Oxford Cambridge and RSA

Friday 13 June 2025 – Afternoon

A Level Further Mathematics A

Y543/01 Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 A particle P of mass 4 kg is moving in a straight line with a speed of $7\,\mathrm{m\,s^{-1}}$ on a smooth horizontal surface.

A constant force of magnitude 12 N acts on P for 5 seconds in the same direction as P's motion.

(a) Find the magnitude of the impulse exerted by the force on P. You must include the units. **[2]**

(b) Hence determine the speed of P at the instant when the force stops acting. **[2]**

(c) By considering the change in the kinetic energy of P, determine the work done by the force over the 5 seconds that it acts. **[2]**

(d) Hence find the distance travelled by P while the force is acting on it. **[2]**

2 Dynamic viscosity is a physical property of liquids and is denoted by η.

The units of η are $\mathrm{Ns/m^2}$.

(a) Determine the dimensions of η. **[3]**

Kinematic viscosity is another physical property of liquids and is denoted by ν.

Kinematic viscosity and dynamic viscosity are related by the formula $\nu = \frac{\eta}{\rho}$ where ρ is the density (mass per unit volume) of the liquid.

(b) By finding the dimensions of ν, state a possible unit for ν. **[3]**

3 In this question you must show detailed reasoning.

A particle P of mass 3.25 kg is moving on a smooth horizontal plane with velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j}$.

(a) Find the kinetic energy of P. **[2]**

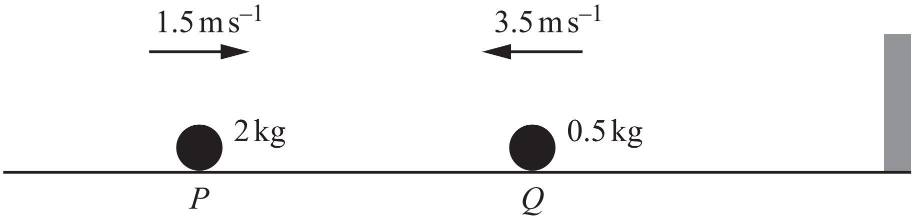
A system of horizontal forces starts to act on P. One of the forces is $\mathbf{F}$ N where $\mathbf{F} = \mathbf{i} - (3+t)\mathbf{j}$ and t is the time, in seconds, after the system of forces starts to act.

(b) Find the power developed by $\mathbf{F}$ at time $t = 0$. **[2]**

(c) Given that the velocity of P does not change at any time, find the **total** power developed by the forces other than $\mathbf{F}$ at time $t = 6$. **[2]**

4 Two particles, P of mass 2 kg and Q of mass 0.5 kg, are moving on a smooth horizontal surface.

Particle P is moving with speed $1.5\,\mathrm{m\,s^{-1}}$ directly towards a vertical wall. Particle Q is between P and the wall and is moving directly towards P with speed $3.5\,\mathrm{m\,s^{-1}}$ (see diagram).



The particles collide directly. After the collision, P moves away from the wall with speed $0.3\,\mathrm{m\,s^{-1}}$. The coefficient of restitution between P and Q is denoted by e.

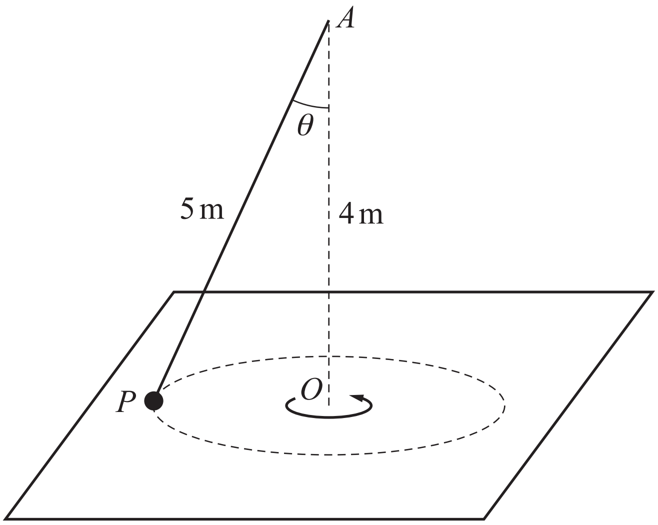
(a) Determine the value of e. **[4]**

After this collision, Q goes on to collide directly with the wall. The coefficient of restitution between Q and the wall is $\frac{1}{10}e$.

