GCE

Further Mathematics B MEI

Y435/01: Extra pure

A Level

Mark Scheme for June 2025

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5. **Crossed-Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
   - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
    To determine the level – start at the highest level and work down until you reach the level that matches the answer
    To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$(a\circ b)\circ c = (3ab)\circ c = 3(3ab)c = 9abc$	**M1**	**2.1**	Attempting to find expressions for $(a\circ b)\circ c$ **and** for $a\circ(b\circ c)$ without $\circ$.	Must reach stage where all brackets are removed for both. Must start correctly but condone one missing $\circ$ for **M1** only. Must be assertion of equality using "=" or "≡" signs only. Must be different, general elements. Condone "0 = 0" proof structure.
			$a\circ(b\circ c) = a\circ(3bc) = 3a(3bc) = 9abc$ $\therefore (a\circ b)\circ c = a\circ(b\circ c)$ www so ($\circ$ is) associative (over the reals).	**A1** **[2]**	**2.1**	Finding correct expressions following some explicit working and noting equality (could be in words – or symbols) between two correct and correctly derived expressions for $(a\circ b)\circ c$ and $a\circ(b\circ c)$. If order of a, b, c changes (when using $\circ$) then **A0**. If equality not noted then **A0**. Condone missing brackets in working if intent clear and correctly used eg $(a\circ b)\circ c = 3ab\circ c$. Must have conclusion indicating understanding of associativity (could be at top).	The expressions stated as equal must be the same. eg "$(a\circ b)\circ c = ... = 3\times 3abc$ and $a\circ(b\circ c) = ... = 9abc$ so $(a\circ b)\circ c = a\circ(b\circ c)$" is not sufficient. $9abc = 9bca$ only allowed on appeal to commutativity of (real) multiplication. Ignore irrelevant, non-contradictory statements. Condone use of incorrect '$\circ$' if clearly distinguishable symbol for general operation and consistently used. Condone poor language if intent clear.
	(b)		$(a\circ e =)\ 3ae = a$ (for all a)	**M1**	**1.1**	Using correct rule for identity to set up an equation for e. a could be numerical or e (ie $3\times 2\times e = 2$ or $3e^2 = e$).	No need to consider both directions. Must be use e or if b, say, b equated to e later. If **M0** then **SCB1** for $3ab = a$.
			$\Rightarrow e = \frac{1}{3}$ or exact equivalent	**A1** **[2]**	**2.2a**	No need to consider specifically the case $a = 0$. Not "$b =$" unless "$b = e$"	No need to state that it's in the reals. If **M0, SCB1** for correct answer with insufficient working (eg verification).

	(c)		If a and b are real numbers then ($a \circ b =$) $3ab$ is also a real number	**B1** **[1]**	**2.4**	Must show knowledge of what is meant by closed. Can be specific or general. Must be clear that we are both starting and ending with real numbers but 3 can be assumed to be meant as real without being explicitly stated as such. Multiplication must be explicitly stated or implied by eg $3ab$; words like "produce" or "generate" or "create" or "output" or "involves" *alone* are insufficient. Can appeal to fact that $\circ$ is only defined on real numbers.	"Multiplication of (three (or two)) real numbers gives a real number." "It is impossible to get a non-real number when multiplying real numbers". Not just "Multiplication is closed under the reals". Not "real numbers will produce real numbers". Not just "a group is closed if...". Not just "$a \circ b \in \mathbb{R}$ so $\mathbb{R}$ is closed". Not "a real multiplied by a real cannot be complex". Not "$\circ$ only consists of multiplication which is closed so $\mathbb{R}$ is closed under $\circ$" since "consists" (or "uses" or similar) is not precisely defined. Ignore irrelevant, non-contradictory statements that are not egregiously incorrect. Condone slightly incorrect wording or notation if this does not materially undermine the point being made.

