



Oxford Cambridge and RSA

Thursday 19 June 2025 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **16** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A
Pure Mathematics

- 1 (a) Express $3x^2 - 12x + 17$ in the form $a(x - b)^2 + c$ where a , b and c are constants. [3]
- (b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. [1]
- (c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. [1]
- 2 (a) The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x -direction.
Write down the equation of the curve after it has been translated. [2]
- (b) The curve $y = 5x^2$ is stretched parallel to the y -axis with scale factor 4.
The point on the curve $y = 5x^2$ with x -coordinate 3 is transformed to the point P .
Write down the coordinates of P . [2]

- 3 (a) Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet. [2]
- (b) Solve the inequality $|2x - 5| < 1$. [2]
- (c) **In this question you must show detailed reasoning.**

Hence find **all** the possible integer values N that satisfy the inequality

$$|2e^{0.1N} - 5| < 1. \quad [3]$$

- 4 Functions f and g are defined for all real values of x by

$$f(x) = \frac{x-k}{2} \text{ and } g(x) = x^2 + kx + 5, \text{ where } k \text{ is a constant.}$$

You are given that the equation $f^{-1}g(x) = 4 - 2kx$ has real distinct roots.

- (a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. [5]
- (b) **In this question you must show detailed reasoning.**

Hence find the set of values of k . Give your answer in set notation. [3]

- 5 (a) Use the substitution $u = \cos x$ to show that

$$\int \frac{1 + \cos x}{\sin x} dx = \ln(1 - \cos x) + c. \quad [4]$$

Fig. 1

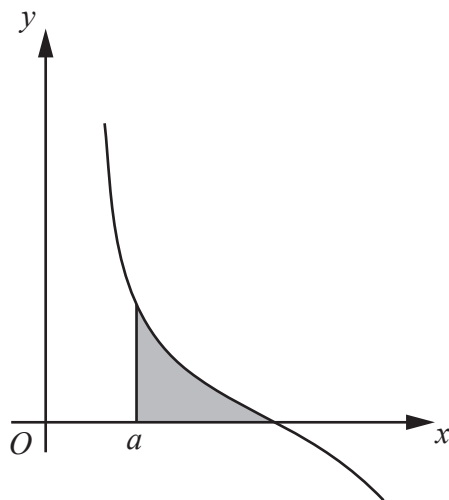


Fig. 1 shows part of the curve $y = \frac{1 + \cos x}{\sin x}$.

The shaded region is bounded by the curve, the x -axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

- (b) Show that the value of a satisfies the equation $a - \cos^{-1}(1 - 2e^{-a}) = 0$. [4]
- (c) Show by calculation that the value of a lies between 1.1 and 1.2. [2]
- (d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1 - 2e^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to **2** decimal places. Show the result of each step of the iterative process. [2]

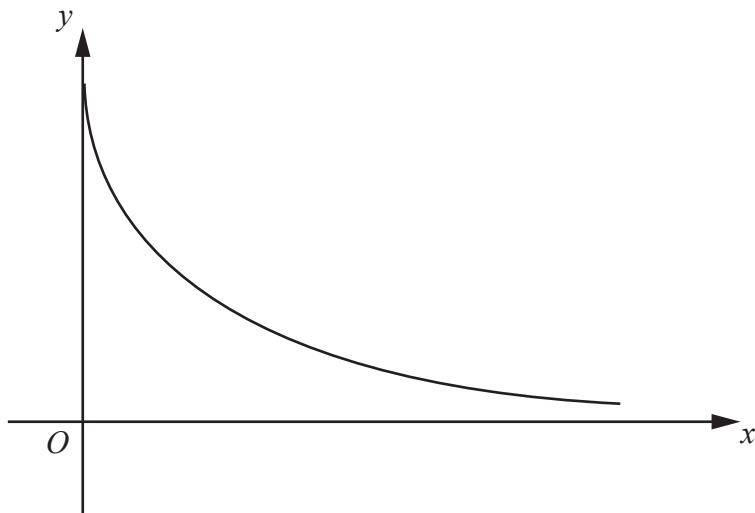
Fig. 2

Fig. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

- (e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part (d), lies in the interval $(-1, 0)$. **[2]**

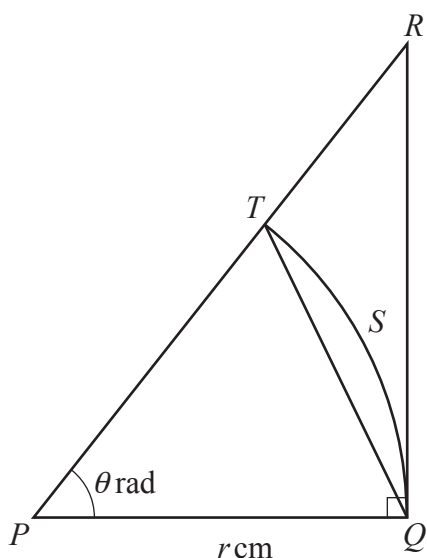
- 6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \text{ and}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B.$$

- (a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right). \quad [1]$$



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

- (b) By considering the areas of triangle PQT , sector $PQST$ and triangle PQR , show that

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}. \quad [4]$$

- (c) (i) Hence write down a similar inequality interval for the expression $\frac{\sin \theta}{\theta}$. [1]

- (ii) Hence state $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$. [1]

- (d) Using parts (a) and (c)(ii), show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. [4]

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let $y = \cos x$, where x is measured in radians.

$$y = \sin\left(\frac{\pi}{2} - x\right)$$

so $\frac{dy}{dx} = \cos\left(\frac{\pi}{2} - x\right)$

but $\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$

therefore $\frac{dy}{dx} = \sin x$.

