Oxford Cambridge and RSA

GCE

Further Mathematics A

Y540/01: Pure Core 1

A Level

Mark Scheme for June 2025

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING

RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5. **Crossed-Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
   - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
    To determine the level – start at the highest level and work down until you reach the level that matches the answer
    To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		Enlargement, scale factor 5, centre at the origin	**B2** **[2]**	1.2 1.1	**B2** for all three correct components, **B1** for any two Accept "enlarge" and accept "stretch" but only if specified in both x and y directions For 'scale factor' accept just 'factor' or 'SF' but not just '5' For 'origin' accept O or (0, 0) allow omission of 'centre' provided intention is clear (e.g. 'about O') If more than two transformations given, then **B0** (unless this is two stretches in both x and y directions)
1	(b)		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	**B1** **[1]**	1.1	cao
1	(c)		$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	**B1** **[1]**	1.1	cao – only penalise the lack of a bracket around the entries once in parts **(b)** and **(c)**

Question			Answer	Marks	AO	Guidance
2	(a)		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{x^2-9}}$	B1 [1]	1.1	Accept any equivalent (un-simplified) forms e.g. $\frac{1}{3}\times\frac{1}{\sqrt{\left(\frac{x}{3}\right)^2-1}}$ or $\frac{1}{\sqrt{x^2-3^2}}$. Invisible brackets e.g. $\frac{1}{3}\times\frac{1}{\sqrt{\frac{x}{3}^2-1}}$ is **B0** unless correctly recovered. ISW if correct un-simplified form is simplified incorrectly
2	(b)		$m_N=-4$	B1	1.1	Correct normal gradient (soi) – possibly BC
			$y=\ln\left(\frac{5}{3}+\sqrt{\left(\frac{5}{3}\right)^2-1}\right)\quad(=\ln 3)$	B1	1.1	For correct un-simplified (or simplified) y-coordinate in logarithmic form when x = 5 – ISW if simplified incorrectly
			$y-\ln 3=-4(x-5)$	M1	1.1	Any complete correct method for the equation of a straight line with x = 5, their y coordinate (which must be of the form $\ln(a)$ where $a>0$) and any non-zero gradient. If using $y=mx+c$ then must explicitly find c using x = 5, any non-zero m and their y which must be of the form as stated above (but allow errors in the evaluation of c)
			$4x+y=20+\ln 3$	A1 [4]	1.1	oe but must be integer values e.g. $8x+2y=40+\ln 9$ and must be of the form $ax+by=c+\ln d$ e.g. allow $-\ln 3-20=-y-4x$

Question			Answer	Marks	AO	Guidance
3	(a)		**DR** $\int \frac{80}{\sqrt{25-x^2}}\,dx = 80\sin^{-1}\left(\frac{1}{5}x\right)$	**B1**	1.1	Correct integration – this mark can be implied by seeing $80\sin^{-1}\left(\frac{1}{5}k\right)$ but www (so if x missing from the integrated expression before the limits were applied then **B0**) – this mark can also be awarded for $\sin^{-1}\left(\frac{1}{5}x\right)$ from $\int \frac{1}{\sqrt{25-x^2}}\,dx$
			$\left(\int_0^k \frac{80}{\sqrt{25-x^2}}\,dx=\right)\quad 80\sin^{-1}\left(\frac{1}{5}k\right)=\frac{40}{3}\pi$	**M1**	2.1	For setting up an equation of the form $a\sin^{-1}(bk)-a\sin^{-1}(0)=\frac{40}{3}\pi$ **or** $a\sin^{-1}(bk)=\frac{40}{3}\pi$ where $a\neq 0, b\neq 0$ or 1 oe (e.g. may have divided both sides by 80). Must have come from their $a\sin^{-1}(bx)$
			$\left(\sin^{-1}\left(\frac{1}{5}k\right)=\frac{1}{6}\pi \Rightarrow\right)\quad k=\frac{5}{2}$	**A1**	1.1	cao – oe e.g. 2.5 - do not need to explicitly see $\sin^{-1}(0)=0$ for full marks
				[3]		
3	(b)		$x^2=-\frac{80^2}{y^2}+25$	**M1**	1.1	Rearranging to make x^2 the subject – allow sign errors only when re-arranging
			$\pi\int_{16}^{20}\left(-\frac{6400}{y^2}+25\right)dy$	**M1**	3.1a	Integral of the form $\pi\int_{16}^{20}\left(\frac{k_1}{y^2}+k_2\right)dy$ for any non-zero k_1, k_2 with correct limits, **M0** if π missing but condone missing dy
			$=20\pi$	**A1**	1.1	BC – must be exact (ISW if correct exact answer seen and then replaced with non-exact). A correct answer with no working (or no incorrect working) can score full marks (as this part is not **DR**)
				[3]		

