GCE

Further Mathematics A

Y543/01: Mechanics

A Level

Mark Scheme for June 2025

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5. **Crossed-Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
   - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
    To determine the level – start at the highest level and work down until you reach the level that matches the answer
    To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
   If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		(12×5 = 60 so impulse is) 60...	**B1**	1.1		May also find this by integrating F wrt t
			...Ns or $kgms^{-1}$	**B1**	1.2	For correct units for impulse. **B0B1** is possible.	
				[2]			
	(b)		"60" = 4v – 4×7	**M1**	1.1	Use of impulse–momentum principle with their impulse, m = 4 and u = 7.	Condone global sign error eg "–60 =" or $mu - mv$ but must be difference between momentums (unless later recovered).
			v = 22 (ms^{-1})	**A1**	1.1	www	**SCB1** for finding a = 12/4 = 3 and using v = 7 + 3×5 = 22, or answer with no working.
				[2]			
	(c)		½×4×7^2 or ½×4×"22"^{2}	**M1**	1.1	Finding, in any numerical form, the initial or their final KE.	98 or 968
			Work done = 870 (J)	**A1**	1.1	Must be some evidence of use of change of KE. **SCB1** for 0.5m(v^2-u^2)=870	d = 7×5 + ½×3×5^2 = 72.5 and W = 12 × 72.5 = 870 is **M0A0**.
				[2]			
	(d)		"870" = 12d	**M1**	1.1	Using "$W = Fd$" with their 'W' from **(c)** and F = 12.	
			d= 72.5 (m)	**A1FT**	1.1	FT their 'WD/12' from **(c)**. Oe e.g. 145/2 **www**	**SCB1FT** use of d = 7×5 + ½×3×5^2 = 72.5 or $22^2 = 7^2 + 2 \times 3 \times d$ to get their d or $d = \frac{(7+22)\times 5}{2}$ seen
				[2]			

Question	Answer	Marks	AO	Guidance

2	**(a)**		$[F] = MLT^{-2}$	**B1**	1.1	Dimensions of force may be embedded	*Do not allow for use of units.*
			So $[\eta]$ = "(MLT^{-2})" (T) (L^{-2})	**M1**	1.1	Combining dimensions correctly using their [F] (may be using base SI units)	Need to see product
			$= ML^{-1}T^{-1}$	**A1**	1.1	Any order but must be product. Do not allow [M] or lower case etc for final mark.	**SCB1 for correct final answer seen if M0 awarded.**
				[3]			
	(b)		$[\rho] = ML^{-3}$	**B1**	1.1	Stating or deriving the dimensions of density soi by $[\nu]$	*May see e.g.* $M^{-1}L^3$ Condone e.g. [M], lower case
			So $[\nu]$ $(= ML^{-1}T^{-1}\ (ML^{-3})^{-1}) = (M^0)L^2\ T^{-1}$	**B1**	1.1	Do not allow [M] or lower case	
			m^2s^{-1} or m^2 / s oe	**B1**	1.1	Any length unit squared (or area unit) per any time unit.	Other possibilities include Nms kg^{-1} or Js kg^{-1}.
				[3]			

Question			Answer	Marks	AO	Guidance	
3	**(a)**		$\mathbf{v}.\mathbf{v} = 4^2 + (-6)^2$ or $\mathbf{v}.\mathbf{v} = 52$	**M1**	1.1	**DR**. Evidence of use of method for calculating scalar product **v.v**.	Or finding the magnitude of the velocity vector using Pythagoras' theorem $\lvert\boldsymbol{v}\rvert = 2\sqrt{13}$
			KE = ½×3.25×52 = 84.5 (J)	**A1**	3.4	If **M0** then **SCB1** for 84.5	*Will condone missing brackets*
				[2]			
	(b)		$P = (\mathbf{i} - 3\mathbf{j}).(4\mathbf{i} - 6\mathbf{j})$	**M1**	3.4	**DR** Use of $P = \mathbf{F}.\mathbf{v}$ with $t = 0$.	*Allow for generalised dot product at point when t=0 substituted in. e.g. P=4+18+6t, when t=0, …*
			$(=1\times4 + (-3)\times(-6)) = 22$ (W)	**A1**	1.1	If **M0** then **SCB1** for 22	*P= 22*
				[2]			
	(c)		$\pm(\mathbf{i} - (3 + 6)\mathbf{j}).(4\mathbf{i} - 6\mathbf{j})$	**M1**	3.4	**DR** Finding $\pm\mathbf{F}$ when $t = 6$ and dotting with **v**.	*Must see use of t=6 for this mark. May use their generalised expression for Power from part b*