	(d)		0 ($\in\mathbb{R}$) does not have an inverse since if $a = 0$ then $3\times0\times b$ (or $3\times0\times a^{-1}$) (= 0 so) cannot equal $\frac{1}{3}$ (or e) as would be required (or equivalent requirement).	**M1**	**2.4**	Error-free and complete argument to show that 0 does not have an inverse in $\mathbb{R}$. eg 0^{-1} would have to equal $(1/3)\div(3\times0)$ or $1/(9\times0)$ which is not defined (or does not exist or is infinite or is not a real number). But not from $0^{-1} = 1/0$. eg (if 0^{-1} exists then) $0\circ0^{-1}$ or $3\times0\times0^{-1} = 0 \Rightarrow e = 0$ which is not the case.	For reference, $a^{-1} = \frac{1}{9a}$ if $a \neq 0$. It is not sufficient to state either 0^{-1} or $1/0$ is not defined. Candidates must demonstrate knowledge of what is meant by a^{-1} in a binary operation context. $a\circ a^{-1}$ or $a\times a^{-1} = 1$ soi is **M0**.
			Inverse axiom not met (so all 4 group axioms not met so is not a group).	**A1**	**2.2a**	or "so not every real number has an inverse". But not just "There is no inverse" nor just "Every real number does not have an inverse". If "Inverse axiom" not explicitly mentioned then it must be made explicit that to be a group it is necessary that all elements have an inverse element. This could be seen first.	Condone slightly incorrect wording or notation if this does not materially undermine the point being made. If **M0** then **SCB1** for "0 does not have an inverse so not every element has an inverse".
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$25u_{n+2} - 35u_{n+1} + 12u_n = 0$ and $u_n = \alpha r^n$ $\Rightarrow 25r^2 - 35r + 12 = 0$ $\Rightarrow r = \frac{3}{5}$ or $r = \frac{4}{5}$ So CF is $u_n = A\left(\frac{3}{5}\right)^n + B\left(\frac{4}{5}\right)^n$ ISW Trial function: try $u_n = \alpha$ $25\alpha - 35\alpha + 12\alpha = 2\alpha = c \Rightarrow \alpha = \frac{1}{2}c$ GS: $u_n = A\left(\frac{3}{5}\right)^n + B\left(\frac{4}{5}\right)^n + \frac{1}{2}c$ oe	**M1** **A1** **B1** **M1** **A1** **[5]**	**1.1** **1.1** **1.1** **1.1** **1.1**	Deriving the auxiliary equation (may be implied by correct solutions) $A0.6^n + B0.8^n$ Condone missing brackets in fractions or missing u_n for **A1** here. If not seen may be seen as part of GS, either as the final answer or the trial function. Correct general form for trial function soi. Correctly substituting in to the RR to find α in terms of c. Could include the CF. Must be "$u_n =$". Not "f(n) =" or similar unless f(n) = u_n defined. Do not condone missing brackets in fractions for **A1** here. Do not ISW. GS must be seen in this part for this mark.	$\frac{3^n A + 4^n B}{5^n}$ Other terms eg dn may be added; **B1** can be awarded when eg $d = 0$ stated (from working or assumption) $\alpha = \frac{1}{2}c$ is not sufficient for A1; must see GS explicitly.
	(b)	(i)	$u_0 = 20 \Rightarrow A + B - 5 = 20$ ($\Rightarrow A + B = 25$) $u_1 = 70 \Rightarrow \frac{3A}{5} + \frac{4B}{5} - 5 = 70$ $\Rightarrow A = -275, B = 300$ $u_n = 300\left(\frac{4}{5}\right)^n - 275\left(\frac{3}{5}\right)^n - 5$	**M1** **M1** **A1** **[3]**	**3.3** **1.1** **1.1**	Using one initial condition to derive an equation in A and B Using the other condition to derive an equation in A and B and attempt to solve for A and B (attempt can be implied by solution) Just obtaining values of A and B is not sufficient for **A1**. Must be "$u_n =$" and fractions bracketed. Do not penalise if penalised for this in part **(a)**. If not **M1M1** then **SCB1** for correct solution.	From GS of the form: $Ap^n + Bq^n + C$, $A, B, C \neq 0$. $3A + 4B = 375$ Could derive u_2 (= 88) from RR and use this (ie $0.36A + 0.64B - 5 = 88$ $\Rightarrow 9A + 16B = 2325$) Or if GS has "+ C" then **M2** for 3 equations in A, B and C (using u_0, u_1 and $u_2 = 88$) and **A1** for final answer.

