



Oxford Cambridge and RSA

Thursday 22 May 2025 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS

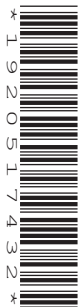
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.



- 1 (a) A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by $\mathbf{M}$. [2]

- (b) Write down the 2×2 matrix that represents a rotation of 90° anticlockwise about the origin. [1]

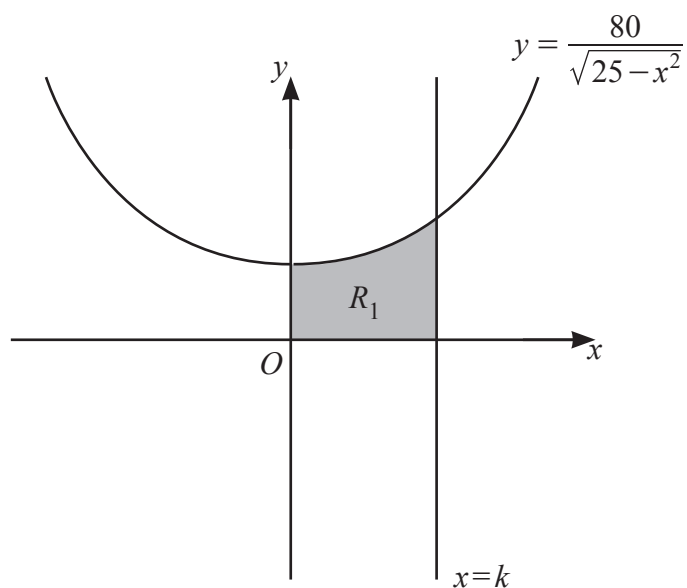
- (c) Write down the 3×3 matrix that represents a reflection in the x - z plane. [1]

- 2 (a) Given that $y = \cosh^{-1}\left(\frac{1}{3}x\right)$, find $\frac{dy}{dx}$ in terms of x . [1]

- (b) Determine an equation of the normal to the curve $y = \cosh^{-1}\left(\frac{1}{3}x\right)$ at the point where $x = 5$.
Give your answer in the form $ax + by = c + \ln d$, where a , b , c and d are integers. [4]

- 3 The region R_1 is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $x = k$ and the coordinate axes, as shown in **Fig. 1**.

Fig. 1



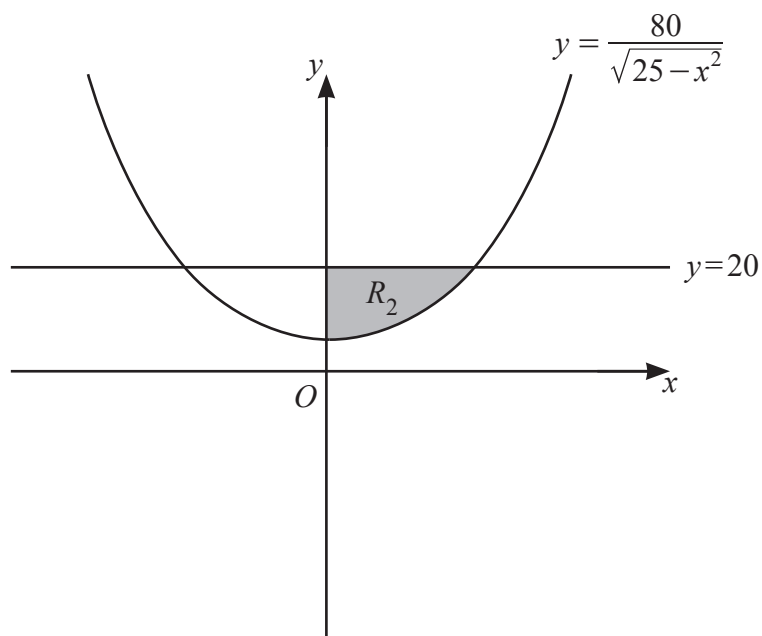
- (a) In this question you must show detailed reasoning.

Given that the area of R_1 is $\frac{40}{3}\pi$, determine the value of k .

[3]

The region R_2 is bounded by the curve $y = \frac{80}{\sqrt{25-x^2}}$, the line $y = 20$ and the y -axis as shown in **Fig. 2**.

Fig. 2



- (b) Find, in an exact form, the volume of the solid formed when R_2 is rotated by 2π radians about the y -axis.

[3]

4 The equation $5x^3 - 4x^2 + 10 = 0$ has roots α , β and γ .

(a) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [2]

(b) By expanding $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, determine the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [3]

(c) By expanding a suitable expression, find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(d) Hence find a cubic equation with integer coefficients that has roots α^2 , β^2 , and γ^2 . [2]

5 A vector equation of the plane Π_1 is $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(a) **Verify** that a cartesian equation of Π_1 is $x - y + 2z = 3$. [1]

For some real constant a , cartesian equations of planes Π_2 and Π_3 are

$$\Pi_2: \quad x \quad - 3z = 1$$

$$\Pi_3: \quad ax - y - z = 4$$

(b) By considering a suitable matrix, show that Π_1 , Π_2 and Π_3 intersect at a single point for all values of a except $a = 2$. [3]

(c) Use the matrix from part (b) to find the coordinates of the point of intersection of Π_1 , Π_2 and Π_3 in the case where $a = 3$. [2]

(d) In the case where $a = 2$, determine the geometrical arrangement of Π_1 , Π_2 and Π_3 . [2]

6 **In this question you must show detailed reasoning.**

The series S_n is given by $S_n = \left(\frac{1}{5} \times \frac{1}{15}\right) + \left(\frac{1}{15} \times \frac{1}{25}\right) + \dots + \left(\frac{1}{10n-5} \times \frac{1}{10n+5}\right)$ for $n \in \mathbb{Z}^+$.

