

GCE

Further Mathematics A

Y544/01: Discrete Mathematics

A Level

Mark Scheme for June 2025

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING

RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.
5. **Crossed-Out Responses**
Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions

Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses

When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).

When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses

When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)

Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)

If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)

Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.
7. There is a NR (**No Response**) option. Award NR (No Response):
 - if there is nothing written at all in the answer space
 - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
 - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).
8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.
10. For answers marked by levels of response: Not applicable in F501
To determine the level – start at the highest level and work down until you reach the level that matches the answer
To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. Annotations

Annotation	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

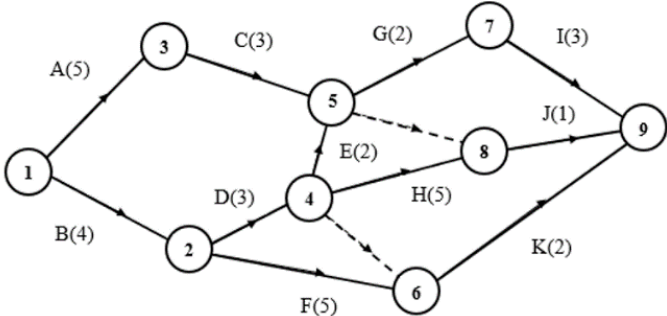
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - If a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as *cao* may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		Travelling Salesperson Problem	B1 [1]	1.2	Travelling salesperson, travelling salesman, TSP, (least weight) Hamiltonian cycle
1	(b)		A C F D E B G A oe 1305 (metres)	M1 A1 [2]	1.1 1.1	Nearest neighbour, working may be on diagram Or implied from arc lengths in order eg 100 + 170 + 185 + 170 + 200 + 280 + 200 (but not having both CF 170 and FC 170) 1305 (or equivalent, eg using km) SC B1 for answer 1305 but no valid method seen
1	(c)		Evidence of Lower Bound Algorithm Choose arcs BC, CF, FD, DE. FG Lower Bound = 900 + AB (120) + AC (100) 1120 (metres)	M1 A1 M1 A1 [4]	3.1b 1.1 1.1 1.1	Delete A, construct a spanning tree for {B, C, D, E, F, G}, need not be minimum spanning tree These arcs in any order, working may be on diagram Or implied from arc lengths 180, 170, 185, 170, 195 in any order (but without repeating arcs, if seen, eg not having both CF 170 and FC 170) Attempt weight of their spanning tree + 120 + 100 (attempt 5 arc weights summed + 220 soi from answer 1120) or 900 + 220 oe seen www 1120 (or equivalent eg using km)
1	(d)		UB is only 185m longer than LB, and LB may not be an achievable tour, so nearest neighbour gives a tour that is fairly efficient, although it may not be optimal	B1 [1]	3.5a	Any written statement that nearest neighbour (or upper bound oe) gives the length of a relatively <u>good</u> tour, <u>but</u> it may or <u>may not</u> be the <u>shortest</u> tour

Question			Answer	Marks	AO	Guidance
2	(a)	(i)	eg. If A is in one set then B, D and F must all be in the other But (eg) arc BD means B and D cannot be in same set	M1 A1	2.2a 2.1	Using a colouring argument oe Connections between arcs of the same colour
			Alternative method 1 D has degree 5 and is directly connected to A, B, C, E and F But eg arc AB means that A and B cannot be in same set	M1 A1		D is connected to all the other vertices Arcs connecting vertices within set {A, B, C, E, F}
			Alternative method 2 eg Each of B, C and D is connected to the other two so all 3 must be in different sets, but there are only 2 sets	M1 A1		Finding a cycle through an odd number of vertices Explaining why this is impossible
				[2]		
2	(a)	(ii)	Remove arcs BD and DF to give bipartite graph with sets {A, C, E}, {B, D, F} $K_{3,3}$ as a subgraph of graph G, hence G is not planar	B1 M1 A1 [3]	2.1 2.21 1.1	Identifying arcs BD and DF Identifying these two sets Correct conclusion using $K_{3,3}$
2	(a)	(iii)	G is a connected graph, no node is directly joined to itself and there are no repeated arcs, so G is also simple, So G is simply connected	B1 [1]	2.2a	Explaining why G is simple, at least saying <u>both</u> that G has no 'loops' (from a vertex to itself) <u>and</u> no multiple (or double) arcs
2	(b)	(i)	Indegree of Node L: 4 Outdegree of Node L: 5	B1 B1 [2]	1.1 1.1	Written on answer line or clearly intended as ans Written on answer line or clearly intended as ans SC B1 for 5 in and 4 out
2		(ii)	Not simply connected, there are multiple arcs from K to J	B1	2.2a	'No, there is a 2 in the table' or 'No, $KJ = 2$ '
			Alternative solution Not simply connected, there is an arc from M to M	B1		No, M is joined to itself
				[1]		