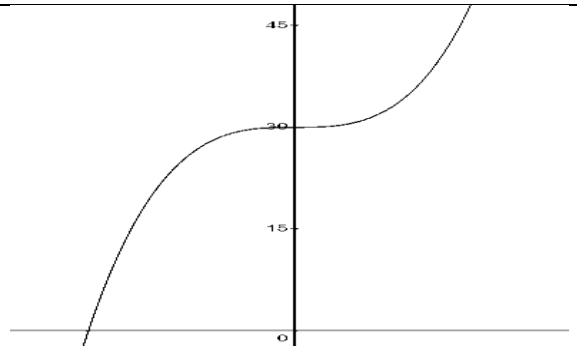
		(ii)	$u_2 = 88$ or $u_3 = 89.2$ or 89 or $u_4 = 82.24$ or 82	**M1**	**3.4**	Correctly calculating u_2 or u_3 or u_4, soi by correct answer. Attempt to use calculus is **M0A0** unless recovered. NB Candidates do not need to have solved the RR to get this part fully correct since they can just use the RR to get subsequent terms. Condone $u_4 = 81.96$.	or u_2, u_3 or u_4 correct from their PS. Mark can be awarded if both bases are between 0 and 1 (inclusive) and the peak is not at 70.
			89	**A1FT** **[2]**	**2.2a**	Must be an integer. 2/2 for correct answer from no working. Answer must be explicitly stated or selected in some manner. If in form $u_n = 89$ then n must be 3.	**FT** their PS of the form $A(0.6)^n + B(0.8)^n + C$, $A, B, C \neq 0$. If "Peak at u_3" then u_3 must be clearly shown as 89. **FT** must include calculated values of u_0 and u_1 if not 20 and 70.

Question			Answer	Marks	AO	Guidance	
		(iii)	$u_{18} = 0$ or 0.4 or awrt 0.38	B1FT	3.1b	0.376391... Condone 0.37.	**FT** correct value from their PS of the form $A(0.6)^n + B(0.8)^n + C$, A, B, $C \neq 0$
			$u_{19} = -1$ or -0.7 or awrt -0.69	M1	3.4	Finding u_{19} correctly using either their solution of the same allowed form or the recurrence relation. Condone -0.6.	$-0.6933...$
			So model predicts –ve holding at end of 19th month (but not at end of 18th month) which is not realistic (since holding must be non-negative)	A1	3.5a	Must follow a negative value for u_{19}. If comment made about u_{18} then must be consistent with their positive u_{18}.	Condone slight inaccuracies in explanation such as "positive" when "non-negative" is meant. Ignore irrelevant comments.
				[3]			
	(c)		$(k =) \lim_{n\to\infty}\left(A\left(\frac{4}{5}\right)^n - B\left(\frac{3}{5}\right)^n + 5\right)$	M1	3.1b	Consideration of the limit as $n \to \infty$ of either the PS or the GS with $"\frac{1}{2}"\times 10$ instead of $\frac{c}{2}$. Condone any values of A and B.	Their PS/GS of the form $A(0.6)^n + B(0.8)^n + C$, A, B, $C \neq 0$ Could see $u_n = 280\left(\frac{4}{5}\right)^n - 265\left(\frac{3}{5}\right)^n + 5.$ eg condone $u_n = 300\left(\frac{4}{5}\right)^n - 275\left(\frac{3}{5}\right)^n + 5.$
			so as $n \to \infty$, $u_n \to 5$ (so $k = 5$) cao	A1	3.2a	5 can be embedded in such a statement or expressed in a statement.	If **M0** then **SCB1** for "5" unambiguously presented as the answer.
				[2]			

Question			Answer	Marks	AO	Guidance	
3	(a)		0 since (for any $a \in M$,) $0 +_9 a = a$	**B1** **[1]**	**2.1**	Or $0 + a \pmod 9 = a$ (for any $a \in M$) Operation considered must be addition modulo 9, not just addition. Allow in words provided that the statement is clear, unambiguous, correct and complete (ie contains the notion that 0 added (mod 9) to any element of M is equal to that element).	Must be for general a or all 9 cases specifically considered or covered by statement such as "and similarly for all members of M". No need to consider both directions. No need to assert $0 \in M$.
	(b)		Order of M is 9 (From Lagrange's theorem) the order of any subgroup is a factor of the order of the group. 5 is not a factor of 9.	**M1** **A1** **[2]**	**1.1** **2.2a**	Correctly stating $\lvert M \rvert$. Or since the order of M is 9 any subgroup must have order 1, 3 or 9, not 5. Must either state or appeal to Lagrange's theorem. Complete and convincing proof without any false statements but condone minor wording issues (eg "5 is not a factor of 9 so cannot be a subgroup of M").	Can be implied by a statement of the type "Order of subgroup must be a factor of order of group. 5 is not a factor of 9. So no subgroup of order 5" provided that the wording is the consistent way round. Do not condone confusion between factor and multiple. In symbols must be $5 \nmid 9$, not $9 \nmid 5$.

