OCR
Oxford Cambridge and RSA

Thursday 12 June 2025 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

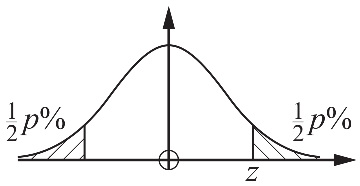
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (23 marks)

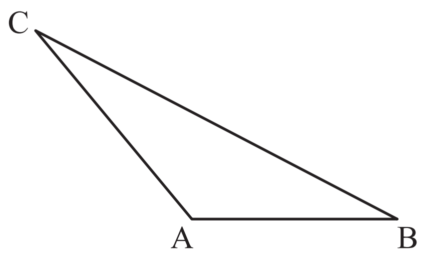
1 The equation of a circle is $(x-4)^2+(y+5)^2-64=0$.

(a) State the coordinates of the centre of the circle. **[1]**

(b) State the radius of the circle. **[1]**

2 Determine the coefficient of x^3 in the expansion of $(1+2x)^{12}$. **[2]**

3 The diagram shows triangle ABC.



Not to scale

AB = 4.7 cm, AC = 6.9 cm and BC = 11.2 cm.

Find the size of angle CAB. Give your answer correct to **3** significant figures. **[2]**

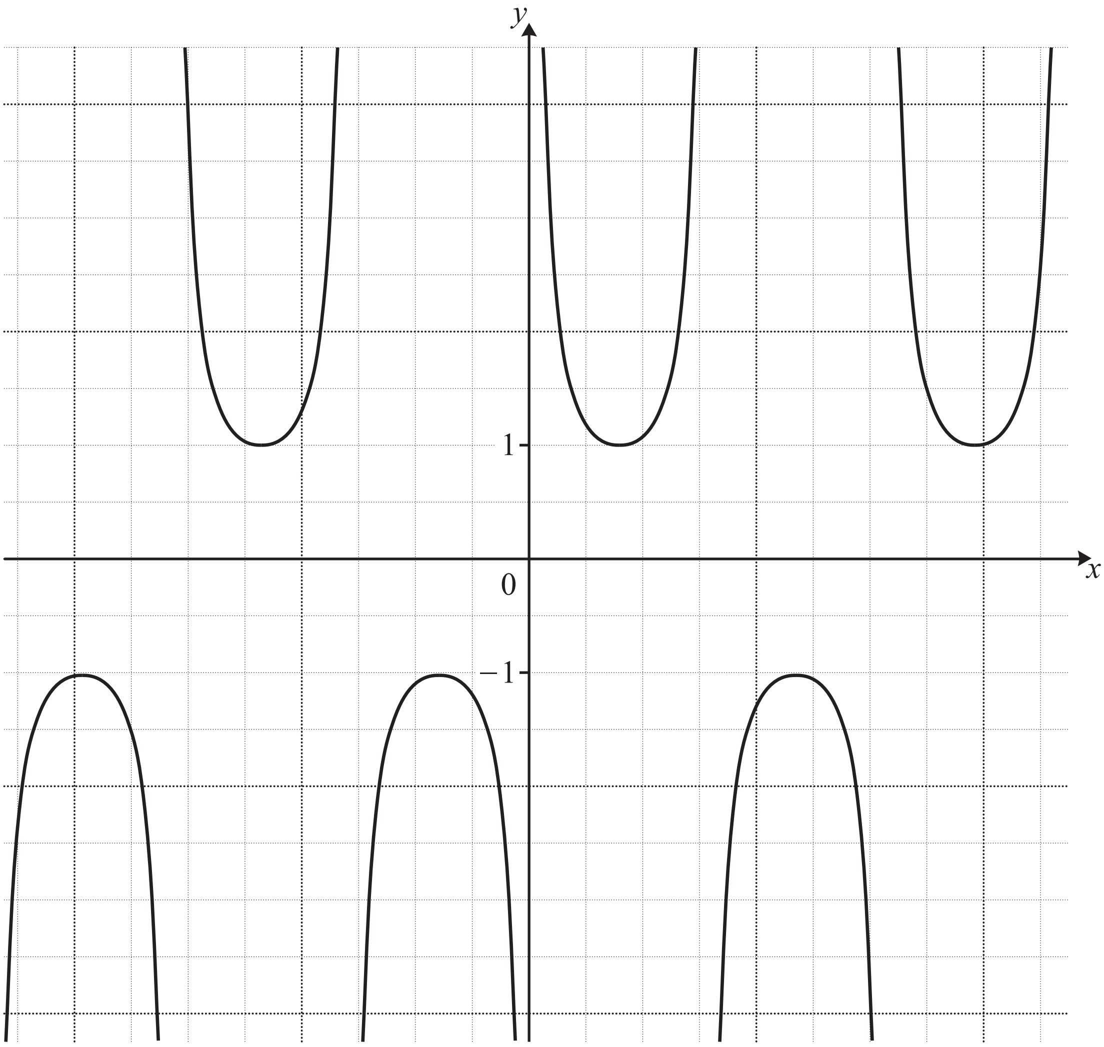
4 **(a)** Use the factor theorem to show that $(x-3)$ is a factor of $4x^3-8x^2-11x-3$. **[1]**