Question		Answer	Marks	AO	Guidance
3	(a)	165, 142, 152, 135, 171, 154, 172, 182 142, 165, 152, 135, 171, 154, 172, 182	B1 [1]	3.1a	Both needed Any indication of given list and given list with the first two elements reversed
3	(b)	List at end of 2nd Pass: 142, 152, 135, 165, 154, 171, 172, 182 List at end of 3rd Pass: 142, 135, 152, 154, 165, 171, 172, 182	B1 B1 FT [2]	1.1 1.1	Fully correct second pass Correct third pass, FT their second pass (bubble)
3	(c)	Bubble sort needs a total of $7 + 6 + 5 + 4 + 3$ = 25 comparisons Shuttle sort uses fewer comparisons so is more efficient	M1 A1 A1 [3]	2.1 1.1 2.4	At least as far as $7 + 6$ seen or implied from 25 or 28 or 'more than 13', 'already 13', oe written Allow M1 for 28 comps (from not stopping early) 25 cao $12 < 25$, so shuttle is more efficient (in this case) An appropriate comparison and conclusion
3	(d)	$\frac{n^2}{300^2} = \frac{60}{2.4 \times 10^{-3}}$ = 50 000 (to 1 sf)	M1 A1 [2]	2.1 2.2b	$O(n^2)$ or quadratic order, used appropriately eg $(n^2 =) 300^2 \times 25000$ or $(n^2 =) 2.25 \times 10^9$ or $(n =) 300 \times \sqrt{\frac{60}{2.4 \times 10^{-3}}}$ or 47434, 47400, 47000 or 50000 seen oe www 50 thousand, www, answer must be to 1 sf

Question			Answer	Marks	AO	Guidance	
4	(a)	(i)		M1	3.1a	Events need not be numbered and durations need not be shown	
			A1			1.1	Activity network with activities on arcs, single start and finish and precedences correct for A, B, C, D
			M1			3.4	Correct network with all arcs directed and exactly two dummies used correctly
			A1			1.1	Evidence of forward and backwards passes, EET increasing along all paths, LET decreasing along all paths, shown at vertices on network or listed
			B1 FT			1.1	All times correct, from a correct network
			Minimum Project Completion Time: 14 hours	[5]		14 stated as minimum completion time or FT from their time at end event	
4	(a)	(ii)	B, D, E, G, I	B1 [1]	1.1	cao	