Question			Answer	Marks	AO	Guidance
4	(a)		$\alpha+\beta+\gamma=\frac{4}{5}$ $\alpha\beta+\beta\gamma+\gamma\alpha=0$ $\alpha\beta\gamma=-2$	B2 [2]	1.1 1.1	**B1** for any two correct
4	(b)		$(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ $=\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)$	M1	2.1	For expanding $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ - must be equivalent to nine terms but allow an error in at most two terms only. Or for using the correct identity $\sum\alpha^2\beta^2=\left(\sum\alpha\beta\right)^2-2\alpha\beta\gamma\sum\alpha$
			$0^2=2\times(-2)\times\frac{4}{5}+\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$	A1FT	1.1	Correct equation/expression for $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ using their results from part **(a)**
			$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=\frac{16}{5}$	A1 [3]	2.2a	
4	(c)		$(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$	B1	1.1	correct expansion of $(\alpha+\beta+\gamma)^2$
			$\alpha^2+\beta^2+\gamma^2=\frac{16}{25}$	B1 [2]	2.2a	www but condone from $\alpha+\beta+\gamma=-\frac{4}{5}$ stated in part **(a)** – this mark is independent of the first **B1** mark
4	(d)		$u^3-\frac{16}{25}u^2+\frac{16}{5}u-(-2)^2(=0)$	M1	2.2a	For a four term cubic equation/expression with two correct coefficients (not including the cubic term) FT their answers to part **(b)** and **(c)**. Condone missing = 0. Can use any single unknown, including x
			$25u^3-16u^2+80u-100=0$	A1 [2]	1.1	Must include = 0. Can use any single unknown, including x but must be integer coefficients **SC B1** for $x=(\pm)\sqrt{u}$ substitution with correct equation with integer coefficients

Question			Answer	Marks	AO	Guidance
5	(a)		$(1+3\lambda+\mu)-(4+\lambda+\mu)+2(3-\lambda)$ $=1+3\lambda+\mu-4-\lambda-\mu+6-2\lambda=3$	B1	2.2a	**AG** substitutes into cartesian equation and obtains 3 or showing that 3 = 3 – must be at least one line of intermediate working from substitution to given answer – as a minimum allow correct expression without any brackets followed by 3. Alternative method would be to find at least three points from the vector equation and show that all three satisfy the cartesian equation
			Alternative method $\begin{pmatrix}3\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$ $\mathbf{r}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\4\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=1-4+6$ $x-y+2z=3$	B1		By direct calculation of cartesian equation Or $\begin{pmatrix}3\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$ so $x-y+2z=1-4+2(3)=3$
				[1]		
5	(b)		$\begin{vmatrix}1 & -1 & 2\\1 & 0 & -3\\a & -1 & -1\end{vmatrix}(=0)$	M1*	1.1	For determinant of a relevant matrix (e.g. rows interchanged but not columns) – no MR of values in this part but allow a slip in no more than 2 (of the 9) values
			$=1\begin{vmatrix}0 & -3\\-1 & -1\end{vmatrix}-(-1)\begin{vmatrix}1 & -3\\a & -1\end{vmatrix}+2\begin{vmatrix}1 & 0\\a & -1\end{vmatrix}$ $(=-3+(-1+3a)+2(-1)=3a-6)$	M1dep*	1.1	Finds determinant of their matrix as far as calculating 2×2 determinants. May expand by any row or column. Ignore sign errors in 2×2 determinant calculations if shown, but cofactors must have correct signs.
			Planes intersect at a **single point** if and only if det $\mathbf{M}\neq0$. Therefore $3a-6\neq0 \Rightarrow a\neq2$.	A1	2.2a	AG - correct determinant followed by 2 and **either** mention of that for a **single point** of intersection **determinant** $\neq$ **0 or** for **no single point** of intersection **determinant** $=0$ - allow substitution of 2 into correct determinant to obtain 0
				[3]		

Question			Answer	Marks	AO	Guidance
5	(c)		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & -3 \\ 3 & -1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$	B1	1.1	This mark is for a clear intention to consider $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & -3 \\ 3 & -1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ so $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & -3 \\ 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ only is **B0**. For reference the correct inverse is $\frac{1}{3}\begin{pmatrix} -3 & -3 & 3 \\ -8 & -7 & 5 \\ -1 & -2 & 1 \end{pmatrix}$. Allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ providing **M** defined (in this part or in part **(b)**)
			$\left(0, -\frac{11}{3}, -\frac{1}{3}\right)$	B1	1.1	BC - values must be exact but allow $x = 0, y = -\frac{11}{3}, z = -\frac{1}{3}$ and condone position vector – this mark is independent of the previous **B1** mark
				[2]		
	(d)		$[1]: x - y + 2z = 3$ $[2]: x - 3z = 1$ $[3]: 2x - y - z = 4$ So [1] + [2] = [3] **or** [3] – [2] = [1]	B1*	1.1	For showing consistency (must explicitly link all three equations together). Note that eliminating x gives $5z - y = 2$ or eliminating z gives $5x - 3y = 11$, but these must be derived twice for **B1**. No MR in this part (so must be using the correct equations). Stating the equation of the line where the three planes meet with no working scores zero marks in this part
			So, equations are consistent (and as none of the planes are coincident so) geometrically, the planes meet in a straight line/they form a sheaf.	B1dep*	3.2a	For "consistent/infinite solutions" **and** either "line" or "sheaf" If **B0 B0** then **SC B1** for implicitly showing consistency (e.g. [1] + [2] = $2x - y - z = 4$ but not explicitly linking this to [3]) together with 'consistent/ infinite solutions' **and** either 'line' or 'sheaf'
				[2]		