(a) Use the method of differences to show that, for all $n \in \mathbb{Z}^+$, $S_n < \frac{1}{50}$. [5]

Let $S_\infty = \lim_{n \rightarrow \infty} S_n$.

(b) Given that, for some value of k , $S_\infty = \frac{1}{2450} + S_k$, find the value of k . [3]

- 7 A 3-D coordinate system, whose units are metres, is set up to model a street containing telephone cables T_1 and T_2 .

The cables are modelled as straight lines with vector equations

$$T_1: \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \text{ and } T_2: \mathbf{r} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

- (a) Show that the cables do not intersect. [3]

To access the cables for maintenance, a ladder can be used. The base of the ladder is placed at a fixed point on the ground.

The ladder is modelled as a straight-line segment. The base of the ladder is modelled as being located at the point $(4, 5, 0)$.

- (b) Determine the minimum length of the ladder required so that it reaches cable T_1 . Give your answer in centimetres to the nearest centimetre. [4]
- (c) Identify a modelling assumption used in part (a) that is unrealistic, **and** which could affect your answer to this part. [1]

8 In this question you must show detailed reasoning.

In this question, arguments of complex numbers are in the interval $[0, 2\pi)$.

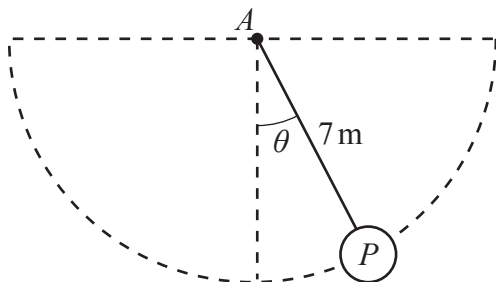
- (a) Given that $z = -5 + 5i$, express z in exact exponential form. [2]

- (b) Given that $w = \frac{36 \cos\left(\frac{1}{7}\pi\right) - 36i \sin\left(\frac{1}{7}\pi\right)}{27 \sin\left(\frac{3}{7}\pi\right) + 27i \cos\left(\frac{3}{7}\pi\right)}$, express w in exact exponential form. [5]

- 9 A pendulum comprises an object P of mass m kg and a string of length 7 m. One end of the string is attached to P and the other end is attached to a fixed point A .

At time t seconds, $t \geq 0$, the string forms an angle of θ radians, measured anti-clockwise, from the downward vertical through A , as shown in the diagram.

When $t = 0$, $\theta = \theta_0 > 0$ and P is released from rest. You may assume that in the subsequent motion P moves along the arc of a circle, centre A and radius 7 m, and that $|\theta| \leq \theta_0$ for all $t \geq 0$.



The motion of P is modelled by the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\sin\theta = 0 \quad (*)$$

In some situations, it is appropriate to approximate $(*)$ with the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\theta = 0 \quad (**)$$

- (a) Explain why it would be appropriate to model the motion of P with the differential equation $(**)$ when $\theta_0 = \frac{1}{15}\pi$ but not when $\theta_0 = \frac{1}{3}\pi$. [1]

You are now given that $\theta_0 = \frac{1}{15}\pi$.

- (b) By finding the particular solution to the differential equation $(**)$, determine the total **distance** travelled by P in the first 6 seconds of the motion according to $(**)$. [6]

An additional force now acts on P . It can be shown that it is now appropriate to model the motion

of P with the differential equation $\frac{d^2\theta}{dt^2} + \frac{k}{m}\frac{d\theta}{dt} + \frac{7}{5}\theta = 0$ where $k > 0$.

- (c) Find the range of values of m , in terms of k , for which the motion of the pendulum is overdamped. [2]

10 In this question you must show detailed reasoning.

- (a)**
- Use de Moivre's Theorem to show that, if
- $\cos 5\theta \neq 0$
- ,

$$\tan 5\theta \equiv \frac{\tan^5 \theta - 10 \tan^3 \theta + 5 \tan \theta}{5 \tan^4 \theta - 10 \tan^2 \theta + 1}. \quad [4]$$

- (b) (i)**
- By considering the equation
- $\tan 5\theta = 1$
- , use the result in part
- (a)**
- to find the exact roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0.$$

Give the roots in the form $t = \tan \phi$ where $0 < \phi < \pi$. [4]

- (ii)**
- By first expressing
- $t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$
- in the form
- $(t-1)^4 = kt^2$
- , where
- k
- is a constant to be determined, show that

$$\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}. \quad [3]$$

END OF QUESTION PAPER

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