			(= ±(1×4 + (–9)×(–6)))= –58 (W)	**A1**	1.1	Must be negative. Mark final answer.	If they calculate with +F, and then change sign, allow minimal justification.
				[2]			

Question			Answer	Marks	AO	Guidance	
4	**(a)**		$2\times1.5 + 0.5\times-3.5 = 2\times-0.3 + 0.5v_Q$	**M1**	3.4	Conservation of momentum. Condone a single sign error.	$e = \frac{v_Q - (-0.3)}{1.5 - (-3.5)}$
			$v_Q = 3.7$ so(speed of Q after collision is 3.7 ms^{-1})	**A1**	1.1		$v_Q = 5e - 0.3$
			$e = \frac{"3.7" - (-0.3)}{1.5 - (-3.5)}$	**M1**	3.4	(Their speed of separation) / (speed of approach)	$2\times1.5 + 0.5\times-3.5 = 2\times-0.3 + 0.5v_Q$
			$= \frac{4}{5}$ or 0.8	**A1**	1.1	oe.	$e = \frac{4}{5}$ or 0.8
				[4]			
	(b)		V_Q = (–)0.1×"0.8"×"3.7" (= –0.296)	**M1**	3.4	Multiplying v_Q by $\pm\frac{1}{10}e$ to find either velocity or speed of Q after hitting wall.	
			0.296 < 0.3 or –0.3 < –0.296 so no further collision.	**A1**	2.2a	Must see comparison and correct conclusion. Comparison must be either $\lvert V_Q\rvert < \lvert v_P\rvert$ or $v_P < V_Q$ oe inequalities but not mixed. eg correct conclusion from 0.296 > –0.3 is **A0**.	Look for same signs. Explicit comparison of numbers seen.
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(a)		↕ $T\cos\theta + C = mg$	**M1***	1.1	Resolving tension and balancing forces in the vertical. Must be contact force, component of tension and weight and no others. NB. $\cos\theta = 4/5$, $\sin\theta = 3/5$	Condone consistent sin/cos confusion and/or sign errors. No need to see value for cos or $C = T$ here. Condone if see e.g. $T\cos\theta + T = mg$
			↔ $T\sin\theta = \frac{mv^2}{3}$	**M1**	1.1	Resolving tension and using NII with correct form of centripetal acceleration. Correct r could be in different form, eg $5\sin\theta$ or implied by later working	Condone consistent sin/cos confusion and/or sign errors. No need to see value for sin or $C = T$ here. Condone use of $m \times 3 \times \omega^2$ if $v = 3\omega$ also seen.
			$C = T$, $\cos\theta = 4/5$ => $T = 5mg/9$ or $49m/9$	**M1dep***	2.1	Using $T=C$ and value of cos to derive T in terms of m (and possibly g). If C found then award mark when $T = C$ soi.	Condone $\cos\theta = 3/5$. May see e.g. T=mg/1.8 May eliminate T for this mark e.g. $(cos\theta + 1)\left(\frac{mv^2}{3sin\theta}\right) = mg$ so $\left(\frac{4}{5} + 1\right)\left(\frac{mv^2}{3(\frac{3}{5})}\right) = mg$
			$((5mg/9) \times (3/5) = mv^2/3$ => $v^2 = g)$ => v = awrt 3.13	**A1**	1.1	$\frac{7\sqrt{5}}{5}$ oe.	Do not allow root (g).
				[4]			