Question			Answer	Marks	AO	Guidance
6	(a)		**DR** $\dfrac{1}{(10r-5)(10r+5)} \equiv \dfrac{A}{10r-5}+\dfrac{B}{10r+5}$ $\Rightarrow A(10r+5)+B(10r-5)\equiv 1$	**M1**	3.1a	For correct identity oe e.g. $P(2r+1)+Q(2r-1)\equiv 1$ following $\dfrac{1}{(2r-1)(2r+1)} \equiv \dfrac{P}{2r-1}+\dfrac{Q}{2r+1}$ - condone if in terms of n – can be implied from correct partial fractions
			$\dfrac{1}{(10r-5)(10r+5)} \equiv \dfrac{1}{10}\left(\dfrac{1}{10r-5}-\dfrac{1}{10r+5}\right)$	**A1**	1.1	oe, allow for $\dfrac{1}{(10r-5)(10r+5)} \equiv \dfrac{A}{10r-5}+\dfrac{B}{10r+5}$ followed by $A=\dfrac{1}{10}, B=-\dfrac{1}{10}$ - condone if in terms of n
			$\sum_{r=1}^{n}\dfrac{1}{10r-5}\times\dfrac{1}{10r+5}=\left(\dfrac{1}{50}-\dfrac{1}{150}\right)+\left(\dfrac{1}{150}-\dfrac{1}{250}\right)+\ldots$ $+\left(\dfrac{1}{100n-150}-\dfrac{1}{100n-50}\right)+\left(\dfrac{1}{100n-50}-\dfrac{1}{100n+50}\right)$	**M1**	2.1	Uses method of differences for their two term partial fractions of the correct form (where their A and B must have different signs). Must show at least the cases for $r=1,2$ and n **or** $r=1,n-1$ and n (so either first two terms and last term **or** first term and last two terms) with subtraction between terms Condone missing or incorrect factor of $\dfrac{1}{50}$
			$=\dfrac{1}{50}\left(1-\dfrac{1}{2n+1}\right)$	**B1**	1.1	oe e.g. $\dfrac{1}{50}-\dfrac{1}{100n+50}$
			$<\dfrac{1}{50}$ (for all n) as $\dfrac{1}{2n+1}>0$ (for all n)	**A1**	2.2a	www - must explaining why the sum is less than $\dfrac{1}{50}$ - must consider the sign of $\pm\dfrac{1}{2n+1}$ correctly (oe) - dependent on all previous marks
				[5]		

Question			Answer	Marks	AO	Guidance
6	(b)		**DR** $S_\infty = \frac{1}{50}$	**B1***	2.2a	soi
			$\left(S_\infty = \frac{1}{2450} + S_k \Rightarrow\right)$			
			$\frac{1}{50} = \frac{1}{2450} + \frac{1}{50}\left(1 - \frac{1}{2k+1}\right)$	**M1dep***	3.1a	Using their S_k from part **(a)** which must be of the form $a - \frac{b}{2n+1}$ where a is non-zero and $b > 0$, in the given equation, to get an equation in k e.g. $\frac{24}{1225} = \frac{1}{50}\left(1 - \frac{1}{2k+1}\right)$
			$k = 24$	**A1** **[3]**	1.1	cao - **SCB1** for the correct answer with no working

Question			Answer	Marks	AO	Guidance
7	(a)		$[1]: 3+5\lambda = 8+2\mu$ $[2]: 1+4\lambda = 2-\mu$ $[3]: 1+\lambda = 4+\mu$	M1	3.4	Attempt to solve any pair of simultaneous equation for either λ or μ. So must obtain a value for either λ or μ. No MR in this part but allow either a single sign error in one equation only **or** one incorrect value in one equation only
			e.g. solving [1] and [2] gives: $\lambda = \frac{7}{13}, \mu = -\frac{15}{13}$	A1	1.1	For finding either λ **or** μ correctly for one pair of equations. For reference, solving [2] and [3] gives: $\lambda = \frac{4}{5}, \mu = -\frac{11}{5}$ and solving [1] and [3] gives: $\lambda = -\frac{1}{3}, \mu = -\frac{10}{3}$
			Substituting $\lambda = \frac{7}{13}, \mu = -\frac{15}{13}$ into [3] gives $1+\frac{7}{13} = \frac{20}{13}$ and $4-\frac{15}{13} = \frac{37}{13}$ so the lines do not intersect (as $\frac{20}{13} \neq \frac{37}{13}$)	A1 **[3]**	2.2a	Substitutes correct λ and μ into third equation and showing inconsistency + conclusion (e.g. 'lines do not intersect', 'no solutions', 'equations not consistent'). Values being compared must be evaluated e.g. $1+\frac{7}{13} \neq 4-\frac{15}{13}$ only is **A0**. For reference (if equations not re-arranged): $\lambda = \frac{4}{5}, \mu = -\frac{11}{5}$ into [1] gives 7 and $\frac{18}{5}$ $\lambda = -\frac{1}{3}, \mu = -\frac{10}{3}$ into [2] gives $-\frac{1}{3}$ and $\frac{16}{3}$ Alternative method is to find one value of λ or μ correctly and then solve a different pair of equations and obtain a different value of either λ or μ (values of λ, μ appear above for checking purposes)