Question		Answer	Marks	AO	Guidance																																																																											
4	(b)	Total float = LET (tail) – EET (head) – duration eg at A $6 - 0 - 5 = 1$ Independent float = $\max\{ \text{EET (tail)} - \text{LET (head)} - \text{duration}, 0\}$ eg at A $5 - 0 - 5 = 0$ Total float = independent float + interfering float All independent floats are zero so all the float is interfering <table><tr><td></td><td>A</td><td>C</td><td>F</td><td>H</td><td>J</td><td>K</td></tr><tr><td>Total float</td><td>1</td><td>1</td><td>3</td><td>1</td><td>1</td><td>3</td></tr><tr><td>Independent</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>Interfering</td><td>1</td><td>1</td><td>3</td><td>1</td><td>1</td><td>3</td></tr></table>		A	C	F	H	J	K	Total float	1	1	3	1	1	3	Independent	0	0	0	0	0	0	Interfering	1	1	3	1	1	3	M1 M1 A1 [3]	1.1 1.2 1.1	(Total) float correct for at least 1 non-critical activity <u>or</u> implied from interfering and independent seen Independent float correct for at least 1 non-critical activity, or say that it is 0, <u>or</u> implied from both total and interfering seen Interfering float correct for all six non-critical activities, A, C, F, H, J and K Not from their EET and LET values (All) interfering floats correct with no evidence of total floats or independent floats $\Rightarrow$ M1 M0 A0																																															
	A	C	F	H	J	K																																																																										
Total float	1	1	3	1	1	3																																																																										
Independent	0	0	0	0	0	0																																																																										
Interfering	1	1	3	1	1	3																																																																										
4	(c)	<table><tr><td>B</td><td>B</td><td>B</td><td>B</td><td>D</td><td>D</td><td>D</td><td>E</td><td>E</td><td>G</td><td>G</td><td>I</td><td>I</td><td>I</td><td></td></tr><tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>C</td><td>C</td><td>C</td><td>H</td><td>H</td><td>H</td><td>H</td><td>H</td><td>J</td><td></td></tr><tr><td></td><td></td><td></td><td></td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>K</td><td>K</td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> Minimum Number of Workers: 3	B	B	B	B	D	D	D	E	E	G	G	I	I	I		A	A	A	A	A	C	C	C	H	H	H	H	H	J						F	F	F	F	F	K	K																																			M1 M1 A1 [3]	1.1 3.1b 2.2a	Activities not shared or split across rows Critical activities (B, D, E, G, I) starting at ending at correct times Non-critical activities (A, C, F, H, J, K) with valid start and finish times 3 from a valid allocation using exactly 3 rows (ie delay H and J by 1 hour so that they fit in after C)
B	B	B	B	D	D	D	E	E	G	G	I	I	I																																																																			
A	A	A	A	A	C	C	C	H	H	H	H	H	J																																																																			
				F	F	F	F	F	K	K																																																																						
4	(d)	(C has) 1 hour of float time (so increasing it by 1 hour would not change the minimum project completion time) $t > 1$	M1 A1 [2]	3.5c 1.1	$5 + (3 + t) > 9$ oe seen May say that $t = 1$ would make (A and C) critical Must be > 1 (t may be implied) not ≥ 1 (not 2, $t \geq 2$, etc), may use set notation, may also say $< \text{infinity}$																																																																											

Question			Answer	Marks	AO	Guidance																									
5	(a)		1	B1 [1]	1.1	cao seen as answer (from PZ, but not ‘ loses -1’)																									
5	(b)		<table><tr><td></td><td>X</td><td>Y</td><td>Z</td><td>Row min</td></tr><tr><td>P</td><td>1</td><td>2</td><td>-1</td><td>-1</td></tr><tr><td>Q</td><td>3</td><td>0</td><td>2</td><td>0</td></tr><tr><td>R</td><td>-1</td><td>4</td><td>-2</td><td>-2</td></tr><tr><td>Col max</td><td>3</td><td>4</td><td>2</td><td></td></tr></table> Row maximin = 0, column minimax = 2 0 ≠ 2 so not stable		X	Y	Z	Row min	P	1	2	-1	-1	Q	3	0	2	0	R	-1	4	-2	-2	Col max	3	4	2		M1 M1 A1 B1 [4]	1.1 1.1 2.2a 2.1	Row minima, -1, 0, -2 Column maxima, 3, 4, 2 oe Or play-safe strategies, Q and Z (only) identified Dependent on both M1 marks Or equivalent valid reasoning, using play-safe values or play-safe strategies and leading to ‘not stable’ oe
	X	Y	Z	Row min																											
P	1	2	-1	-1																											
Q	3	0	2	0																											
R	-1	4	-2	-2																											
Col max	3	4	2																												
5	(c)		Best outcomes for A for each of B’s choices: (Q, X), (R, Y), (Q, Z) Best outcomes for B for each of A’s choices: (P, Z), (Q, Y), (R, Z) No cell is the best for both players	M1 A1 [2]	2.2a 1.1	If part (c) is done in (b) must be stated as being (c) At least one set of outcomes written or unambiguously indicated on a table Correct conclusion from both sets correct or equivalent valid reasoning																									
5	(d)		Cell: (Q, Y) New Pay-off Value: 2	B1 B1 dep [2]	3.1b 1.1	Allow YQ oe cao																									