- (e) Identify the error made by the student. [1]

Section B
Mechanics

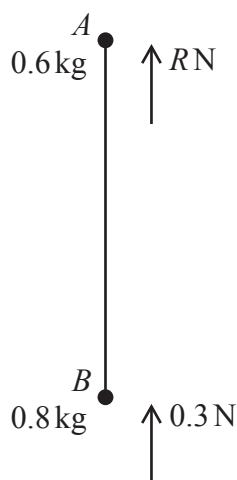
- 7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i} + 3\mathbf{j})\text{ N}$ and $(a\mathbf{i} + 3b\mathbf{j})\text{ N}$. The acceleration of P is $(2\mathbf{i} - 3\mathbf{j})\text{ m s}^{-2}$.

(a) Find the value of a and the value of b . [3]

At time $t = 0$ seconds the velocity of P is $\mathbf{u}\text{ m s}^{-1}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i} - 6\mathbf{j})\text{ m s}^{-1}$.

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{u}$. [2]

8



Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A .

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of $R\text{ N}$ and the air resistance acting on B has a constant magnitude of 0.3 N . The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is 9.2 m s^{-2} and the tension in the string is $T\text{ N}$.

(a) By applying Newton's second law to B , find the value of T . [2]

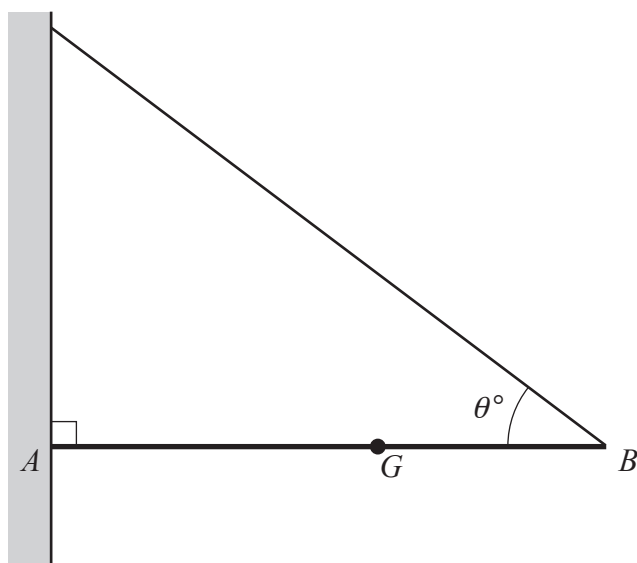
(b) Hence, or otherwise, find the value of R . [2]

9 In this question you must show detailed reasoning.

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v \text{ m s}^{-1}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is 15 m s^{-2} . [6]

10



The diagram shows a **non-uniform** rod AB of mass 4 kg and length 2 m . The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B . The other end of the string is attached to the wall at a point vertically above A . The string is inclined at an angle of θ° to the horizontal, where $\sin \theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB .

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m .

- (a) Show that the distance AG is 1.2 m . [1]
- (b) Find the tension in the string. [2]
- (c) Determine the magnitude of the contact force exerted on the rod by the wall at A . [5]

11 In this question you should take the acceleration due to gravity to be 10 m s^{-2} .

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j}) \text{ m s}^{-1}$.

The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})\text{m}$ relative to O .

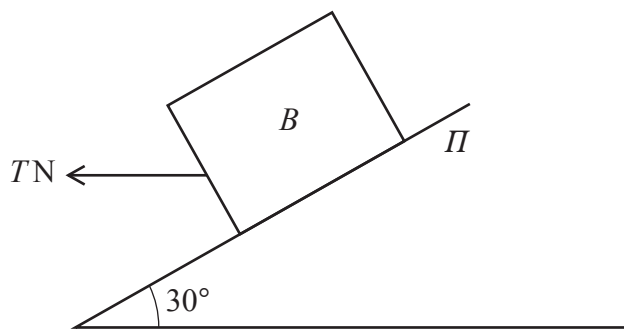
- (a) Show that $y = 6x - 1.25x^2$. [5]
- (b) Determine the maximum height of P above the ground during its motion. [3]
- (c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.
 - the speed of P
 - the direction of motion of P [5]

In reality P does not move only under the influence of gravity but is also subject to air resistance.

- (d) Explain how this would affect your answer to part (b). [1]

Turn over for question 12

12

Fig. 1

A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude $T\text{ N}$, acting above a line of greatest slope, is applied to B (see **Fig. 1**).

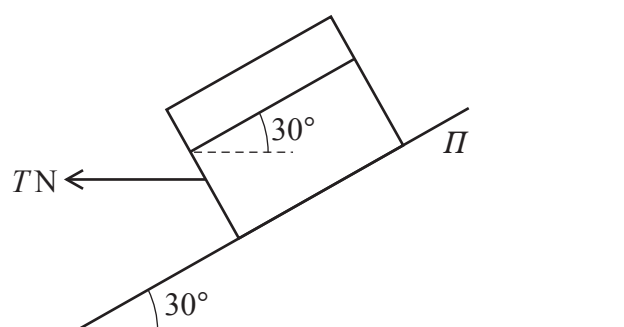
The coefficient of friction between B and the plane is 0.8 .

(a) Show that the value of T is 14.9 , correct to **3** significant figures.

[6]

For the remainder of the question, you should use this value of T .

Fig. 2



Block B is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg , and the lower block has a mass of 6 kg . The two blocks are held at rest with the lower block on Π . The horizontal force of magnitude $T\text{ N}$ is now applied to the lower block (see Fig. 2).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration 3.5 m s^{-2} .

- (b) Determine the coefficient of friction between the two blocks. [4]
- (c) Show that the lower block does **not** move. [3]

END OF QUESTION PAPER

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