Oxford Cambridge and RSA

Wednesday 4 June 2025 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

1881157028

QP

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series
$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series
$$S_n = \frac{a(1-r^n)}{1-r}$$
$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series
$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$
where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles
$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration
$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$
$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations
$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

1 Express each of the following in the form px^q, where p and q are constants.

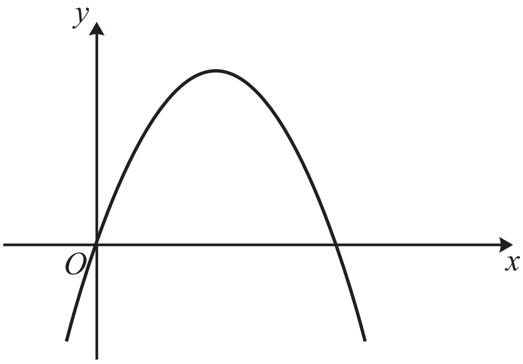
(a) $\dfrac{2}{\sqrt[4]{x}}$ **[1]**

(b) $(5x\sqrt{x})^3$ **[2]**

(c) $\sqrt{2x^3} \times \sqrt{8x^5}$ **[2]**

(d) $x^5(27x^6)^{\frac{1}{3}}$ **[3]**

2



The diagram shows the curve $y = ax(x-b)$, where a and b are constants and $b > 0$.

(a) Given that the curve has a stationary point at $x = 3$, state the value of b. **[1]**

(b) Given also that the stationary point at $x = 3$ is a maximum, state what can be deduced about the value of a. **[1]**

(c) Find the y-coordinate of the stationary point, giving your answer in terms of a. **[1]**

(d) State the range of values of x for which the curve is increasing. **[1]**

3 **(a)** A sequence has terms $u_1, u_2, u_3, \ldots$ defined by $u_1 = 3$ and $u_{n+1} = u_n^2 - 5$ for $n \geqslant 1$.

(i) Find the values of u_2, u_3 and u_4. **[2]**

(ii) Describe the behaviour of the sequence. **[1]**

(b) The second, third and fourth terms of a geometric progression are 12, 8 and $\frac{16}{3}$.

Determine the sum to infinity of this geometric progression. **[3]**

4 The position vector of the point A is $0.5\mathbf{i}-0.5\mathbf{j}$.

(a) Determine whether $0.5\mathbf{i}-0.5\mathbf{j}$ is a unit vector. **[2]**

(b) Find the direction of $0.5\mathbf{i}-0.5\mathbf{j}$, giving your answer in degrees. **[2]**

The position vector of the point B is $3.5\mathbf{i}+c\mathbf{j}$, where c is a constant.

(c) Given that $|\overrightarrow{AB}|=5$, determine the possible values of c. **[3]**

5 Scientists are comparing h, the average height of a child in cm, with t, the age of the child in years. They suggest the model $h=at+b$ for $t \geqslant 2$, where a and b are constants.

They find that the average height of a 2 year old child is 87 cm and the average height of a 5 year old child is 108 cm.

(a) Find the values of a and b that are consistent with the scientists' findings. **[4]**

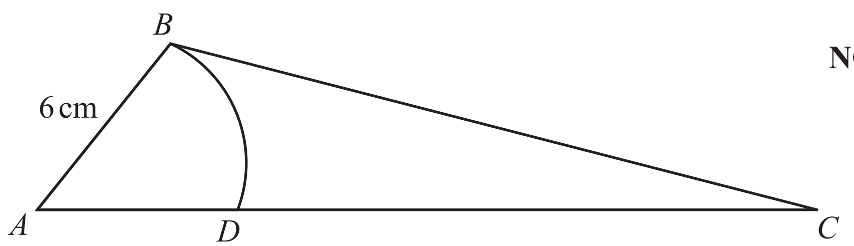
(b) **(i)** Sam is 4 years old.

Use the model to predict Sam's height. **[2]**

(ii) Comment on the accuracy of this prediction. **[1]**

(c) Suggest **one** possible limitation of this model when predicting the average height of a 12 year old child. **[1]**

6



NOT TO SCALE

The diagram shows a triangle ABC. The arc BD is part of a circle with centre A and radius 6 cm.

The area of the sector ABD is 14.4 cm^2.

(a) Show that angle BAD is 0.8 radians. **[1]**

The area of the triangle ABC is **three** times the area of the sector ABD.

(b) Find the length AC. **[2]**

(c) Find the perimeter of the region BCD. **[5]**

7 In this question you must show detailed reasoning.

A curve has parametric equations $x = t^3 + t^2,\ y = t^2 + 2t$ for all real values of t.

The curve passes through the point P with coordinates (2, 3).

(a) Show that the equation of the tangent to the curve at the point P can be written as $5y = 4x + 7$. **[5]**

(b) Determine the coordinates of the point where the tangent to the curve at P meets the curve again. **[4]**

8 (a) Find the first **three** terms in the expansion of $(2+x)^{-3}$ in ascending powers of x. **[4]**

(b) Hence find the first **three** terms in the expansion of $\dfrac{\sqrt{1+4x}}{(2+x)^3}$ in ascending powers of x. **[4]**

(c) Determine the range of values of x for which the expansion in part **(b)** is valid, giving a reason for your answer. **[2]**

9 (a) Express $3\cos 2x + 4\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(b) In this question you must show detailed reasoning.

Solve the equation $\dfrac{3\cos 2x + 4\sin 2x + 6}{3\cos 2x + 4\sin 2x - 1} = 4$ for $0° < x < 360°$. **[7]**

10 The graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x-axis **followed by** a translation.

(a) **(i)** State the scale factor of the stretch. **[1]**

(ii) Give full details of the translation. **[2]**

Alternatively the graph of $y = e^x$ can be transformed to the graph of $y = e^{2x-1}$ by a stretch parallel to the x-axis and a stretch parallel to the y-axis.

(b) State the scale factor of the stretch parallel to the y-axis. **[1]**

The point P lies on the curve $y = e^{2x-1}$ and has x-coordinate of $\frac{1}{2}$.

(c) Show that the tangent to the curve $y = e^{2x-1}$ at P has equation $y = 2x$. **[4]**

(d) Find the **exact** area enclosed by the curve $y = e^{2x-1}$, the tangent to the curve at P and the y-axis. **[4]**

11 A student is attempting to prove that $\sqrt{5}$ is irrational.
The first three lines of their proof are shown below.

Assume that $\sqrt{5}$ is rational, so it can be written as $\sqrt{5} = \frac{a}{b}$.	**Line 1**
Squaring both sides gives $5 = \frac{a^2}{b^2}$. Hence $a^2 = 5b^2$.	**Line 2**
Hence a must be a multiple of 5, so $a = 5k$, for some integer k.	**Line 3**

(a) State any conditions required on **Line 1**. **[2]**

(b) Explain why **Line 2** means that a must be a multiple of 5. **[2]**

(c) Complete the proof to show that $\sqrt{5}$ is irrational. **[3]**

Turn over for question 12

12 **(a)** Express $\dfrac{2+4x}{x(1+x)(1-x)}$ in partial fractions. **[4]**

The gradient of a curve is given by $\dfrac{dy}{dx} = \dfrac{2+4x}{x(1+x)(1-x)\tan y}$ and the curve passes through the point $\left(\frac{1}{2}, \frac{1}{4}\pi\right)$.

(b) Show that the equation of the curve can be written in the form $\cos y = f(x)$, where $f(x)$ is fully simplified. **[7]**

END OF QUESTION PAPER