(b) Hence show that $4x^3-8x^2-11x-3=(x-3)(ax+b)^2$, where a and b are integers to be determined. **[2]**

(c) Sketch the graph of $y=4x^3-8x^2-11x-3$ on the axes in the Printed Answer Booklet. **[2]**

5 Prove that the sum of the first n positive odd numbers is a square number. **[3]**

6 The diagram shows part of the graph of $f(x) = \operatorname{cosec} 2x$.



(a) State which of the following is the equation of the curve $y = \dfrac{1}{f(x)}$.

$y = \cos 2x$ $\quad$ $y = \operatorname{cosec}(-2x)$ $\quad$ $y = -\operatorname{cosec} 2x$ $\quad$ $y = \sin 2x$ **[1]**

(b) State the exact equations of the asymptotes of $y = \operatorname{cosec} 2x$ for $0 \leqslant x \leqslant \pi$. **[1]**

7 Some people have blood type A and some people have brown hair. A person may have both, just one, or neither of these characteristics.

A person is selected at random.

A is the event 'the person has blood type A'.
B is the event 'the person has brown hair'.

You are given the following probabilities.

$\mathrm{P}(\mathrm{A} \cap \mathrm{B}') = 0.3382$, $\mathrm{P}(\mathrm{A}' \cap \mathrm{B}) = 0.0682$ and $\mathrm{P}((\mathrm{A} \cup \mathrm{B})') = 0.5518$.

(a) Show this information on the Venn diagram in the Printed Answer Booklet. **[2]**

(b) Determine whether A and B are independent events. **[4]**

(c) Explain whether A and B are mutually exclusive events. **[1]**

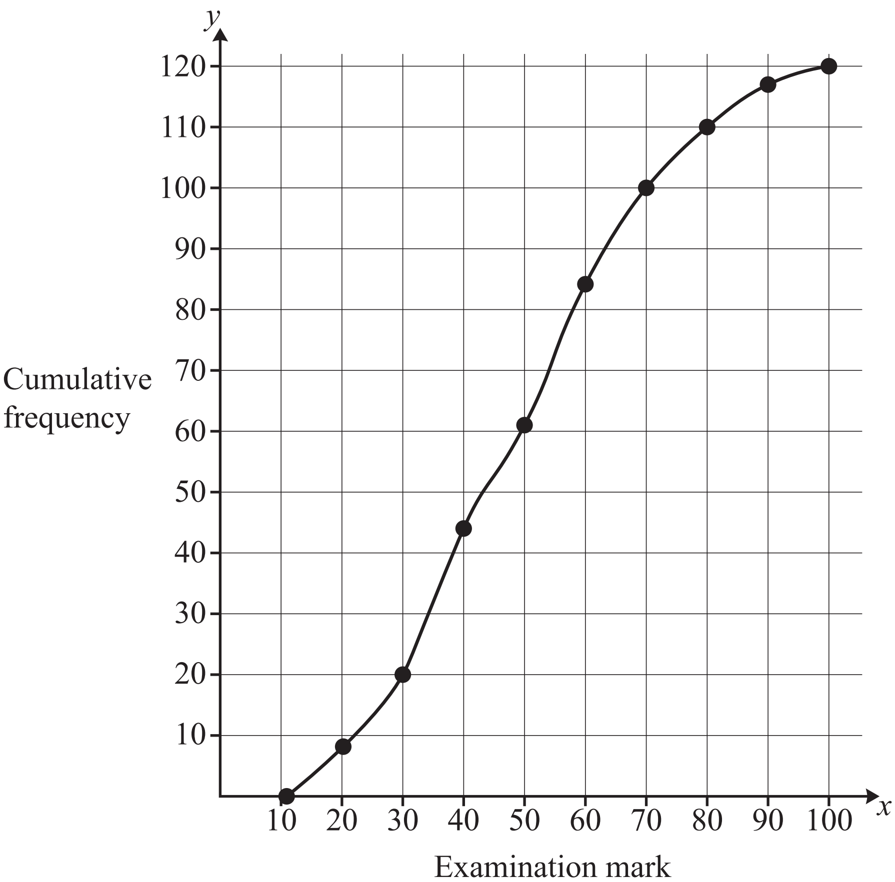
Section B (77 marks)