Question			Answer	Marks	AO	Guidance												
5	(e)		Z (strictly) dominates X , whichever strategy A chooses, B always does better playing Z rather than X, delete X $-1 < 1, 2 < 3, -2 < -1$	B1 [1]	2.2a	Statement that Z dominates X or that X is dominated by Z or that X can be deleted oe supported by these 3 comparisons oe seen or described in full												
5	(f)	(i)	<table border="1"><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>Y</td><td>-2</td><td>-m</td><td>-4</td></tr><tr><td>Z</td><td>1</td><td>-2</td><td>2</td></tr></table> Let B play Y with probability p and Z with prob $1-p$ If A plays P, $E = -2p + (1-p) = 1 - 3p$ If A plays Q, $E = -mp - 2(1-p) = (2 - m)p - 2$ If A plays R, $E = -4p + 2(1-p) = 2 - 6p$ Optimum is when $1 - 3p = (2 - m)p - 2 = 0.4$ $1 - 3p = 0.4 \Rightarrow p = 0.2$ Optimal Mixed Strategy for Player B: play Y 1/5 of the time and Z 4/5 of the time		P	Q	R	Y	-2	-m	-4	Z	1	-2	2	M1 M1 A1 M1 A1 A1 [6]	2.1 3.1a 1.1 3.3 3.4 3.2a	Reduced game rewritten with player B on rows and payoffs for B, seen or implied from pay-off calcs Condone m instead of $-m$ in cell (Y, Q) for this mark Any letter used for variable, need not be defined here Attempt three expected pay-offs (or values) for B, must involve only 1 unknown probability All three correct, oe, need not be simplified Attempt to solve any pair of (their) equations $p = 0.2$ at optimum, cao cao, both, in context
	P	Q	R															
Y	-2	-m	-4															
Z	1	-2	2															
			Alternative solution 1 (max 3 marks) <table border="1"><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>X</td><td>-1</td><td>-3</td><td>1</td></tr><tr><td>Y</td><td>-2</td><td>-m</td><td>-4</td></tr></table> Let B play X with probability p and Y with prob $1-p$ If A plays P, $E = -p - 2(1-p) = p - 2$ If A plays Q, $E = -3p - m(1-p) = (m - 3)p - m$ If A plays R, $E = p - 4(1-p) = 5p - 4$ Any pair from $p - 2 = (m - 3)p - m = 5p - 4 = 0.4$		P	Q	R	X	-1	-3	1	Y	-2	-m	-4	M1 M1 M1		Deleting column Z instead of column X Reduced game rewritten with player B on rows and payoffs for B, seen or implied from pay-off calcs Condone m rather than $-m$ in cell (Y, Q) Any letter used for variable, need not be defined here Attempt three expected pay-offs (or values) for B, need not be simplified, but must involve only 1 unknown probability Attempt to solve any pair of (their) equations
	P	Q	R															
X	-1	-3	1															
Y	-2	-m	-4															

