OCR
Oxford Cambridge and RSA

Thursday 19 June 2025 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Insert

Time allowed: 2 hours

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INSTRUCTIONS

- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

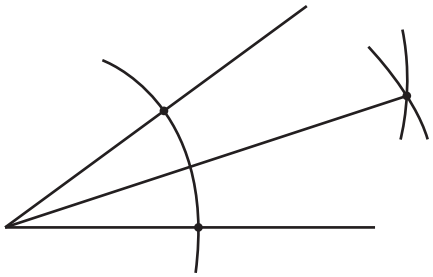
INFORMATION

- This Insert contains the article for Section B.
- This document has **4** pages.

The trisectrix of Maclaurin

To bisect an angle means to divide it into two equal angles. It is possible to bisect an angle using a ruler and compasses. The construction is illustrated in **Fig. C1**.

Fig. C1



To trisect an angle means to divide into three equal angles. It is impossible to trisect an angle using a ruler and compasses alone but mathematicians found other methods.

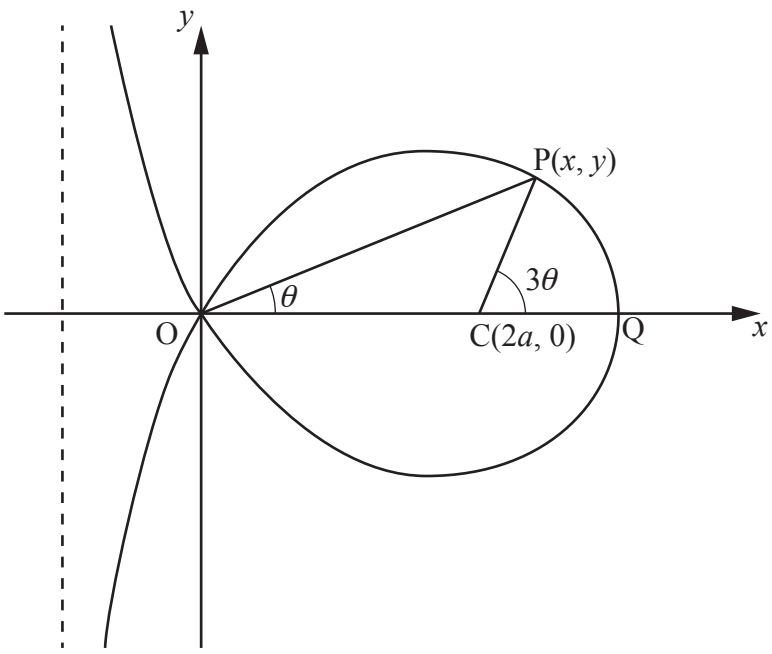
In 1742, Colin Maclaurin first studied a curve which can be used to trisect an angle. The curve is called the Trisectrix of Maclaurin.

The equation of the curve in cartesian form is $y^2 = \frac{x^2(3a-x)}{a+x}$, where a is a constant. The curve is shown in **Fig. C2** below. Only values of $a > 0$ are considered in this article.

It can be shown that everywhere on the curve $\frac{3a-x}{a+x} \geqslant 0$. It follows that $3a-x \geqslant 0$ and $a+x > 0$.

The curve has the trisection property illustrated in **Fig. C2**.

Fig. C2



The point C has coordinates $(2a, 0)$ and Q is the point where the curve crosses the positive x-axis. The point P is a general point (x, y) on the loop of the curve. The origin of the coordinate system is denoted by O.

The dashed line is an asymptote to the curve.

If the line CP makes an angle 3θ with the positive x-axis then the line OP makes an angle θ with the positive x-axis. Angles are measured anticlockwise from the positive x-axis.

To use the curve to trisect an angle, place the angle in position QCP. One arm of the angle (CQ) lies along the x-axis. The other arm of the angle meets the curve at the point P. Angle QOP is one third of angle QCP.

Deriving the cartesian equation of the trisectrix

Using the formula for $\tan(A+B)$ in terms of $\tan A$ and $\tan B$, it can be shown that $\tan 3\theta = \dfrac{t(3-t^2)}{1-3t^2}$, where $t = \tan\theta$.

From this and **Fig. C2** it follows that $\dfrac{x-2a}{y} = \dfrac{1-3t^2}{t(3-t^2)}$.

Hence $\dfrac{1}{t} - \dfrac{2a}{y} = \dfrac{1-3t^2}{t(3-t^2)}$.

It then follows that $\dfrac{2a}{y} = \dfrac{1}{t} - \dfrac{1-3t^2}{t(3-t^2)} = \dfrac{2+2t^2}{t(3-t^2)}$ so $\dfrac{a}{y} = \dfrac{1+t^2}{t(3-t^2)}$ and this gives the parametric equation for y, $y = \dfrac{at(3-t^2)}{1+t^2}$.

The equation for x follows from $\tan\theta = \dfrac{y}{x}$. Together with the parametric equation for y, this leads to $x = \dfrac{a(3-t^2)}{1+t^2}$.

The parameter t can be eliminated from the parametric equations to derive the cartesian equation $y^2 = \dfrac{x^2(3a-x)}{a+x}$, as stated in line 8.