(b) Determine whether there will be a subsequent collision between P and Q. **[2]**

5 One end of a light inextensible string of length 5 m is attached to a fixed point A. The other end is attached to a particle P of mass m kg which lies on a smooth horizontal plane.

With the string taut, P moves on the plane with constant speed $v\,\mathrm{m\,s^{-1}}$ in a circular path. The centre of the circular path is the point O which is 4 m vertically below A. The string makes an angle of θ with the downward vertical through A (see diagram).



(a) Given that the magnitude of the contact force between P and the surface is equal to the tension in the string, determine the value of v. **[4]**

(b) Given instead that P is about to lose contact with the surface, determine the angular velocity of P. **[3]**

6 One end of a light inextensible string of length r m is attached to a fixed point O. The other end is attached to a particle P of mass m kg.

Particle P hangs in equilibrium vertically below O with the string taut. An initial horizontal impulse is applied to P so that it instantaneously starts moving horizontally with a speed of $u\,\mathrm{m\,s^{-1}}$.

Particle P subsequently moves in complete circles in a vertical plane containing O.

When the line OP makes an angle of θ with the downward vertical through O, the speed of P is $v\,\mathrm{m\,s^{-1}}$ and the tension in the string is T N.

(a) Show that $T = \frac{m}{r}\left(u^2 + \alpha gr + \beta gr\cos\theta\right)$ where α and β are constants to be determined. **[6]**

(b) Determine, in terms of m, g and r, the minimum initial impulse required to ensure that P describes complete vertical circles around O with the string remaining taut. **[3]**

7 A particle P of mass 36 kg moves in a straight line on a smooth horizontal plane. At time t seconds the displacement of P from a fixed point O is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. Initially, P is at rest at O.

At time $t = 0$, P is acted on by a horizontal force, directed along the line. The magnitude of the force is proportional to $\sqrt{9 - v^2}$. When $v = 2.4$, the magnitude of the force is 32.4 N. When the velocity of P reaches $3\,\mathrm{m\,s^{-1}}$, the force ceases to act.

(a) Show that while the force is acting the motion of P is modelled by the differential equation $\frac{1}{\sqrt{9-v^2}}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}$. **[3]**

(b) Hence find v in terms of t, showing that this expression for v is valid for $0 \leqslant t \leqslant \pi$. **[5]**

(c) Determine the total distance travelled by P in the interval $0 \leqslant t \leqslant 10$. Give your answer in an exact form. **[4]**

8 In this question you must show detailed reasoning.

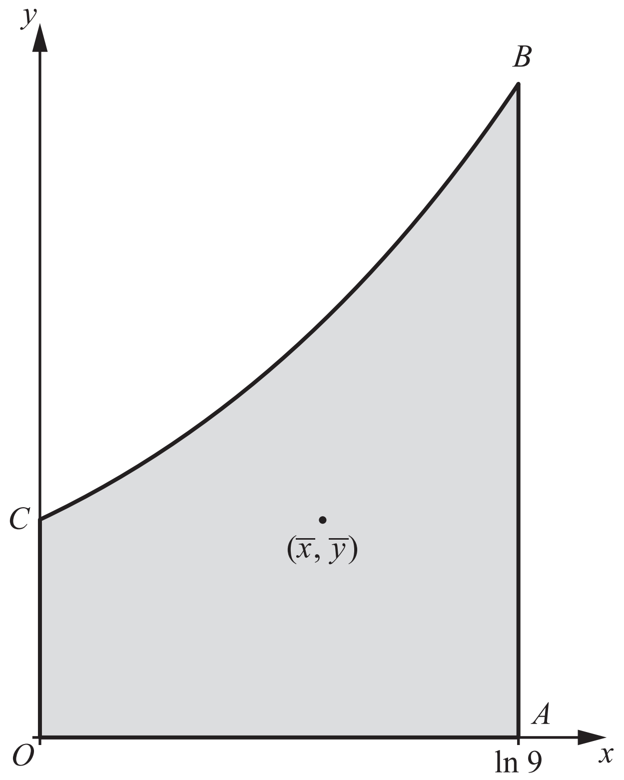
The units of the coordinate system in this question are metres.