Question			Answer	Marks	AO	Guidance												
5	(f)	(i)	Alternative solution 2 <table><tr><td></td><td>Y</td><td>Z</td></tr><tr><td>P</td><td>2</td><td>-1</td></tr><tr><td>Q</td><td>m</td><td>2</td></tr><tr><td>R</td><td>4</td><td>-2</td></tr></table> Let B play Y with probability p and Z with prob $1-p$ If A plays P, $E = 2p - (1-p) = 3p - 1$ If A plays Q, $E = mp + 2(1-p) = 2 + (m - 2)p$ If A plays R, $E = 4p - 2(1-p) = 6p - 2$ Optimum is when $3p - 1 = 2 + (m - 2)p = -0.4$ $3p - 1 = -0.4 \Rightarrow p = 0.2$ Optimal Mixed Strategy for Player B: play Y 1/5 of the time and Z 4/5 of the time		Y	Z	P	2	-1	Q	m	2	R	4	-2	M1 M1 A1 M1 A1 A1		Reduced game with table not transposed seen or implied from pay-off calcs Any letter used for variable, need not be defined here Attempt three expected pay-offs (or values) for B, must involve only 1 unknown probability All three correct, oe, need not be simplified Attempt to solve (their) equations $p = 0.2$ at optimum. cao cao, both, in context
			Y	Z														
		P	2	-1														
Q	m	2																
R	4	-2																
	Alternative solution 3 (max 3 marks) <table><tr><td></td><td>X</td><td>Y</td></tr><tr><td>P</td><td>1</td><td>2</td></tr><tr><td>Q</td><td>3</td><td>m</td></tr><tr><td>R</td><td>-1</td><td>4</td></tr></table> Let B play X with probability p and Y with prob $1-p$ If A plays P, $E = p + 2(1-p) = 2 - p$ If A plays Q, $E = 3p + m(1-p) = m + (3 - m)p$ If A plays R, $E = -p + 4(1-p) = 4 - 5p$ Any pair from $2 - p = m + (3 - m)p = 4 - 5p = -0.4$		X	Y	P	1	2	Q	3	m	R	-1	4	M1 M1 M1		Deleting column Z instead of column X Reduced game with table not transposed seen or implied from pay-off calcs Any letter used for variable, need not be defined here Attempt three expected pay-offs (or values) for B, need not be simplified, but must involve only 1 unknown probability Attempt to solve any pair of (their) equations		
	X	Y																
P	1	2																
Q	3	m																
R	-1	4																
5	f	(ii)	$(2 - m)(0.2) - 2 = 0.4 \Rightarrow m = -10$	B1	3.4	-10 from correct working												
			Alternative solution $(m - 2)(0.2) + 2 = -0.4$ $\Rightarrow m = -10$	B1		Table not transposed -10 from correct working												
				[1]														

Question			Answer	Marks	AO	Guidance																																								
6	(a)		Maximise $P = 16x + 20y + 14z$	B1 [1]	3.3	<u>Max</u> $16x + 20y + 14z$ (or any positive multiple of this)																																								
6	(b)		<table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>RHS</td></tr><tr><td>1</td><td>-16</td><td>-20</td><td>-14</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>4</td><td>3</td><td>5</td><td>1</td><td>0</td><td>0</td><td>200</td></tr><tr><td>0</td><td>6</td><td>5</td><td>4</td><td>0</td><td>1</td><td>0</td><td>160</td></tr><tr><td>0</td><td>2</td><td>11</td><td>4</td><td>0</td><td>0</td><td>1</td><td>144</td></tr></table>	P	x	y	z	r	s	t	RHS	1	-16	-20	-14	0	0	0	0	0	4	3	5	1	0	0	200	0	6	5	4	0	1	0	160	0	2	11	4	0	0	1	144	B1 B1 FT M1 A1 A1 [5]	2.5 3.3 3.3 1.1 1.1	Correct structure for columns P, r, s, t Objective row correct (in row 1) FT their objective from part (a) x, y, z and RHS correct for any one constraint row x, y, z and RHS correct for all three constraint rows Constraint rows in any order A fully correct table
P	x	y	z	r	s	t	RHS																																							
1	-16	-20	-14	0	0	0	0																																							
0	4	3	5	1	0	0	200																																							
0	6	5	4	0	1	0	160																																							
0	2	11	4	0	0	1	144																																							