	(b)		$\updownarrow T\cos\theta = mg$	**M1**	2.1	Resolving tension and balancing forces in the vertical. Must be component of tension and weight and no others but could see contact force C and then $C = 0$ later.	Condone consistent sin/cos confusion and/or sign errors. No need to see value for cos here.
			$\leftrightarrow T\sin\theta = 3m\omega^2$	**M1**	1.1	Resolving tension and using NII with correct form of centripetal acceleration. If $\frac{v^2}{r}$ used then mark awarded when $v = r\omega$ also used . r could be in different form, eg $5\sin\theta$ or implied by later working	Condone consistent sin/cos confusion and/or sign errors. No need to see value for sin here. .
			$(mg\tan\theta = 3m\omega^2 \Rightarrow \omega^2 = g/4)$ $\Rightarrow \omega =$ awrt 1.57 (rad s^{-1})	**A1**	1.1	$\frac{7\sqrt{5}}{10}$ oe. (not 0.5 root(g)) **www**	Could see eg $v = 4.6957...$ and then $\omega = v/3$ leading to answer. But if ω or ω^2 never found then max **M1M0A0**.
				[3]			

Question			Answer	Marks	AO	Guidance	
6	**(a)**		Gain in PE = $mgr(1 - \cos\theta)$ J	**B1**	1.1	**DR** or $-mgr$ and $-mgr\cos\theta$ seen on opposite sides of energy equation.	Could be the PE at θ if zero PE level set at lowest point. May award for mgh if h= $r(1-\cos\theta)$ clearly substituted in later on.
			$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$	**M1**	1.1	Use of energy conservation with all terms seen (or gain in PE = loss in KE).	Must see +ve KE terms on each side (or difference on one side) but condone sign error for PE term(s).
			$\therefore v^2 = u^2 - 2gr(1-\cos\theta)$	**A1**	1.1	Could have m in each term	

			$T - mg\cos\theta = \frac{mv^2}{r}$	**M1***	3.1b	Resolving the weight and using NII in the radial direction with a correct expression for the centripetal acceleration.	Condone $T + mg\cos\theta = \frac{mv^2}{r}$ Do not condone sin/cos confusion.
			$\therefore T = \frac{m(u^2 - 2gr(1-\cos\theta))}{r} + mg\cos\theta$	**M1dep***	2.1	Eliminating v^2 using their expressions **(Allow if just quoted expression for v^2)**	$\therefore u^2 = v^2 + 2gr(1-\cos\theta)$ $\therefore v^2 = u^2 - 2gr(1-\cos\theta)$
			$\therefore T = \frac{m(u^2 - 2gr(1-\cos\theta)) + rmg\cos\theta}{r}$ $= \frac{m}{r}\left(u^2 - 2gr + 2gr\cos\theta + gr\cos\theta\right)$ $= \frac{m}{r}\left(u^2 - 2gr + 3gr\cos\theta\right)$ (so $\alpha = -2$ and $\beta = 3$)	**A1**	1.1	**www ISW if correct expression found with factor m/r then alpha and beta extracted incorrectly**	
				[6]			
	(b)		$(T > 0 \Rightarrow) u^2 - 2gr + 3gr\cos\theta > 0$ for all θ	**B1FT**	3.1b	Use of their $T > 0$, may be in terms of alpa and beta. Allow > or ≥ or =, throughout, cos theta may be replaced with a numerical value.	**ALT method** For candidates who restart Use of T=0 in N2L at top of motion $(T+)mg = \frac{mv^2}{r}$ leading to $v^2_{min} = gr$ oe for v or KE
			Worst case scenario is $\theta = \pi$ or $\cos\theta = -1 \Rightarrow u^2 - 2gr - 3gr > 0$ $\Rightarrow u^2 > 5gr$ so $u_{min} = \sqrt{(5gr)}$	**M1**	3.1b	Using correct boundary value of cos to find minimum value of u^2 (or u). Do not allow from KE or $v^2 > 0$ unless clear comes from their T For students who measure theta from the upward vertical cos theta = 1 or theta = 0 seen used. If $\beta < 0$, then $\theta = 0, \cos(\theta) = 1$ used	From energy considerations $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2r)$ $\frac{1}{2}mu^2 = \frac{1}{2}m\sqrt{gr}^2 + mg(2r)$ $\Rightarrow u^2 > 5gr$ so $u_{min} = \sqrt{(5gr)}$
			So minimum impulse required is $m\sqrt{5gr}$	**A1**	2.2a		

				[3]			