Question			Answer	Marks	AO	Guidance
7	(b)		$\left(\begin{pmatrix}3\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\1\end{pmatrix}-\begin{pmatrix}4\\5\\0\end{pmatrix}\right)\bullet\begin{pmatrix}5\\4\\1\end{pmatrix}\ (=0)$	**M1***	3.3	Scalar product of direction component of T_1 with vector from (4, 5, 0) to general point on $T_{1.}$ No MR in this part but condone a single numerical slip in a value and sign errors only (but must be subtracting the general point from (4, 5, 0)). No evaluation of the scalar product needed for this mark
			Alternative for first M mark $D^2=(5\lambda-1)^2+(4\lambda-4)^2+(\lambda+1)^2$ $\Rightarrow 2D\times\frac{\mathrm{d}D}{\mathrm{d}\lambda}=10(5\lambda-1)+8(4\lambda-4)+2(\lambda+1)$	**M1***		Finds distance (or distance squared) from (4, 5, 0) to general point on T_1 **and** then differentiates with respect to λ - condone a single numerical slip in a value and sign errors only when setting up the expression for the distance (or distance squared) (but must be subtracting the general point from (4, 5, 0))
			$(5(5\lambda-1)+4(4\lambda-4)+(\lambda+1)=0\Rightarrow)$ $\lambda=\frac{10}{21}$	**A1**	1.1	Solves to correctly find λ at closest point to (4,5,0) on $T_{1.}$ Need not find coordinates of point
			$\left\|\begin{pmatrix}3\\1\\1\end{pmatrix}+\frac{10}{21}\begin{pmatrix}5\\4\\1\end{pmatrix}-\begin{pmatrix}4\\5\\0\end{pmatrix}\right\|=\sqrt{\left(\frac{29}{21}\right)^2+\left(-\frac{44}{21}\right)^2+\left(-\frac{31}{21}\right)^2}$	**M1dep***	3.4	Finding the magnitude from (4,5,0) to the closest point on T_1 using their value of λ following through their $\begin{pmatrix}3\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}5\\4\\1\end{pmatrix}-\begin{pmatrix}4\\5\\0\end{pmatrix}$
			= 291 (cm)	**A1** **[4]**	1.1	3 sf answer must be seen somewhere. Accept 291 with no units, but not 2.91 m or 2.91

Question			Answer	Marks	AO	Guidance
7	(b)		**Alternative method 1**			
			$\dfrac{\left\lvert\left(\begin{pmatrix}3\\1\\1\end{pmatrix}-\begin{pmatrix}4\\5\\0\end{pmatrix}\right)\times\begin{pmatrix}5\\4\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}5\\4\\1\end{pmatrix}\right\rvert}$	**M1***		For using the formula $D=\dfrac{\lvert\overrightarrow{AP}\times\mathbf{d}\rvert}{\lvert\mathbf{d}\rvert}$ where A is (4, 5, 0), P is any point on T_1 and **d** is the direction vector of T_1. No MR in this part but condone a single numerical slip in a value and sign errors only. No evaluation of the vector product needed for this mark
			$\begin{pmatrix}-1\\-4\\1\end{pmatrix}\times\begin{pmatrix}5\\4\\1\end{pmatrix}=\begin{pmatrix}-8\\6\\16\end{pmatrix}$	**A1**		For calculating correct vector product $\pm\begin{pmatrix}-8\\6\\16\end{pmatrix}$
			$=\dfrac{\sqrt{(-8)^2+6^2+16^2}}{\sqrt{5^2+4^2+1^2}}$	**M1dep***		For $=\dfrac{\sqrt{a^2+b^2+c^2}}{\sqrt{5^2+4^2+1^2}}$ where a, b and c are the components of their vector product
			= 291 (cm)	**A1**		3 sf answer must be seen somewhere. Accept 291 with no units, but not 2.91 m or 2.91

Question			Answer	Marks	AO	Guidance
7	(b)		**Alternative method 2**			
			$\left(\begin{pmatrix}4\\5\\0\end{pmatrix}-\begin{pmatrix}3\\1\\1\end{pmatrix}\right)\bullet\begin{pmatrix}5\\4\\1\end{pmatrix}=\left\|\begin{pmatrix}4\\5\\0\end{pmatrix}-\begin{pmatrix}3\\1\\1\end{pmatrix}\right\|\times\left\|\begin{pmatrix}5\\4\\1\end{pmatrix}\right\|\times\cos\theta$	**M1***		Using $\mathbf{a}\bullet\mathbf{b}=\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$ with the direction component of T_1 and a vector from (4, 5, 0) to any point on T_1. No MR in this part but condone a single numerical slip in a value and sign errors only. No evaluation is needed for this mark
			Using (3, 1, 1) gives $\cos\theta=\pm\dfrac{10\sqrt{21}}{63}$	**A1**		Correct expression for the cosine of the angle between T_1 and the line from (4, 5, 0) to any point on T_1 – for reference the most common point on T_1 is (3, 1, 1) but any point could have been used. If (3, 1, 1) then accept $\cos\theta=\pm0.73$ or better $(\pm0.72739...)$ - if the angle found then accept an angle of 43 or better (43.3317…) or 137 or better (136.668…)
			$d=\sqrt{(4-3)^2+(5-1)^2+(0-1)^2}\sin(43.3317...)$	**M1dep***		Correct expression for the required distance using their angle and the magnitude of the distance between (4, 5, 0) and their point on T_1
			= 291 (cm)	**A1**		3 sf answer must be seen somewhere. Accept 291 with no units, but not 2.91 m or 2.91
				[4]		
7	(c)		e.g. the cables are modelled as **lines** (so they have zero width) but they will have **width** (and so may intersect) e.g. the cables are modelled as **straight/lines** but (they are likely to not be straight, as) they might **bend** (under gravity, so may intersect)	**B1** **[1]**	3.5b	For identifying a modelling assumption that would affect the answer to part **(a)**. Also, other reasonable answers. **B0** for 'the lines are modelled as being infinite' only Allow modelling assumption about how the cable might impact on the answer to part **(b)** e.g. the cables are modelled as rigid, and the ladder may cause them to bend. However, an answer which does not refer to the cables scores **B0**