A designer is constructing a sign *OABC* which is to be made from a flat board.

The designer creates two possible models for the sign, Model 1 and Model 2. In both models the sign is modelled by a uniform lamina.

In Model 1, the lamina occupies the region bounded by the x-axis, the lines $x = 0$ and $x = \ln 9$ and the curve with equation $y = e^{0.5x}$.

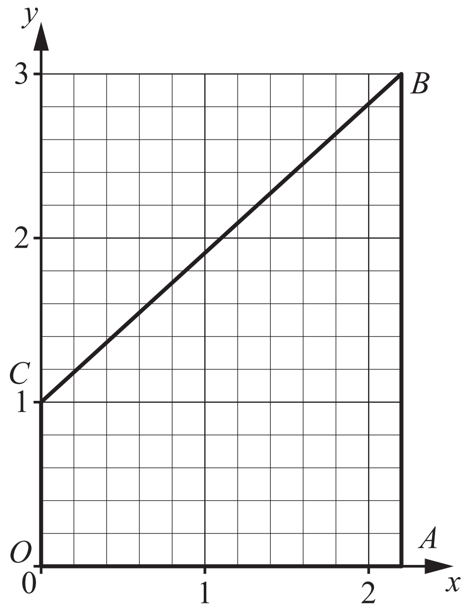
The vertices of the sign are the points $O(0, 0)$, $A(\ln 9, 0)$, $B(\ln 9, 3)$ and $C(0, 1)$. The centre of mass of the lamina in Model 1 is at the point with coordinates $(\bar{x}, \bar{y})$ (see diagram).



(a) **(i)** Show that $\bar{x} = 3\ln 3 - 2$. **[4]**

(ii) Find the value of $\bar{y}$. **[2]**

Model 2 is a simplified model for the sign. In this model, the lamina is in the shape of a trapezium with vertices at the same points O, A, B and C as in Model 1 (see diagram).



The centre of mass of the lamina in Model 2 is at the point with coordinates $(\overline{X}, \overline{Y})$.

(b) Find the value of $\overline{X}$ and the value of $\overline{Y}$. **[3]**

The sign is placed on a rough plane inclined at 50° to the horizontal. The line OA rests on the plane with O below A. The line OA lies along a line of greatest slope of the plane.

You may assume that all forces acting on the sign lie in the vertical plane containing OA and the plane is sufficiently rough to prevent sliding.

(c) **(i)** You are given that $\overline{x}$ and $\overline{y}$ from **Model 1** are both within 0.04 m of the actual coordinates of the centre of mass of the sign.

Determine whether the sign will topple. Justify your answer. **[3]**

(ii) Use your answer to part **(b)** to explain whether **Model 2** would have predicted the same result as in part **(c)(i)**. **[1]**

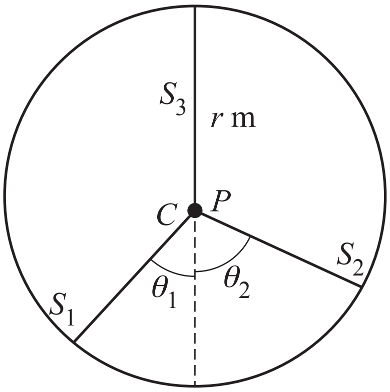
9 In this question you must show detailed reasoning.

A rigid circular frame of centre C and radius r m is fixed in a vertical plane. A particle P of mass 1.25 kg is attached to one end of each of three elastic strings, S_1, S_2 and S_3.

The natural length of each string is $\frac{2}{3}r$. The modulus of elasticity of S_1 is 20 N. The other ends of the strings are attached to the circular frame in such a way that P remains in equilibrium at C.

The string S_3 is attached to the frame at the point vertically above C while S_1 and S_2 are attached at points such that S_1 makes an angle of θ_1, and S_2 an angle of θ_2, with the downward vertical through C, one on each side of this vertical (see diagram).

You are given that $\sin\theta_1 = \frac{3}{5}$ and $\sin\theta_2 = \frac{12}{13}$.



Find, in terms of r, the total elastic potential energy stored in all three strings. **[8]**

END OF QUESTION PAPER