Question			Answer	Marks	AO	Guidance	
7	**(a)**		$(\text{F}=)\, k\left(9-2.4^2\right)^{\frac{1}{2}} = 32.4$	**M1**	3.3	**DR** Substituting in $v = 2.4$ and $F = 32.4$ soi by k=18 seen.	
			$k = 18$	**A1**	2.1		
			$18\left(9-v^2\right)^{\frac{1}{2}} = 36\dfrac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \dfrac{1}{\sqrt{9-v^2}}\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1}{2}$	**A1**	3.3	Clear use of $F = ma$ with $a = \mathrm{d}v/\mathrm{d}t$ leading to AG	Clear substitution of m and dv/dt seen
				[3]			
	(b)		$\displaystyle\int \frac{1}{\sqrt{9-v^2}}\mathrm{d}v = \frac{1}{2}t + c$	**M1**	3.4	Separating the variables and correctly integrating to obtain bt .	c may be omitted here.
			$\sin^{-1}\dfrac{v}{3} = \dfrac{1}{2}t + c$	**M1**	3.4	Integrating other side to obtain $p\sin^{-1}qv$.	c may be omitted here.
			$c = 0$	**B1**	1.1	Explicitly substituting $t = 0$, $v = 0$ into a soln of the form $p\sin^{-1}qv$ which has a constant of integration to obtain $c = 0$.	NB failure to explicitly find $c = 0$ only loses this mark. At a minimum. $\sin^{-1}0 = 0 + c \Rightarrow c = 0$ or ksin(0-c)=0
			$v = 3\sin\left(\dfrac{1}{2}t\right)$	**A1**	2.1	**AG www**	
			$v = 3 \Rightarrow \sin\left(\dfrac{1}{2}t\right) = 1 \Rightarrow t = \pi$, must identify as first positive solution so DE applies in $0 \le t \le \pi$.	**A1**	2.4	**AG** May be seen on a sketch oe reference to one-one function in that domain	Allow t= pi => v=3 , as long as t=pi is identified as the first solution to give v=3.
				[5]			
	(c)		$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 3\sin\left(\dfrac{1}{2}t\right) \Rightarrow x = C - 6\cos\left(\dfrac{1}{2}t\right)$	**M1**	3.4	Integrating wrt t to obtain $-p\cos qt$	C may be omitted (or seen as c) $\left[-6\cos\left(\dfrac{1}{2}t\right)\right]_{\ldots}^{\ldots}$
			$t = 0, x = 0 \Rightarrow C = 6$ so $x = 6\left(1-\cos\left(\dfrac{1}{2}t\right)\right)$	**A1**	2.2a		Note candidates may alternatively use definite integration between 0 and pi and not find C = 6

			$\Rightarrow t = \text{pi}, x = 6$				$\left[-6\cos\left(\frac{1}{2}t\right)\right]_0^{\pi}$ $= -6\cos(\pi) - -(6\cos(0)) = 6$
			$10 - \pi$ seconds at 3 ms^{-1} is $3(10 - \pi)$ m	**B1**	3.1b	Finding distance for constant velocity portion of journey.	
			So total dist travelled is $36 - 3\pi$ m	**B1**	3.4	oe $3(12 - \pi)$	
				[4]			

Question			Answer	Marks	AO	Guidance	
8	**(a)**	**(i)**	$A = \int_0^{\ln 9} e^{0.5x}dx = \frac{1}{0.5}\left[e^{0.5x}\right]_0^{\ln 9}$ $= 2\left(e^{0.5\ln 9} - e^0\right) = 2\left(e^{\ln 3} - 1\right) = 4$	**B1**	3.3	**DR** so some intermediate working must be shown. Must be from correct integral with correct limits.	In this series condone 6-2(= 4 soi) seen as minimal working.
			$\int_{\ldots}^{\ldots} xe^{0.5x}dx = \left[2xe^{0.5x}\right]_{\ldots}^{\ldots} - \int_{\ldots}^{\ldots} 2e^{0.5x}dx$ $= \left[2xe^{0.5x} - 4e^{0.5x}\right]_{\ldots}^{\ldots}$	**M1**	1.1	**DR**. Correctly integrating $xf(x)$ using integration by parts. Ignore limits here. Could have $+ c$.	
			$\therefore \int_0^{\ln 9} xe^{0.5x}dx = \left[2xe^{0.5x} - 4e^{0.5x}\right]_0^{\ln 9}$ $= \left(2\ln 9e^{0.5\ln 9} - 4e^{0.5\ln 9}\right) - \left(0 - 4e^0\right)$ $= 6\ln 9 - 12 + 4 = 6 \times 2\ln 3 - 8 = 12\ln 3 - 8$	**A1**	1.1	oe e.g. 6ln9-8	Must see substitution of limits with 4 terms seen. Must see $e^{0.5\ln(9)}$ with x clearly substituted in both times. Condone e^0 written as 1. Condone use of previous $\frac{1}{0.5}\left[e^{0.5x}\right]_0^{\ln 9}$ =4 if full substitution already shown.