8 At the end of the summer term all 120 Year 8 pupils at Oakmount College sit the same mathematics examination.

The marks obtained by the pupils in the 2024 examination are summarised in the table.

Mark	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	8	12	24	17	23	16	10	7	3

Software is used to produce the cumulative frequency curve shown below.



(a) It is proposed that the 30 pupils with the best marks in the examination are placed in the top set in Year 9.

Use the copy of the cumulative frequency curve in the **Printed Answer Booklet** to determine an estimate of the lowest mark needed for a pupil to be placed in the top set. **[2]**

(b) The person who will teach the top set states that pupils will **not** manage in this class unless they obtain at least 75 marks in the examination.

Use the copy of the cumulative frequency curve in the **Printed Answer Booklet** to determine an estimate of how many pupils will be in the top set if the lowest mark is set at 75. **[2]**

9 The position vectors of the points A and B are $\overrightarrow{OA} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ respectively.

(a) Write down the vector $\overrightarrow{AB}$. **[1]**

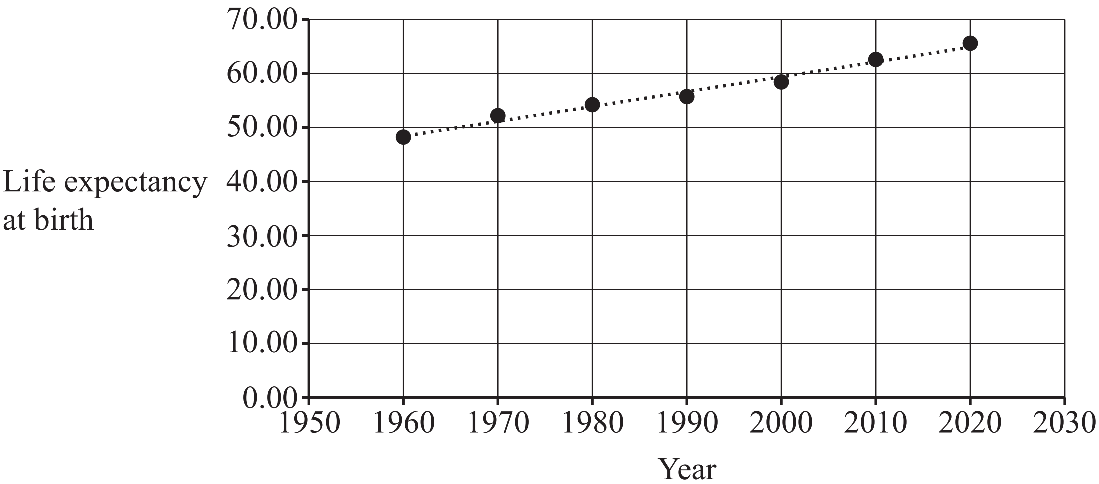
The point C lies on AB such that $2\overrightarrow{AC} = \overrightarrow{CB}$.