Question			Answer	Marks	AO	Guidance
8	(a)		**DR** $\lvert z\rvert = \sqrt{5^2+5^2} \quad \left(=5\sqrt{2}\right)$ **or** $\arg(z) = \pi - \tan^{-1}\left(\frac{5}{5}\right) \quad \left(=\frac{3}{4}\pi\right)$ **or** $5\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{3}{4}\pi}$	B1*	1.1	For completely correct working for either $\lvert z\rvert$ **or** $\arg(z)$ **or** for the correct answer of $5\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{3}{4}\pi}$ (accept $\sqrt{50}$ and any exact equivalent for $\frac{3}{4}\pi$) For $\lvert z\rvert$ must see either $\sqrt{25+25}$ or $\sqrt{(-5)^2+5^2}$ or $\sqrt{5^2+5^2}$ but not just $\sqrt{50}$ or $5\sqrt{2}$ For $\arg(z)$ must see either $\pi - \tan^{-1}\left(\frac{5}{5}\right)$ or $\frac{1}{2}\pi + \tan^{-1}\left(\frac{5}{5}\right)$ or $\pi + \tan^{-1}\left(-\frac{5}{5}\right)$ or equivalent with $\tan\theta = \frac{5}{5}$
			$5\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{3}{4}\pi}$ from complete working	B1dep* **[2]**	1.1	www must have shown complete working for both $\lvert z\rvert$ **and** $\arg(z)$ as **DR** – accept $\sqrt{50}$ and any exact equivalent for $\frac{3}{4}\pi$

Question			Answer	Marks	AO	Guidance
8	(b)		$\lvert w\rvert = \frac{36}{27}$ $\left(=\frac{4}{3}\right)$	B1	3.1a	**DR** Correct modulus of $\lvert w\rvert$ seen anywhere (check final answer)
			$\cos\left(\frac{1}{7}\pi\right) - \mathrm{i}\sin\left(\frac{1}{7}\pi\right) = \cos\left(-\frac{1}{7}\pi\right) + \mathrm{i}\sin\left(-\frac{1}{7}\pi\right)$ or $\mathrm{e}^{-\mathrm{i}\frac{1}{7}\pi}$	M1	2.1	Correctly re-writes $\cos\left(\frac{1}{7}\pi\right) - \mathrm{i}\sin\left(\frac{1}{7}\pi\right)$ in either modulus-argument or exponential form, or states that the argument of the numerator is $-\frac{1}{7}\pi$ or $\frac{13}{7}\pi$
			$\sin\left(\frac{3}{7}\pi\right) + \mathrm{i}\cos\left(\frac{3}{7}\pi\right) = \mathrm{i}\left(\cos\left(\frac{3}{7}\pi\right) - \mathrm{i}\sin\left(\frac{3}{7}\pi\right)\right)$ or $\mathrm{i}\mathrm{e}^{-\mathrm{i}\frac{3}{7}\pi}$ or $\mathrm{e}^{\mathrm{i}\frac{1}{2}\pi}\mathrm{e}^{-\mathrm{i}\frac{3}{7}\pi}$ or $\dfrac{\mathrm{i}}{-\cos\left(\frac{3}{7}\pi\right) + \mathrm{i}\sin\left(\frac{3}{7}\pi\right)}$	M1	3.1a	Correct first step to re-write $\sin\left(\frac{3}{7}\pi\right) + \mathrm{i}\cos\left(\frac{3}{7}\pi\right)$ in either modulus-argument (so in terms of c + is) or exponential form e.g. $\cos\left(\frac{1}{2}\pi - \frac{3}{7}\pi\right) + \mathrm{i}\sin\left(\frac{1}{2}\pi - \frac{3}{7}\pi\right)$ or $-\mathrm{i}\left(-\cos\left(\frac{3}{7}\pi\right) + \mathrm{i}\sin\left(\frac{3}{7}\pi\right)\right)$ If argument of $\sin\left(\frac{3}{7}\pi\right) + \mathrm{i}\cos\left(\frac{3}{7}\pi\right)$ stated as $\frac{1}{14}\pi$ without one step of correct intermediate working, then **M0**
			$\dfrac{\mathrm{e}^{-\mathrm{i}\frac{1}{7}\pi}}{\mathrm{e}^{\mathrm{i}\frac{1}{2}\pi} \times \mathrm{e}^{-\mathrm{i}\frac{3}{7}\pi}} = \dfrac{\mathrm{e}^{-\mathrm{i}\frac{1}{7}\pi}}{\mathrm{e}^{\mathrm{i}\frac{1}{14}\pi}} = \mathrm{e}^{-\mathrm{i}\frac{3}{14}\pi}$ or $\dfrac{\mathrm{e}^{\mathrm{i}\frac{13}{7}\pi}}{\mathrm{e}^{\mathrm{i}\frac{1}{2}\pi} \times \mathrm{e}^{-\mathrm{i}\frac{3}{7}\pi}} = \dfrac{\mathrm{e}^{\mathrm{i}\frac{13}{7}\pi}}{\mathrm{e}^{\mathrm{i}\frac{1}{14}\pi}} = \mathrm{e}^{\mathrm{i}\frac{25}{14}\pi}$	A1	1.1	Correct simplified $\arg w$ **which follows directly from their working** – allow as a minimum $\dfrac{\mathrm{e}^{-\mathrm{i}\frac{1}{7}\pi}}{\mathrm{e}^{\mathrm{i}\frac{1}{14}\pi}} = \mathrm{e}^{-\mathrm{i}\frac{3}{14}\pi}$ (oe) provided $\frac{1}{14}\pi$ in denominator derived convincingly (see previous **M** mark). Note that $\arg w = -\frac{3}{14}\pi$ stated with no working is **A0**
			$w = \frac{4}{3}\mathrm{e}^{\mathrm{i}\frac{25}{14}\pi}$	A1 **[5]**	2.2a	Dependent on all previous marks – allow $\frac{36}{27}\mathrm{e}^{\mathrm{i}\frac{25}{14}\pi}$ and any positive multiple of $\frac{25}{14}$. Common incorrect answers are $\frac{4}{3}\mathrm{e}^{-\mathrm{i}\frac{4}{7}\pi}$ or $\frac{4}{3}\mathrm{e}^{\mathrm{i}\frac{10}{7}\pi}$ - these score **B1 M1** only