	(c)		eg 1 1, 2, 4, 5, 7, 8	**M1** **A1** **[2]**	**1.1** **1.1**	Any generator Allow eg <1> or {1} for **M1** only. All six and no others even if congruent to a generator modulo 9 (eg 10). Any order. Ignore repeats	eg "Every element except 0" is **M1A0**. If in table with orders then either the generators must be highlighted or it must be explained that elements of order 9 are generators.
	(d)	**(i)**	(H =) {0, 3, 6} cao	**B1** **[1]**	**2.5**	Notation must be correct but condone different bracket types (ie ( ) or [ ]). Any order. Do not accept <3> or <6> or ({0, 3, 6}, $+_9$).	Not {3, 6, 9}.
		(ii)	By Lagrange's Theorem, a proper. non-trivial subgroup of M can only have order 3. So such a subgroup could only contain 0, 3 and 6 since all other elements have order 9 (which means it is H).	**M1** **A1** **[2]**	**2.1** **2.2a**	Making a true and relevant, substantive statement relating to a second such subgroup of M. eg "The only elements of M which do not have order 9 are 0, 3 and 6" Completing the argument. Argument must be comprehensive, including consideration of "proper" and "non-trivial" and not contain false or contradictory statements (eg "The subgroup is not closed" unless this is stated as a contradiction). It can be assumed without stating that each subgroup of M is generated by a single element.	eg "1, 2, 4, 5, 7 & 8 have order 9 so (by Lagrange) cannot be in a proper subgroup of M." eg "A subgroup containing any of the generators of M has order 9 (and so is not proper)" Do not condone confusion of "proper" and "trivial". Condone a complete argument based on subgroups generated by each single element for **M1A1**. eg consideration of "proper" and "non-trivial" could be explicit or implicit (eg statement that the order of such a subgroup must be 3).

Question			Answer	Marks	AO	Guidance	
4	(a)		$\frac{\partial z}{\partial x}$ or $\frac{\partial f}{\partial x} = 3x^2 - 12y^2$	B1	1.1	Condone poor notation for **B1B1** provided intent clear. But must be distinguishable so not eg $\partial z =$.	Allow correctly embedded in grad. Do not allow implied misreads.
			$\frac{\partial z}{\partial y}$ or $\frac{\partial f}{\partial y} = -24xy + 192y$	B1	1.1		If **B0B0** then **SCB1** for both RHSs correct.
			SPs where **both** $\frac{\partial z}{\partial x} = 0$ **and** $\frac{\partial z}{\partial y} = 0$	M1	1.1	Showing required condition for SP. Can be implied by seeing both their derivatives set to 0 and an attempt to solve simultaneously	
			$-24xy + 192y = 0$ (=> $y = 0$) or $-24x + 192 = 0$	M1	1.1	Setting both derivatives equal to 0 and eliminating one unknown or subbing the known value of x or y into the other equation.	or eg $3x^2 = 12y^2$ => $x = (\pm)2y$ $(\mp)48y^2 + 192y = 0$ or $(\mp)48y + 192 = 0$ or $(\mp)12x^2 + (\pm)96x = 0$
			$x = 8$ => $12y^2 = 3\times64 = 192$ => $y = (\pm)4$	A1	1.1	Finding a correct non-zero x, y pair correctly	
			=> other SPs are (8, 4, 542), (8, –4, 542)	A1	1.1	Finding z correctly for each correct x, y pair and no other SPs found or attempted (ignore presence of (0, 0, 30)).	Must be coordinates or $x =$, $y =$, $z =$ as unambiguous triplets, not position vectors.
				[6]			