(b) Find the vector $\overrightarrow{OC}$. **[2]**

10 The pre-release material contains information about life expectancy at birth for countries of the world at 10-year intervals from 1960 until 2020.

The life expectancy at birth of the population of Sudan for this period, together with a line of best fit, is shown in **Fig. 10.1**.

Fig. 10.1



The equation of the line of best fit is $L = 0.275X - 490$, where X is the year and L is the life expectancy at birth, measured in years.

(a) Use the equation of the line of best fit to estimate the life expectancy at birth in Sudan in 1975. **[1]**

(b) Explain whether this estimate is likely to be close to the true value of life expectancy at birth in Sudan in 1975. **[1]**

(c) Use your knowledge of the pre-release material to explain whether your answer to part **(a)** is likely to be a close approximation to the life expectancy at birth in the United Kingdom in 1975. **[1]**

The pre-release material also gives the median age of the population in countries of the world.

The table shows the median age of the population and the life expectancy at birth in 2020 for some countries in Africa.

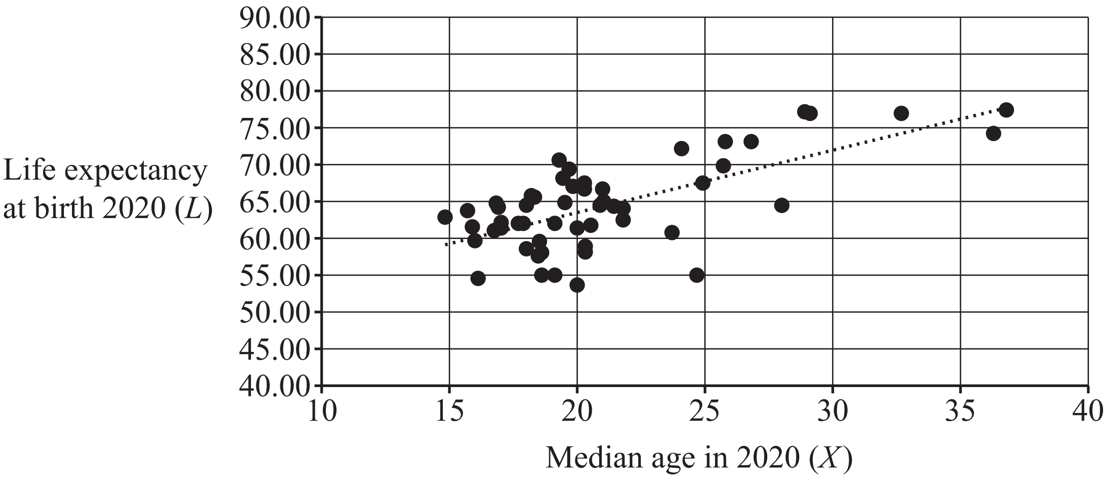
Country	**Median age**	**Life expectancy at birth 2020**
Nigeria	18.6	55.02
Rwanda	19.7	69.33
Saint Helena, Ascension and Tristan da Cunha	43.2	#N/A
Sao Tome and Principe	19.3	70.58
Senegal	19.4	68.21

(d) Explain how the data in the table should be cleaned before these data can be included in a scatter diagram for life expectancy at birth against median age for the countries in Africa. **[1]**

Fig. 10.2 shows a scatter diagram for the cleaned data of life expectancy at birth against median age in 2020 for the countries in Africa. It also shows a line of best fit.

The product moment correlation coefficient for these data is 0.680.

Fig. 10.2



(e) A student decides to use the line of best fit to estimate the life expectancy at birth in 2020 for Saint Helena, Ascension and Tristan da Cunha.

