoning as they have simply stated '$\div\, c^5$' without explicitly showing this step.

Question 10 (b) (i)

(b) **(i)** By considering the equation $\tan 5\theta = 1$, use the result in part **(a)** to find the exact roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0.$$

Give the roots in the form $t = \tan\phi$ where $0 < \phi < \pi$. **[4]**

Some candidates were able to score 2 marks in this part for correctly obtaining the quintic equation $t^5 - 5t^4 - 10t^3 + 10t^2 + 5t - 1 = 0$ from setting the equation in part (a) equal to 1 and correctly stating all five roots of the equation $\tan 5\theta = 1$ in the required range. However, many did not articulate correctly why only four of these roots were the basis of the corresponding roots of the given quartic equation nor did they state these roots correct in terms of tan.

Question 10 (b) (ii)

(ii) By first expressing $t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$ in the form $(t-1)^4 = kt^2$, where k is a constant to be determined, show that

$$\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}.$$ **[3]**

It was rare in this part to see candidates make much progress apart from correctly stating/showing that $k = 20$. Of those that did, very few candidates derived the two correct quadratic equations $t^2 - \left(2 \pm \sqrt{20}\right)t + 1 = 0$, with many only having the quadratic equation with the positive square root. Therefore, it was extremely rare for all four roots to be found together with a correct explanation that $\tan\left(\frac{9}{20}\pi\right)$ was the largest positive root of the four.

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Access to Scripts

We've made it easier for Exams Officers to download copies of your candidates' completed papers or 'scripts'. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular professional development courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is the exam paper builder platform for a range of our qualifications. Find out more.

Free for all OCR centres with an Interchange account, it allows unlimited users per centre. We require an Interchange username to verify your centre's users for ExamBuilder.

If you do not have an Interchange account, please contact your centre administrator (usually the Exams Officer) to request a username.

Once you have an Interchange username, use the form on this page to sign up for ExamBuilder, or ask an existing user at your centre to create an ExamBuilder account for you.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals (examined units only).

Find out more.

Tell us what you think

Your feedback plays an important role in how we develop, market, support and resource qualifications now and into the future. We want you and your students to enjoy and get the best out of our qualifications and resources, but to do that we need your honest opinions to tell us whether we're on the right track or not.

You can email your thoughts to resources.feedback@ocr.org.uk or visit our feedback page to learn more about how you can help us improve our resources.

Designing and testing in collaboration with you and your students

Helping young people develop an ethical view of the world

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