	(b)			**B1**	**1.1**	Cubic graph, horizontal at z-axis and 30 labelled (or specified) as z-intercept. Graph must intersect x-axis and show/imply correct large x behaviour.	
			Moving along the section from the SP you go up in one direction and down in another so it is neither a minimum nor a maximum so must be a saddle point.	**B1** **[2]**	**2.4**	Explanation must be based on shape of section and not from calculus (but could be based on gradient either side). Section must be correct except possibly having a positive gradient at $x = 0$ or missing 30 (not both). Allow just "Stationary point of graph is a point of inflection so stationary point of S is a saddle point". If stationary point of section not mentioned in this argument then graph must be horizontal at $x = 0$.	"z can either increase or decrease when x changes". Not "z can either go positive or negative when x changes." "$f(x + h, 0) > f(x, 0)$ (or 30) and $f(x – h, 0) < f(x)$ (or 30)" with no need for $x = 0$ (could just be $f(x)$ rather than $f(x, y)$). Ignore additional slightly incorrect statements if not contradictory. **SCB1** if **B0B0** but the only fault with the graph is either a positive gradient at $x = 0$ or missing 30 (but not both) and "Saddle point" is stated.

Question	Answer	Marks	AO	Guidance	
(c)	When $x = 3$ and $y = 2$, $\frac{\partial z}{\partial x} = 3(3)^2 - 12(2)^2 = -21$	**M1**	**1.1**	Attempting to calculate f_x where $x = 3$ and $y = 2$.	For this **M1M1A1** the substitution may be made at a later point; the marks are to be awarded at the point where substitution is made.
	and either $z = 3^3 - 12(3)(2)^2 + 96(2)^2 + 30 = 297$ or $\frac{\partial z}{\partial y} = -24(3)(2) + 192(2) = 240$	**M1**	**1.1**	Attempting to calculate either f_y or z (not a) where $x = 3$ and $y = 2$.	
	So $\mathbf{n} = \begin{pmatrix} -21 \\ 240 \\ -1 \end{pmatrix}$ soi or $\mathbf{n} = \begin{pmatrix} -21 \\ ... \\ -1 \end{pmatrix}$ and $z = 297$ soi	**A1**	**1.1**	Forming grad at the point and identifying it as the normal or using it as the direction vector in the vector equation of a line. Do not award **A1** if this vector is not seen as the normal to the surface or if it is used in the equation of a plane.	If grad is never seen then it can be implied by correct x and either y or z equations.
	$\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 297 \end{pmatrix} + \lambda \begin{pmatrix} -21 \\ 240 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 2 \\ 297 \end{pmatrix} + \lambda \begin{pmatrix} -21 \\ 240 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ 482 \\ 295 \end{pmatrix}$ or $(\lambda =) \frac{x-3}{-21} = \frac{y-2}{240} = \frac{z-297}{-1}$ or $\frac{a-3}{-21} = \frac{482-2}{240}$ or $\frac{295-297}{-1}$	**M1**	**1.1**	Forming equation of normal using their numerical position and grad vectors (this may be implied by later substitution or by correct x and either y or z equations). One of 240 and 297 could be non-numerical. Could use (a, 482, 295) as point on the line to form the equation of the line and then use the fact that (3, 2, 297) is on the line.	If **M0A0M0** then **SCB1** for $\mathbf{n} = \begin{pmatrix} "3x^2 - 12y^2" \\ "-24xy + 192y" \\ -1 \end{pmatrix}$ and further **SCB1** for $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 297 \end{pmatrix} + \lambda \begin{pmatrix} "3x^2 - 12y^2" \\ "-24xy + 192y" \\ -1 \end{pmatrix}$
	$(2 + 240\lambda = 482 \Rightarrow \lambda = 2 \Rightarrow) a = 3 - 42 = -39$	**A1** **[5]**	**2.1**	or $297 - \lambda = 295$	

