OCR
Oxford Cambridge and RSA

Tuesday 3 June 2025 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

192020543 1

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2025 [603/1325/0]
DC (ST/CB) 357886/3
OCR is an exempt Charity

1 **In this question you must show detailed reasoning.**

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$.

Determine, in either order

- **a.b**
- **a**×**b**. **[3]**

2 You are given that $-7-5\mathrm{i}$ is one root of the equation $x^3 + 10x^2 + 18x - 296 = 0$.

(a) Write down another complex root of the equation $x^3 + 10x^2 + 18x - 296 = 0$. **[1]**

(b) Using your answer to part **(a)**, express $x^3 + 10x^2 + 18x - 296$ as a product of a real linear factor and a real quadratic factor. **[3]**

3 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 4 & 8 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & -3 \\ 1 & 2 \end{pmatrix}$.

(a) Find the matrix **AB**. **[1]**

(b) Verify that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$. **[2]**

(c) Use matrices **A** and **B** to demonstrate that matrix multiplication is **not** commutative. **[2]**

The transformation represented by matrix **A** is denoted by T.

(d) Show that the point (2, −5) is **not** an invariant point under T. **[2]**

(e) Find the matrix which represents the inverse transformation of T. **[2]**

© OCR 2025

4 **In this question you must show detailed reasoning.**

Determine the sum of all cube numbers from 216 to 512 000 inclusive. **[4]**

5 **In this question you must show detailed reasoning.**

(a) Use an algebraic method to determine the two square roots of $-3+(4\sqrt{7})\mathrm{i}$.

Give your answers in the form $a+b\mathrm{i}$ where a and b are exact. **[5]**

(b) State the relationship between the two arguments of the two square roots found in part **(a)**. **[1]**

6 One of the regions bounded by two polar curves, C_1 and C_2, is used to model the face of a flat earring.

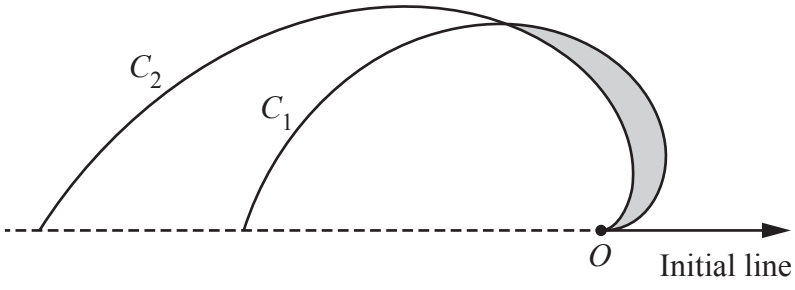
The polar equations of C_1 and C_2 are

$C_1: r = 2\theta$

$C_2: r = \theta^2$

where $0 \leqslant \theta \leqslant \pi$.

The curves C_1 and C_2 are shown in the diagram below with the region used to model the earring shaded.



You are given that C_1 and C_2 intersect at the pole O.

(a) Find the other point of intersection of C_1 and C_2. Give your answer in polar coordinates. **[2]**

(b) **In this question you must show detailed reasoning.**

Determine the area of the face of the earring. **[4]**

7 **In this question you must show detailed reasoning.**

(a) Express $\dfrac{-4x^2+5x-17}{x^3-x^2+3x-3}$ in partial fractions. **[6]**

(b) Hence determine the exact value of $\displaystyle\int_{\sqrt{3}}^{3} \frac{-4x^2+5x-17}{x^3-x^2+3x-3}\,\mathrm{d}x$. **[4]**

8 A function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x\sinh 2x$.

(a) Prove by induction that $\dfrac{\mathrm{d}^{2n}\mathrm{f}}{\mathrm{d}x^{2n}} = 4^n(x\sinh 2x + n\cosh 2x)$ for $n \geqslant 0$ where $\dfrac{\mathrm{d}^0\mathrm{f}}{\mathrm{d}x^0}$ is defined as being equal to $\mathrm{f}(x)$. **[6]**

(b) Using the formula given in part **(a)**, determine the exact value of the coefficient of x^8 in the Maclaurin series for $x\sinh 2x$. **[3]**

(c) Use the Maclaurin series for e^x to verify your answer to part **(b)**. **[3]**

9 When light hits a certain photo-sensitive cell, at time $t = 0$, there is an electrical response in the cell which is denoted by $y(t)$ where both y and t are measured in suitable units. A student wishes to model this response, y.

In an attempt to model y, the student sets up the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6\frac{\mathrm{d}y}{\mathrm{d}t} + 9y = 10\,\mathrm{e}^{-3t} \quad (*)$$

which is subject to the following conditions.

- $y = 0$ when $t = 5$
- $y \geqslant 0$ for all $t \geqslant 0$

(a) By substituting into a suitable differential equation, **verify** that $y = (A + Bt)\mathrm{e}^{-3t}$ is a complementary function of the differential equation (*). **[2]**

(b) Determine the particular solution for y in terms of t. **[8]**

(c) Using your answer to part **(b)**, find the value of y immediately after the light hits the cell. **[1]**

The cell can be considered to be operating properly if $y < 4$ when $t = 1$.

(d) Discuss whether the cell can be inferred to be operating properly. **[2]**

10 **(a)** Find $\frac{1}{2}e^{-u}$ as a percentage of $\sinh u$ for the following values of u.

- $u = 2$
- $u = 5$ **[1]**

(b) Find, as a percentage of $\sinh u$, the difference between $\sinh u$ and $\cosh u$ for the following values of u.

- $u = 2$
- $u = 5$ **[1]**

A function f is defined for all integers n by $f(n) = \sinh(0.01n) - 5\cosh(0.005n) - 9\tanh n$.

(c) **In this question you must show detailed reasoning.**

With the help of suitable approximations, use an algebraic method to determine the smallest value of n for which $f(n) > 100$. You should **verify** your answer, once found, by direct calculation. You may assume that the required value of n is large. **[6]**

END OF QUESTION PAPER

© OCR 2025

BLANK PAGE

© OCR 2025

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025