Question			Answer	Marks	AO	Guidance																																								
6	(c)	(i)	$200 \div 3 = 200/3 = 66.7$ $160 \div 5 = 160/5 = 32$ $144 \div 11 = 144/11 = 13.1$	B1 [1]	1.1	3 correct calculations (or as fractions or decimals) Need not show a pivot choice																																								
6	(c)	(ii)	<table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>r</th><th>s</th><th>t</th><th>RHS</th></tr><tr><td>1</td><td>-136/11</td><td>0</td><td>-74/11</td><td>0</td><td>0</td><td>20/11</td><td>2880/11</td></tr><tr><td>0</td><td>38/11</td><td>0</td><td>43/11</td><td>1</td><td>0</td><td>-3/11</td><td>1768/11</td></tr><tr><td>0</td><td>56/11</td><td>0</td><td>24/11</td><td>0</td><td>1</td><td>-5/11</td><td>1040/11</td></tr><tr><td>0</td><td>2/11</td><td>1</td><td>4/11</td><td>0</td><td>0</td><td>1/11</td><td>144/11</td></tr></table> Pivot Column: y Value of Pivot Element: 11	P	x	y	z	r	s	t	RHS	1	-136/11	0	-74/11	0	0	20/11	2880/11	0	38/11	0	43/11	1	0	-3/11	1768/11	0	56/11	0	24/11	0	1	-5/11	1040/11	0	2/11	1	4/11	0	0	1/11	144/11	M1 M1 FT A1 [3]	1.1 3.4 1.1	Structure correct Basis cols P and 3 others eg y, r, s (not still r, s, t) Non-negative values in RHS column Value of P has increased New pivot row, fractions or decimals (2 dp or 3 sf) FT their table from (b) and their pivot choice Fully correct, allow mixed fractions but not decimals for the exact answer
P	x	y	z	r	s	t	RHS																																							
1	-136/11	0	-74/11	0	0	20/11	2880/11																																							
0	38/11	0	43/11	1	0	-3/11	1768/11																																							
0	56/11	0	24/11	0	1	-5/11	1040/11																																							
0	2/11	1	4/11	0	0	1/11	144/11																																							
6	(d)		No negative values in Row 1	B1 [1]	2.4	Non-negative or ≥ 0 , oe Allow positive or > 0 (imply non-basis columns)																																								
6	(e)		$x = 4, y = 0, z = 34, r = k, s = 0, t = 0$ $4x + 3y + 5z + r = 200 \Rightarrow 16 + 170 + k = 200$ $\Rightarrow k = 14$	M1 A1 [2]	3.1a 1.1	Reading off values for x, y, z values stated or show substitution into the machine time constraint eg $4(4) + 0 + 5(34) \leq 200$ May imply $y = 0$ if no other value is seen 14 from correct reasoning																																								

Question		Answer	Marks	AO	Guidance
7	(a)	Number of multiples of 2 = 50 Number of multiples of 3 = 33 Number of multiples of 5 = 20	M1	1.1	Attempt to count the number of multiples of 2, 3, 5 ie 50, 33, 20 (at most 1 slip)
		Number of multiples of 2 and 3 (or of 6) = 16 Number of multiples of 2 and 5 (or of 10) = 10 Number of multiples of 3 and 5 (or of 15) = 6	M1	1.1	Attempt to count the number of multiples of pairs and multiples of triple ie 16, 10, 6, 3 (at most 1 slip)
		Number of multiples of 2, 3 and 5 = 3			
		$50 + 33 + 20 - 16 - 10 - 6 + 3 = 74$	A1	1.1	Use Inclusion-Exclusion Principle www to get 74 Calculation or formula seen and all correct Allow $103 - 32 + 3 = 74$
			[3]		
7	(b)	3 cards each with one digit $9 \times 8 \times 7 = 504$	M1	2.2a	9 cards with 1 digit (1 to 9) 504 seen oe eg 9P3 or $9 \times 8 \times 7$ or $9C3 \times 3!$
		1 card with two digits followed by 1 card with 1 one digit 1 card with one digit followed by 1 card with two digits $(90 \times 9) + (9 \times 90) = 1620$	M1	1.1	90 cards with 2 digits (10 to 99) and 9 with 1 digit Either 1620 or 810 seen oe eg 90×18 , $90 \times 9 \times 2$ or 90×9
		Or 1 card with three digits $504 + 1620 + 1 = 2125$	A1	1.1	1 card with 3 digits (100) 2125 as final answer, cao
			[3]		

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