			$\bar{x} = \dfrac{12\ln 3 - 8}{4} = 3\ln 3 - 2$	**B1**	3.3	AG so calculation must be shown but can be awarded after **M0A0** for clear indication of use of $\bar{x} = \dfrac{\int x\,\mathrm{f}(x)\mathrm{d}x}{\int \mathrm{f}(x)\mathrm{d}x}$ in this context	Condone $4\bar{x} = \cdots$ so $\bar{x} = \cdots$
				[4]			
	(a)	**(ii)**	$\bar{y} = \dfrac{\frac{1}{2}\int_0^{\ln 9} \left(\mathrm{e}^{0.5x}\right)^2 \mathrm{d}x}{4}$ oe	**M1**	3.3	**DR.** Clear demonstration of use of $\bar{y} = \dfrac{\frac{1}{2}\int \left(\mathrm{f}(x)\right)^2 \mathrm{d}x}{\int \mathrm{f}(x)\mathrm{d}x}$ in this context.	i.e. include the ½ $\bar{y} = \dfrac{\frac{1}{2}\int_0^{\ln 9} \left(\mathrm{e}^{0.5x}\right)^2 \mathrm{d}x}{\int_0^{\ln 9} \left(\mathrm{e}^{0.5x}\right) dx}$ and attempt integration of numerator
			$\therefore \bar{y} = \dfrac{\frac{1}{2}\left[\mathrm{e}^x\right]_0^{\ln 9}}{4} = \dfrac{9-1}{8} = 1$	**A1**	1.1	If **M0** then **SCB1** for correct answer www.	Minimally two terms seen on substitution of limits.
				[2]			

	(b)		$\bar{X} = \dfrac{\ln 9 \times \frac{\ln 9}{2} + \ln 9 \times \frac{2}{3}\ln 9}{\ln 9 + \ln 9}$	**M1**	3.3	Forming $\dfrac{\sum m_i x_i}{\sum m_i}$. Condone one incorrect distance used. Allow $(2\ln 9)\bar{X} = \cdots$ Ratio of areas is 1:1 so 'ln9's might be omitted $\bar{X} = \dfrac{1 \times \frac{\ln 9}{2} + 1 \times \frac{2\ln 9}{3}}{2}$ Could also be by subtraction $\bar{X} = \dfrac{3\ln 9 \times \frac{\ln 9}{2} - \ln 9 \times \frac{\ln 9}{3}}{3\ln 9 - \ln 9}$ May see $\bar{X} = \dfrac{\int_0^{\ln 9} x\left(\frac{2x}{\ln 9} + 1\right)\mathrm{d}x}{\int_0^{\ln 9}\left(\frac{2x}{\ln 9} + 1\right)\mathrm{d}x}$ *Table methods – m,x need to be combined soi by answer* COM triangle is e.g. $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}0 + 2.2 + 2.2\\ 1 + 1 + 3\end{pmatrix} = \begin{pmatrix}22/15\\ 5/3\end{pmatrix}$ or exact equivalent using ln 9. Award method mark when used in complete moments equation.	**ALT METHOD** – use if $\bar{Y}$ considered most completely $\bar{Y} = \dfrac{\ln 9 \times \frac{1}{2} + \ln 9 \times \left(1 + \frac{2}{3}\right)}{\ln 9 + \ln 9}$ Allow $(2\ln 9)\bar{Y} = \cdots$ or $\bar{Y} = \dfrac{1 \times \frac{1}{2} + 1 \times \left(1 + \frac{2}{3}\right)}{2}$ Or by subtraction $\bar{Y} = \dfrac{3\ln 9 \times \frac{3}{2} - \ln 9 \times \left(3 - \frac{2}{3}\right)}{3\ln 9 - \ln 9}$ Could be from $\dfrac{\frac{1}{2}\int_0^{\ln 9}\left(\frac{2x}{\ln 9} + 1\right)^2 \mathrm{d}x}{\int_0^{\ln 9}\left(\frac{2x}{\ln 9} + 1\right)\mathrm{d}x}$ Must see correct limits (ln9 or 2.2)