Question			Answer	Marks	AO	Guidance
8	(b)		**Alternative method**			
			$\lvert w\rvert=\frac{36}{27}\quad\left(=\frac{4}{3}\right)$	B1		Correct modulus of $\lvert w\rvert$ seen anywhere (check final answer)
			$\frac{\cos\left(\frac{1}{7}\pi\right)-\mathrm{i}\sin\left(\frac{1}{7}\pi\right)}{\sin\left(\frac{3}{7}\pi\right)+\mathrm{i}\cos\left(\frac{3}{7}\pi\right)}\times\frac{\sin\left(\frac{3}{7}\pi\right)-\mathrm{i}\cos\left(\frac{3}{7}\pi\right)}{\sin\left(\frac{3}{7}\pi\right)-\mathrm{i}\cos\left(\frac{3}{7}\pi\right)}$	M1		Multiplying numerator and denominator by a correct suitable conjugate For reference: $\frac{\cos\left(\frac{1}{7}\pi\right)\sin\left(\frac{3}{7}\pi\right)-\sin\left(\frac{1}{7}\pi\right)\cos\left(\frac{3}{7}\pi\right)-\mathrm{i}\left(\sin\left(\frac{1}{7}\pi\right)\sin\left(\frac{3}{7}\pi\right)+\cos\left(\frac{1}{7}\pi\right)\cos\left(\frac{3}{7}\pi\right)\right)}{\sin^2\left(\frac{3}{7}\pi\right)+\cos^2\left(\frac{3}{7}\pi\right)}$
			$=\sin\left(\frac{3}{7}\pi-\frac{1}{7}\pi\right)-\mathrm{i}\cos\left(\frac{3}{7}\pi-\frac{1}{7}\pi\right)$	M1		For correctly writing the numerator as a single term in sine and a single term in cosine – the angle must be seen as two separate terms e.g. $-\sin\left(\frac{1}{7}\pi-\frac{3}{7}\pi\right)-\mathrm{i}\cos\left(\frac{1}{7}\pi-\frac{3}{7}\pi\right)$
			$=\cos\left(\frac{1}{2}\pi-\frac{2}{7}\pi\right)-\mathrm{i}\sin\left(\frac{1}{2}\pi-\frac{2}{7}\pi\right)$ $=\cos\left(-\frac{3}{14}\pi\right)+\mathrm{i}\sin\left(-\frac{3}{14}\pi\right)$	A1		Correct simplified $\arg w$ with at least two terms shown e.g. $\frac{1}{2}\pi-\frac{2}{7}\pi$. Note that $\arg w=-\frac{3}{14}\pi$ stated or implied with no working is **A0**
			$w=\frac{4}{3}\mathrm{e}^{\mathrm{i}\frac{25}{14}\pi}$	A1		Dependent on all previous marks – allow $\frac{36}{27}\mathrm{e}^{\mathrm{i}\frac{25}{14}\pi}$ and any positive multiple of $\frac{25}{14}$. A common incorrect answer is $\frac{4}{3}\mathrm{e}^{-\mathrm{i}\frac{4}{7}\pi}$ or $\frac{4}{3}\mathrm{e}^{\mathrm{i}\frac{10}{7}\pi}$ - these scores **B1 M1** only
				[5]		