Question	Answer	Marks	AO	Guidance	
(d)	$x^3 - 12xy^2 + 96y^2 + 30 = 542$ $\Rightarrow y^2 = \frac{512 - x^3}{12(8-x)}$ or $\frac{512 - x^3}{96 - 12x}$	**M1**	**3.1a**	Setting $z = 542$ and rearranging to $y^2 =$ or $y =$.	$y = (\pm)\sqrt{\frac{512 - x^3}{12(8-x)}}$ or $\sqrt{\frac{512 - x^3}{96 - 12x}}$
	$512 - x^3 = (8 - x)(64 + 8x + x^2)$	**M1**	**1.1**	Attempting to factorise $512 - x^3$ with $(8 - x)$ as a factor.	Or attempts equivalent polynomial division.
	so $x = 8 \Rightarrow y = \pm\sqrt{\frac{64 + 8x + x^2}{12}} = \pm\sqrt{\frac{3 \times 64}{12}} = \pm 4$	**A1**	**1.1**	Allow embedded ie $(8, \pm 4)$.	If **M0M0** then **SCB1** for either explicitly quoting the result from part **(a)** or recalculating $y = \pm 4$ from the derivatives without justifying that these are the points on the curve.
	Alternative method: $x^3 - 12xy^2 + 96y^2 + 30 = 542$ $\Rightarrow x^3 - 12xy^2 + 96y^2 - 512 = 0$ $\Rightarrow (x - 8)(...) = 0$	**M1**		Setting $z = 542$, either rearranging to = 0 and attempting to factorise with $x - 8$ oe as a factor or similar attempt to factorise both sides. If in different parts must have same factor must 'look' the same in both (but condone difference in sign of factor eg $(x - 8)$ and $(8 - x)$).	Or **M1** for $x = 8 + h$ substituted in and expanding, **M1** for $z = 542$, cancelling 0^{th} order terms and considering $h \to 0$ and **A1** for finding limit of y^2 and correct completion after completely correct working. But **M0** if done entirely numerically (eg finding y when $x = 7.99$).
	$\Rightarrow (x - 8)(x^2 + 8x + 64 - 12y^2) = 0$	**M1**		Correctly factorising.	
	$\therefore x = 8 \Rightarrow 64 + 64 + 64 - 12y^2 = 0$ $\therefore 12y^2 = 192$ $\therefore y = \pm 4$	**A1**		Allow embedded ie $(8, \pm 4)$. Disappearance of $(x - 8)$ factor does not need to be explained.	
		[3]			

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	**DR** $\begin{vmatrix} a-\lambda & b & 0 \\ -b & a-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$ $\Rightarrow (1-\lambda)\left((a-\lambda)^2 - -b\times b\right)\ (=0)$	**M1**	**1.1**	Expanding det($\mathbf{A} - \lambda\mathbf{I}$) (= 0) to a non-determinant form. or eg (from 1st row): $(a-\lambda)\left((a-\lambda)(1-\lambda)-0\right)$ $-b\left(-b(1-\lambda)-0\right)=0$	$\mathbf{A} - \lambda\mathbf{I}$ must be correct for **M1**.
			$(1 - \lambda)(\lambda^2 - 2\lambda a + 1) = 0$ or $(1 - \lambda)((a - \lambda)^2 - a^2 + 1) = 0$ or $\lambda^3 - (1 + 2a)\lambda^2 + (1+ 2a)\lambda - 1 = 0$ or $-\lambda^3 + (1 + 2a)\lambda^2 - (1+ 2a)\lambda + 1 = 0$	**A1**	**1.1**	Must not be in terms of b for this **A1**. So do not condone $(1 - \lambda)(\lambda^2 - 2\lambda a + a^2 + b^2) = 0$ or $\lambda^3 - (1 + 2a)\lambda^2 + (1+ 2a)\lambda$ $- a^2 - b^2 = 0$ here. Must be an equation. Must reach characteristic equation either in standard (polynomial) form or factorised form.	
			Either $\lambda = 1$ from factorised form or observing $1^3 - (1 + 2a)\times 1^2 + (1 + 2a) - 1 =$ $1 - (1 + 2a) + (1 + 2a) - 1 = 0$ for any a, from subbing $\lambda = 1$	**A1** **[3]**	**2.1**	Condone cancelling out other factor to reach $\lambda = 1$. Must not be from selecting values for a and b. Condone error in their quadratic (in λ) in their other factor. Condone incorrect cubic in λ, f(λ), if f(1) = 0.	Equation could be in terms of a and b here so **M1A0A1** is possible.