			$= \dfrac{\left(\frac{3}{6}+\frac{4}{6}\right)\ln 9}{2} = \dfrac{7\ln 9}{12} or \dfrac{7\ln 3}{6}$	**A1**	1.1	Accept awrt $\bar{X}$ =77/60 = 1.28…	$\bar{Y} = \dfrac{\ln 9 \times \left(\frac{1}{2}+1+\frac{2}{3}\right)}{\ln 9 + \ln 9} = \dfrac{13}{12}$
			$\bar{Y} = \dfrac{\ln 9 \times \frac{1}{2} + \ln 9 \times \left(1+\frac{2}{3}\right)}{\ln 9 + \ln 9} = \dfrac{13}{12}$	**B1**	3.3	Accept $\bar{Y}$ =1.08…	$\bar{X} = \dfrac{\ln 9 \times \frac{\ln 9}{2} + \ln 9 \times \frac{2}{3}\ln 9}{\ln 9 + \ln 9} = \dfrac{7\ln 3}{6}$
				[3]			
	(c)	**(i)**	Limiting case is when *G* is directly above *O*.	**M1**	3.1b	May be implied by working. May be implied by using 50° (or 40 degrees) and setting up a contact force at a point a distance above *O* and then using moments to show that this distance is positive.	Ie. About to topple when G is above O – with use of this seen in a calculation
			Worst case scenario is $\theta = \tan^{-1}\left(\dfrac{3\ln 3 - 2 - \varepsilon}{"1" + \varepsilon}\right)$ $\left(= \tan^{-1}\left(\dfrac{1.2558...}{1.04}\right) = 50.37...°\right)$	**M1**	3.4	ie trying to make θ as small as possible to give the lowest angle at which toppling might occur. ε is their version of the error. Signs of ε must both be correct. $\bar{x}$ and $\bar{y}$ must be the right way round for their angle. Or using angle to the vertical $\theta = \tan^{-1}\left(\dfrac{"1" + \varepsilon}{3\ln 3 - 2 - \varepsilon}\right)$ $(= 39.63...°)$	May find resultant moment, or compare two moments. Condone mg not seen. $mg((3\ln 3 - 2.04)\cos 50° - (1.04)\sin 50°)$ $(= 0.01055mg...)$ Or calculates both distances: $(3\ln 3 - 2 - 0.04) = 1.2558...$ $\tan 50° \times (1 + 0.04) = 1.2394...$ Or 1.2558tan(40)=1.0537>1.04
			50.37... > 50 and so no matter what the error in $\bar{x}$ and $\bar{y}$ the sign will **not topple** may see 39.6<40 using angle to the vertical	**A1**	3.2a	Must be correct justification, from correct values, for not toppling.	Or $d > 0$ from correct values $(3\ln 3 - 2.04)\cos 50° - (1.04)\sin 50°$ $= 0.01055...$ which is a clockwise moment so it won't topple Or 1.25>1.239… so it **won't topple**