Question			Answer	Marks	AO	Guidance
9	**(a)**		$\sin\theta \approx \theta$ is only valid for small angles	**B1** **[1]**	3.5b	States $\boldsymbol{\sin\theta \approx \theta}$ (or in words), and this is only valid for **small** angles (oe e.g. 'close to zero')

Question			Answer	Marks	AO	Guidance
	(b)		$\theta = A\cos\left(\frac{\sqrt{35}}{5}t\right) + B\sin\left(\frac{\sqrt{35}}{5}t\right)$ or $\theta = R\cos\left(\frac{\sqrt{35}}{5}t + \varepsilon\right)$	B1	3.4	Correct general solution. Allow exact equivalents e.g. $\theta = A\cos\left(\sqrt{1.4}t\right) + B\sin\left(\sqrt{1.4}t\right)$, $\theta = A\cos\left(\sqrt{1.4}t\right)$. Allow 1.18 or better (1.183215…) for $\sqrt{1.4}$
			When $t = 0, \theta = \frac{1}{15}\pi \Rightarrow A = \frac{1}{15}\pi$ and $\dot{\theta} = -\frac{\sqrt{35}}{5}A\sin\left(\frac{\sqrt{35}}{5}t\right) + \frac{\sqrt{35}}{5}B\cos\left(\frac{\sqrt{35}}{5}t\right)$ and when $t = 0, \dot{\theta} = 0 \Rightarrow B = 0$	M1*	3.4	Use correct initial conditions $\theta = \frac{1}{15}\pi, \dot{\theta} = 0$ when $t = 0$ to find constant(s) from a GS of the correct form. Allow θ_0 for $\frac{1}{15}\pi$, and allow stating $B = 0$ without corresponding working. If differentiation seen, then must see $\sin \to \pm\cos$ and $\cos \to \pm\sin$ together with a change in coefficients.
			$\theta = \frac{1}{15}\pi\cos\left(\frac{\sqrt{35}}{5}t\right)$	A1	1.1	cao www – allow $\theta = \theta_0\cos\left(\frac{\sqrt{35}}{5}t\right)$ provided $\theta_0 = \frac{1}{15}\pi$ stated explicitly in their working. The correct particular solution (with no incorrect working seen) scores the first three marks. Allow $\theta = 0.209\cos(1.18t)$ or better.
			$T = \frac{2\pi}{\sqrt{1.4}} = 5.310...$ $\Rightarrow \theta = \frac{1}{15}\pi\cos\left(\sqrt{1.4}\times(6 - 5.310...)\right)$ **or** for $\theta = \frac{1}{15}\pi\cos\left(\frac{\sqrt{35}}{5}\times 6\right)$	M1*	3.1b	Correct method to find θ after one complete oscillation **or** for θ when $t = 6$. Follow through their PS which must be of the form $\theta = \frac{1}{15}\pi\cos(pt)$ where $p > 0$ and $p \neq 1$. For reference if correct then $\theta = 0.1434...$
			Distance = $7\left(\frac{4}{15}\pi + \frac{1}{15}\pi - \theta\right)$	M1dep*	3.4	For their numerical $7\left(\frac{4}{15}\pi + \frac{1}{15}\pi - \theta\right)$ **or** $7\left(\frac{4}{15}\pi + \theta\right)$ only
			6.33 (m)	A1 [6]	2.1	awrt 6.33 (6.32603…)

Question			Answer	Marks	AO	Guidance
9	(c)		$\left(\frac{k}{m}\right)^2 - 4\left(\frac{7}{5}\right)(>0)$	M1	3.1b	For consideration of correct discriminant (allow any inequality or equals)
			$0 < m < \frac{\sqrt{35}}{14}k$	A1	1.1	cao – oe but must be exact
				[2]		

Question			Answer	Marks	AO	Guidance
10	(a)		**DR** $\cos 5\theta + \mathrm{i}\sin 5\theta = (c + \mathrm{i}s)^5$ $= c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$ $\tan 5\theta \left(= \dfrac{\mathrm{Im}(c+\mathrm{i}s)^5}{\mathrm{Re}(c+\mathrm{i}s)^5} \right) = \dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ $\tan 5\theta = \dfrac{\dfrac{5c^4s}{c^5} - \dfrac{10c^2s^3}{c^5} + \dfrac{s^5}{c^5}}{\dfrac{c^5}{c^5} - \dfrac{10c^3s^2}{c^5} + \dfrac{5cs^4}{c^5}}$ $= \dfrac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}$	**M1*** **M1dep*** **M1** **A1** **[4]**	1.1 1.1 1.1 2.1	De Moivre's theorem with $n = 5$ Assume $c = \cos\theta$ and $s = \sin\theta$ (so condone not explicitly stated) Expanding $(c + \mathrm{i}s)^5$ to obtain six terms - coefficients must be numerical and correct (so binomial coefficients must be evaluated) and correctly in terms of i only (so not powers of i). Allow at most one sign error and at most one term with an incorrect index Correctly taking real and imaginary parts of their expanded $(c + \mathrm{i}s)^5$ and dividing correctly (numerator and denominator must contain the correct number of terms and no i's). **Dependent on both previous M marks** **AG -** www Must see **mathematically** each term in both the numerator and the denominator being divided by c^5 oe (e.g. multiplying by c^{-5}) so $\dfrac{c^{-5}\left(5c^4s - 10c^2s^3 + s^5\right)}{c^{-5}\left(c^5 - 10c^3s^2 + 5cs^4\right)}$ or $\dfrac{\dfrac{5c^4s - 10c^2s^3 + s^5}{c^5}}{\dfrac{c^5 - 10c^3s^2 + 5cs^4}{c^5}}$ is fine but $\dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4} \div c^5$ or $\dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4} \div \dfrac{c^5}{c^5}$ etc. or in words (e.g. 'divide both by c^5') is **A0** **Note that** $\dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4} = \dfrac{\dfrac{5s}{c} - \dfrac{10s^3}{c^3} + \dfrac{s^5}{c^5}}{1 - \dfrac{10s^2}{c^2} + \dfrac{5s^4}{c^4}}$ is **A0** (as **AG**) Final answer must be in terms of tan not t