	(a)	**(ii)**	**DR** $(1-\lambda) \neq 0 \Rightarrow \lambda^2 - 2\lambda a + 1 = 0$	**M1**	**1.1**	Reducing characteristic equation to a quadratic equation by removing the factor $(1-\lambda)$. Must not have b. Characteristic equation could be wrong. Should be equation but this can be implied by correct method of solution	Can be implied by sight of factorised form of characteristic equation and then unsimplified, correct solution or quadratic equation in completed square form $(\lambda-\alpha)^2 + 1 - \alpha^2 = 0$.
			$\lambda = \dfrac{-(-2a) \pm \sqrt{(2a)^2 - 4\times1\times1}}{2\times1} = a \pm \sqrt{a^2-1}$	**A1**	**1.1**	AG. Or by completing the square: $(\lambda - a)^2 + 1 - a^2 = 0$ $\lambda - a = \pm\sqrt{(a^2-1)}$ to AG. Must reach AG. Could be shown by substitution but $\pm$ must be correctly used. Allow one recovered slip in working. Condone confusion of a, b, c from quadratic formula with a and b in question if dealt with without error.	By substitution of AG: $\lambda^2 - 2a\lambda + 1$ $= \left(a \pm \sqrt{a^2-1}\right)^2 - 2a\left(a \pm \sqrt{a^2-1}\right) + 1$ $= a^2 + (a^2-1) \pm 2a\sqrt{a^2-1}$ $-2a^2 \mp 2a\sqrt{a^2-1} + 1$ $= 2a^2 - 1 - 2a^2 + 1 = 0$ Must be both roots but condone one root for **M1** only. Could be into characteristic equation if full details seen.
			$a \pm \sqrt{a^2-1} = 1 \Rightarrow \pm\sqrt{a^2-1} = 1 - a$ $\Rightarrow a^2 - 1 = a^2 - 2a + 1 \Rightarrow 2a = 2 \Rightarrow a = 1$ But $a \neq 1$ (given) so this is not possible. So other eigenvalues are not 1.	**B1** **[3]**	**2.1**	Complete argument but can come from $\lambda = a \pm \sqrt{a^2-1}$ (given). Or by contradiction from $\lambda = 1 \Rightarrow a = 1$ properly reasoned.	

Question	Answer	Marks	AO	Guidance	

(b)	**DR**
	If (a is real and) $a^2 > 1$:
	$\Rightarrow 1 - a^2 < 0 \Rightarrow b = \pm\sqrt{1-a^2}$ is non-real (or imaginary) so **A** has non-real elements.
	
	If (a is real and) $0 \leq a^2 < 1$:
	$\Rightarrow a^2 - 1 < 0 \Rightarrow \lambda = a \pm \sqrt{a^2-1}$ is non-real so (two of the) eigenvalues are non-real.
	
	If a is non-real then **A** has non-real elements.
		B1
		
		B1
		
		B1
			3.1a
			
			3.1a
			
			3.2a
				Ignore consideration of impossible cases (ie $\lvert a\rvert = 1$.)
				Condone just $a > 1$ for this mark.
				
				Must assert that λ is not real (ie just "$\sqrt{(1 - a^2)}$ is not real" is insufficient).
				
				B1B1B1 can only be awarded if *all* possibilities (real and non-real) for the value of a have been considered.
					In this question part the conclusion must be that **A** or λ is non-real. But if "imaginary" is used as synonymous with "non-real" in any conclusion then only penalise once. Condone use of "complex" for "non-real" throughout.
					
					Also, arguments must be complete (except where allowed by MS) and correct. Generalised conclusions drawn from specific values of a, b or λ gain no credit.