				[3]			
	(c)	**(ii)**	$\theta = \tan^{-1}\left(\frac{\bar{X}}{\bar{Y}}\right) = \tan^{-1}\left(\frac{1.2817...}{1.0833...}\right) = 49.79...°$ (< 50°) and so the simplified model predicts **toppling** which means that it **does not** predict the same result as in **(c)(i)**.	**B1**	3.5a	May see awrt 49.8° or 40.2° without epsilon. Allow for candidates who have included +- 0.04 correctly $\theta = \tan^{-1}\left(\frac{\bar{X}-\varepsilon}{\bar{Y}+\varepsilon}\right)$ $= \tan^{-1}\left(\frac{1.2417...}{1.1233...}\right) = 47.865...°$ (< 50°) so it **does topple** so **does not** predict the same result as in **(c)(i)**. May see awrt 47.9° or 42.1° with epsilon used	May see comparison of $\bar{X} = \frac{7ln3}{6} \approx 77/60 = 1.28\ldots$ and $\bar{Y}\tan(50) = \frac{13}{12}\tan(50) = 1.29..$ (difference is -0.0093…) Or $\bar{X}\tan(40)$ compared with $\bar{Y}$ So it **does topple** so **does not** predict the same result as in **(c)(i)**. $\bar{X} - \varepsilon = 1.24\ ...$ $(\bar{Y} + \varepsilon)\tan(50) = \frac{337}{300}\tan(50)$ $= 1.338\ ...$ (difference is -0.0987…) So it **does topple** so **does not** predict the same result as in **(c)(i)**.
				[1]			

Question			Answer	Marks	AO	Guidance	
9			$T_1 = \dfrac{20 \times \frac{1}{3}r}{\frac{2}{3}r}$ or $T_2 = \dfrac{\lambda_2 \times \frac{1}{3}r}{\frac{2}{3}r}$ or $T_3 = \dfrac{\lambda_3 \times \frac{1}{3}r}{\frac{2}{3}r}$	**B1**	3.1b	**DR** Use of $T = \dfrac{\lambda x}{l}$ with correct x and l for at least one string.	$T_1 = 10$ or $T_i = \dfrac{\lambda_i}{2}$
			$\leftrightarrow \quad T_1 \sin\theta_1 = T_2 \sin\theta_2$	**M1**	1.1	Resolving T_1 and T_2 and balancing forces in the horizontal. Allow sign errors. Two forces only.	T_1 could be incorrect. $10 \times \dfrac{3}{5} = T_2 \times \dfrac{12}{13}$ $\leftrightarrow \quad \dfrac{1}{2}\lambda_1 \sin\theta_1 = \dfrac{1}{2}\lambda_2 \sin\theta_2$
			$\updownarrow \quad 1.25g + T_1 \cos\theta_1 + T_2 \cos\theta_2 = T_3$	**M1**	2.1	Resolving T_1 and T_2 and balancing forces in the vertical. Allow sign errors. Exactly four forces.	T_i could be incorrect. $12.25 + 10 \times \dfrac{4}{5} + \dfrac{13}{2} \times \dfrac{5}{13} = \dfrac{\lambda_3}{2}$ $\dfrac{\lambda_3}{2} = 12.25 + 8 + 2.5 = 22.75$
			$\lambda_3 = 45.5$	**B1**	1.1	$T_3 = 22.75$	
			$\lambda_2 = 13$	**B1**	1.1	$T_2 = 6.5$	Could be embedded in working.

			$E_1 = \dfrac{20\times\left(\frac{1}{3}r\right)^2}{2\times\frac{2}{3}r}$ or $E_2 = \dfrac{13\times\left(\frac{1}{3}r\right)^2}{2\times\frac{2}{3}r}$ or $E_3 = \dfrac{45.5\times\left(\frac{1}{3}r\right)^2}{2\times\frac{2}{3}r}$	**M1**	2.1	Use of $E = \dfrac{\lambda x^2}{2l}$ with correct x and l for at least one string, using 20, or their declared λ_2 or λ_3 as modulus of elasticity. May be seen as $E_i = \dfrac{T_i x}{2}$ $E_1 = \dfrac{10\times(\frac{1}{3}r)}{2}$ $E_2 = \dfrac{6.5\times(\frac{1}{3}r)}{2}$ $E_3 = \dfrac{22.75\times(\frac{1}{3}r)}{2}$	$E_1 = \dfrac{5}{3}r$ or $E_2 = \dfrac{13}{12}r$ or $E_3 = \dfrac{91}{24}r$
			$E_1 = \dfrac{5}{3}r$ and/or $E_2 = \dfrac{13}{12}r$ and/or $E_3 = \dfrac{91}{24}r$	**A1**		Two correct values for EPE terms seen	
			So total elastic PE is $\frac{157r}{24}$ J	**A1**	1.1	Allow ISW	
				[8]			

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