Explain whether this estimate is likely to be reliable. **[1]**

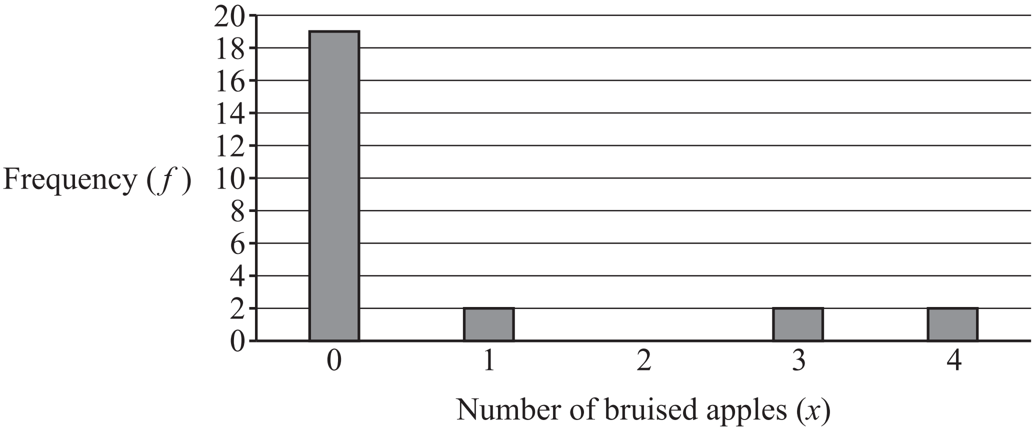
11 Apples are sold in packets of 4. Ling and Sam are investigating the frequency, f, of the number of bruised apples in a packet, x. They each collect a random sample of packets, note the number of bruised apples in each packet and draw a diagram to represent their data.

Ling's results are shown in **Table 11.1** and **Fig. 11.1**.

Table 11.1

x	0	1	2	3	4
f	19	2	0	2	2

Fig. 11.1

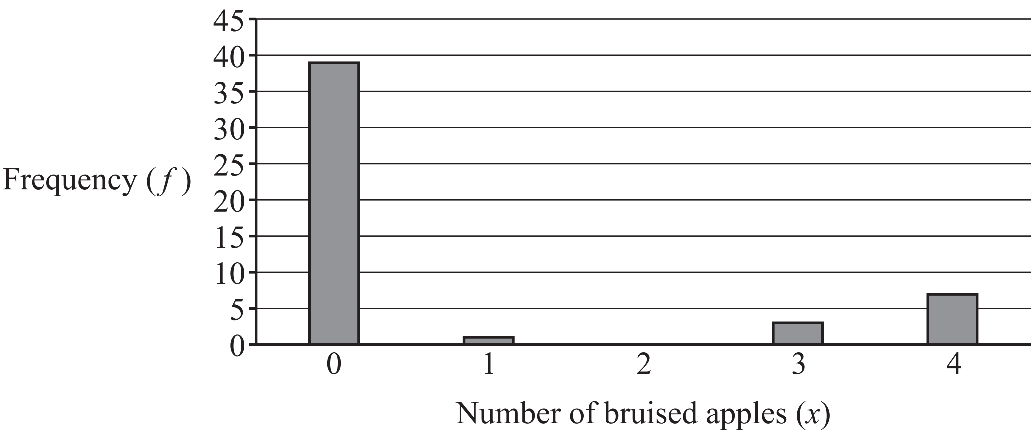


Sam's results are shown in **Table 11.2** and **Fig. 11.2**.

Table 11.2

x	0	1	2	3	4
f	39	1	0	3	7

Fig. 11.2



(a) Explain why Sam's diagram is likely to be a better representation of the true distribution of the number of bruised apples in a packet than Ling's diagram. **[1]**

(b) Calculate the mean number of bruised apples per packet for Sam's data. **[1]**

Sam thinks that the distribution of bruised apples may be modelled by a binomial distribution.

(c) Use your answer to part **(b)** to calculate the value of p, the probability that an apple selected at random is bruised, for Sam's model. **[1]**

(d) Calculate the theoretical frequency distribution of the number of bruised apples per packet for Sam's model, giving your answers correct to **2** decimal places. **[3]**

(e) Comment on whether Sam's model appears to be a good fit for the data. **[1]**

12 In this question you must show detailed reasoning

Solve the equation $3\sec^2\theta + 2\tan\theta - 4 = 0$ for $-180° < \theta < 180°$. **[6]**

13 According to the Office for National Statistics, in 2022 the proportion of people in England and Wales aged 16 or over who were married or in a civil partnership was 0.494.

In 2024 a researcher collected a random sample of 78 people aged 16 or over.