	Alternative method 1: $\lambda = a \pm \sqrt{a^2-1} = a \pm \sqrt{-b^2} = a \pm b\text{i}$ since $a^2 + b^2 = 1$ (given). If a is real and b is real (then $b \neq 0$ since $a \neq \pm 1$) so λ is non-real so (two of the) eigenvalues are non-real. Otherwise, either a or b (or both) is (are) non-real and so **A** has non-real elements.	**B1** **B1** **B1**		NB Beware the false argument that "therefore λ is non-real" (since this may be real if a and/or b are themselves non-real). **B1B1B1** can only be awarded if $b = 0$ is convincingly discounted.	or $\lambda = \pm\sqrt{1-b^2} \pm b\text{i}$. Allow left as $a \pm \sqrt{-b^2}$ if properly dealt with. If b is real (then $b \neq 0$ since $a \neq \pm 1$) and $-1 < b < 1$ then (two of the) eigenvalues are not real. Otherwise either $b^2 > 1 \Rightarrow a$ is not real or b is not real.
	Alternative method 2: If λ not real then some of the eigenvalues are not real. If λ is real: $\lambda = a \pm \sqrt{a^2-1} = a \pm \sqrt{-b^2} = a \pm b\text{i}$ since $a^2 + b^2 = 1$ (given)... ...so either $b = 0$ and a is real, which cannot be true since $b = 0 \Rightarrow a^2 = 1$, or either a or b (or both) are not real and so **A** has non-real elements.	**B1** **B1** **B1**		Allow left as $a \pm \sqrt{-b^2}$ if properly dealt with. Possibility of $b = 0$ must be explicitly eliminated.	Or "If λ is real and a is not real then some elements of **A** are not real"... "...and if λ is real and a is real then $a^2 > 1$ so b^2 is negative so b is not real so some elements of **A** are not real". NB $a^2 \geq 1$ is **B0** unless $a^2 = 1$ then discounted.

			Alternative method 3: By contradiction. Assume all eigenvalues are real and all elements of **A** are real. Since a and λ are real, either $a^2 > 1$ => (since $a^2 + b^2 = 1$) $b^2 < 0$ => b not real so contradiction... ...or $a = 1$ which is not possible (given). So our assumption must be wrong so either some eigenvalues are not real or some elements of **A** are non-real.	**B1** **B1** **B1**		Must be a proper conclusion. Possibility of $a = 1$ ($b = 0$) must be explicitly eliminated.	
				[3]			
	(c)	**(i)**	**DR** (Since **e** is an eigenvector of **A** its direction is unaffected by the rotation which means that) the axis of rotation must be in the direction of **e** (so **A** must represent a rotation about the z-axis). A rotation will not affect a vector in the direction of the axis of rotation and so the magnitude of **e** is unaffected by the transformation (so its associated eigenvalue must be 1). or The axis of rotation is a line of invariant points.	**B1** **B1** **[2]**	**2.1** **2.4**	ie an indication that **e** must be in the direction of the axis of the rotation that **A** is known to represent. Condone eg "**e** is the axis of rotation" ie an indication that in a rotation any vector in the direction of the axis of rotation must be invariant so the e-val must be 1. Not just "an invariant line".	For both marks, ignore true but irrelevant comments.

Question			Answer	Marks	AO	Guidance	
	(c)	(ii)	**DR** In a rotation (by an acute angle) the only axis which remains invariant is the axis of rotation; any other direction is changed by the rotation. (But other e-vals are not 1 so they must be in a different direction from the rotation axis.) So if another e-vec were real it would represent another axis which was invariant which is not possible.	**B1** **[1]**	**2.4**	Or "If there were another real eigenvector then, since its e-value is not 1, its length would change under **A** but this is not possible since **A** is a rotation. A 180° rotation could be associated with an e-val of –1 but this is an acute-angled rotation". or "In a rotation all other lines (apart from the axis) will move".	Ignore true but irrelevant or non-geometric comments. Condone eg "There is only one invariant line in a rotation" but not just "In a rotation there is only one axis of rotation."
	(c)	(iii)	**DR** $(\mathbf{e}=)\begin{pmatrix}0\\0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\0\\z\end{pmatrix}$ for any z. So equation of line is $\mathbf{r}=\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$	**B1** **B1FT** **[2]**	**1.1** **2.2a**	or any non-zero multiple. If correct, can be implied by 2nd **B1**. **FT** their non-zero, numerical **e** previously given as eigenvector. Condone use of λ as parameter. Condone eg $\mathbf{r}=\begin{pmatrix}0\\0\\1\end{pmatrix}+\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}0\\0\\0\end{pmatrix}+\mu\begin{pmatrix}0\\0\\1\end{pmatrix}$ or $\mathbf{r}=\mu\begin{pmatrix}0\\0\\2\end{pmatrix}$	**r** can be written $\begin{pmatrix}x\\y\\z\end{pmatrix}$.

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