Question			Answer	Marks	AO	Guidance
10	(b)	(i)	**DR** $5t^4-10t^2+1=t^5-10t^3+5t$ $\Rightarrow t^5-5t^4-10t^3+10t^2+5t-1\,(=0)$	**M1***	2.1	Equates result from part **(a)** to 1, multiplies through by denominator and then rearranges to get all terms on one side. Condone using tan rather than t - Allow sign errors only.
			$\theta=\frac{1}{20}\pi,\frac{5}{20}\pi,\frac{9}{20}\pi,\frac{13}{20}\pi,\frac{17}{20}\pi$	**B1**	1.1	Finds all five solutions to $\tan 5\theta=1$ in the interval $0<\theta<\pi$ - ignore any solutions outside of this interval but if any incorrect in this interval, then **B0**
			$\tan\left(\frac{5}{20}\pi\right)=1$ so $(t-1)$ is a factor of $t^5-5t^4-10t^3+10t^2+5t-1$ $\Rightarrow(t-1)(t^4-4t^3-14t^2-4t+1)$	**B1dep***	3.1a	Correct justification that $(t-1)$ is a factor of the correct quintic in t using the result that $\tan\left(\frac{5}{20}\pi\right)=1$ **and** re-writing quintic as $(t-1)(t^4-4t^3-14t^2-4t+1)$ oe (e.g. by long division) **or** re-writes quintic as $(t-1)(t^4-4t^3-14t^2-4t+1)$ oe (e.g. by long division) and relates the factor $(t-1)$ to $\tan\left(\frac{5}{20}\pi\right)$ (in essence this mark is for justifying that the root $\tan\left(\frac{5}{20}\pi\right)$ of the correct quintic equation is not a root of the given quartic equation therefore this mark is dependent on having derived the correct expression $t^5-5t^4-10t^3+10t^2+5t-1\,(=0)$)
			$\tan\left(\frac{1}{20}\pi\right),\tan\left(\frac{9}{20}\pi\right),\tan\left(\frac{13}{20}\pi\right),\tan\left(\frac{17}{20}\pi\right)$	**A1** **[4]**	2.2a	These exact four roots only (so must not include $\tan\left(\frac{5}{20}\pi\right)$) – this mark is not dependent on the previous **B1** mark – a correct quintic equation/expression followed by these four roots only scores **M1 A1** only. These four roots only without the correct quintic equation/expression seen is no marks

Question			Answer	Marks	AO	Guidance
	(b)	(ii)	**DR** $(t-1)^4 = t^4 - 4t^3 + 6t^2 - 4t + 1$ so if $t^4 - 4t^3 - 14t^2 - 4t + 1 = 0 \Rightarrow (t-1)^4 = 20t^2$	**B1**	2.2a	www for correctly finding the value of k – allow for either $(t-1)^4 = 20t^2$ or $k = 20$ stated with no working
			$(t-1)^2 = \pm\sqrt{20}t$ $\Rightarrow t^2 - \left(2 \pm \sqrt{20}\right)t + 1 \; (= 0)$	**M1**	3.1a	For correct three term quadratic expressions(s)/equation(s) (in t), follow through their positive value of k. Condone missing $\pm$ - so for either $t^2 - \left(2 \pm \sqrt{k}\right)t + 1$ or $t^2 - \left(2 + \sqrt{k}\right)t + 1$ or $t^2 - \left(2 - \sqrt{k}\right)t + 1$ with their k
			$\left(t - \left(1 \pm \sqrt{5}\right)\right)^2 - \left(1 \pm \sqrt{5}\right)^2 + 1 = 0$ $\Rightarrow t = 1 + \sqrt{5} \pm \sqrt{5 + 2\sqrt{5}}$ or $t = 1 - \sqrt{5} \pm \sqrt{5 + 2\sqrt{5}}$ $\tan\left(\frac{9}{20}\pi\right)$ is positive and the largest root (of the quartic equation) so $\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}$	**A1** **[3]**	2.3	**AG** Obtains all four roots www and explains that $\tan\left(\frac{9}{20}\pi\right)$ is positive and the largest root and so $\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}$

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