The researcher decided to conduct a hypothesis test at the 1% level to see if there was any evidence to suggest that the proportion of people aged 16 or over who are married or in a civil partnership was lower in 2024.

(a) State the hypotheses for the test. You must define the parameter used to carry out the test. **[2]**

(b) Determine the critical region for the test. **[3]**

The researcher found that 26 of the 78 people in the random sample were married or in a civil partnership.

(c) Use your answer to part **(b)** to carry out the hypothesis test. **[3]**

(d) State the probability that the null hypothesis is incorrectly rejected. **[1]**

14 The equation of a curve is $y = \frac{16}{x^2} + \frac{3}{x}$.

(a) Determine the coordinates of the point where the curve cuts the x-axis. **[2]**

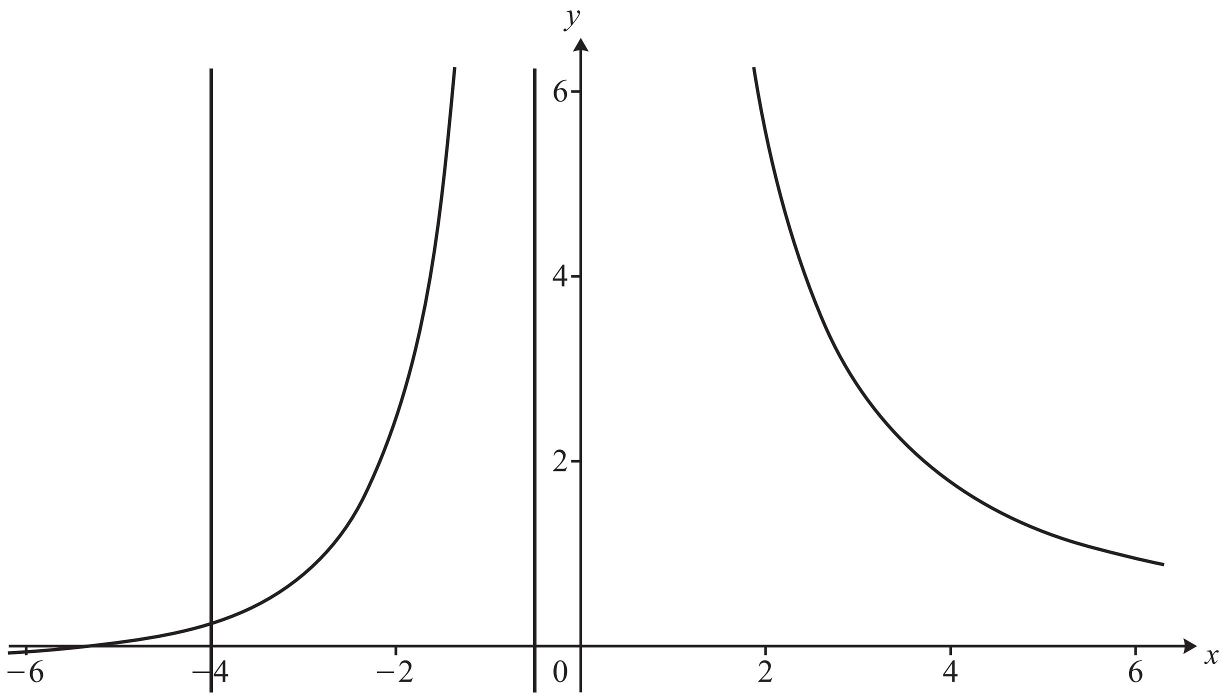
(b) **(i)** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **[2]**

(ii) Hence determine the exact coordinates of the stationary point on the curve $y = \frac{16}{x^2} + \frac{3}{x}$. **[2]**

(c) **(i)** Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **[2]**

(ii) Hence determine the set of values of x for which the curve $y = \frac{16}{x^2} + \frac{3}{x}$ is concave downwards. **[2]**

The diagram shows parts of the curve $y = \frac{16}{x^2} + \frac{3}{x}$, the line $x = -4$ and the line $x = -\frac{1}{2}$.



(d) Determine the exact area bounded by the curve $y = \frac{16}{x^2} + \frac{3}{x}$, the x-axis and the lines $x = -4$ and $x = -\frac{1}{2}$. Give your answer in the form $a + b\ln 2$, where a and b are constants to be determined. **[5]**

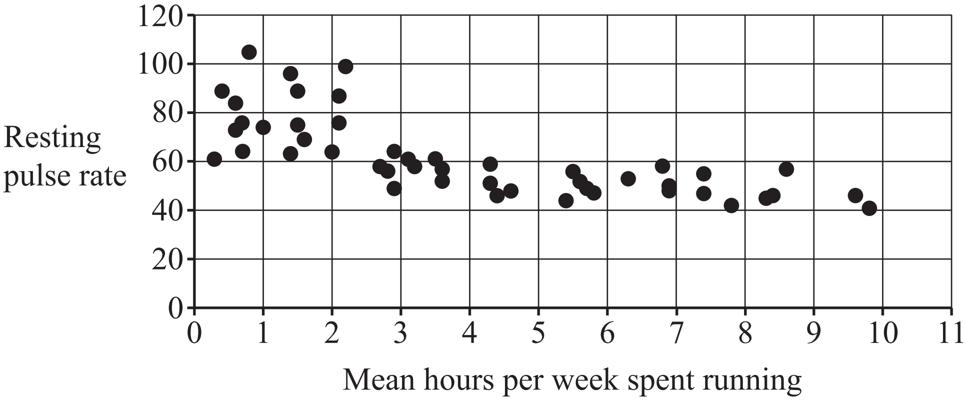
15 A personal trainer is investigating whether, in the general population, there is any association between resting pulse rate in beats per minute and mean hours per week spent running.

One week he collects a sample by asking 25 of his clients for the relevant data.

(a) **(i)** State the name of the sampling technique used by the personal trainer. **[1]**

(ii) Explain why this sampling technique might introduce bias. **[1]**

A biologist researching the same topic collects a random sample of size 47. She represents the data using the scatter diagram shown below.



(b) Describe the association between resting pulse rate and mean hours per week spent running. **[1]**

(c) The biologist identifies two distinct regions on the scatter diagram. Identify these regions on the copy of the scatter diagram in the **Printed Answer Booklet** by drawing an appropriate **vertical** line between them. **[1]**

According to medical research, the normal resting pulse rate for an adult is between 60 and 110 beats per minute.

The biologist separates the data into a group to the left of the vertical line on the scatter diagram and a group to the right of the vertical line on the scatter diagram. She calculates Spearman's rank correlation coefficient, r_s, and the associated p-value for each group.

The results are shown in the table.

Group	r_s	p-value
To the left of the vertical line	0.20936	0.41998
To the right of the vertical line	–0.61349	0.00031

(d) With reference to the two groups identified by the biologist and to the values in the table, explain what may be inferred about the association between resting pulse rate and mean number of hours per week spent running. **[6]**

16 In this question you must show detailed reasoning

The population of puffins on a remote island is estimated to be $P = 58.5$, where P is measured in thousands of birds.

Just after this estimate is made there is a serious oil spill in the area. It is noted that there is a rapid decline in the population of puffins on the island.

The situation is modelled by the differential equation

$\frac{\mathrm{d}P}{\mathrm{d}t} = (72t - 108)\mathrm{e}^{-0.8t}$, where t is the time in years after the pollution incident.

(a) Determine an expression for P in terms of t. **[7]**

(b) Determine whether, according to the model, P will recover to 58.5. You may assume that $\lim_{t\to\infty} t\mathrm{e}^{-0.8t} = 0$. **[2]**

17 In this question you must show detailed reasoning

Jars of honey contain a nominal mass of 340 grams of honey.

The jars are filled by an automated process so that the actual mass of honey in a jar, X grams, is Normally distributed with mean 350 grams. This is to minimize the chance of a jar which contains less than 340 grams of honey being sold.

During a quality control check, a random sample of 50 jars of honey are checked.

Summary statistics for the check are as follows.

$\sum x = 17\,367 \quad \sum x^2 = 6\,036\,597 \quad n = 50$

Carry out a hypothesis test at the 1% level to see if there is any evidence to suggest that the mean mass of honey in the jars is less than 350 grams. **[9]**

END OF QUESTION PAPER