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Examiners' report

AS LEVEL

FURTHER MATHEMATICS B (MEI)

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics B (MEI)

H635

For first teaching in 2017

Y413/01 Summer 2025 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y413 series overview

June 2025 was the sixth summer examination series for the Linear H635 AS Further Maths B (MEI) optional paper Y413 (Modelling with Algorithms). It was extremely pleasing to note that most candidates were very well prepared for the examination making good attempts on all parts of all questions. Candidates seemed to have enough time to answer the questions, and nearly all seemed to make careful use of the Printed Answer Booklet, with very few candidates needing to use the Additional Answer Space and not many instances of candidates using the incorrect space for their response.

Based on the responses seen by examiners in this series the following general points should be considered by centres in preparation for candidates for future sittings of this paper.

- Diagrams should be completed in pencil so that candidates can rub out and replace their answer rather than trying to correct in pen on what is likely to be a diagram. This was especially true in Question 1 (Prim's in tabular form), 3 (b) (i) (Dijkstra's algorithm) and 7 (d) (linear programming) in which it was difficult at times for examiners to know what work candidates were genuinely trying to correct or what was in fact their final attempt.
- This unit is primarily a methods examination and so it is vital that candidates make their method and application of any corresponding algorithm clear; spotting and writing down the solution will rarely gain any credit.
- The specification (Section 2b) contains a section on the meanings of several instructions that will be used in examination questions –
  - the word 'determine' in this mathematical context requires that justification should be given for any results found and not just the writing down or stating of the answer.
  - The bold statement '**In this question you must show detailed reasoning**' indicates that a full analytical method containing sufficient detail to allow the line of their argument to be followed.
- Candidates are reminded to use the number of marks available as the main guide to how detailed their answer should be and not the space given in the Printed Answer Booklet. It is always hoped that the PAB should provide enough space for the candidate's response, which often includes additional space so candidates can correct any errors.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• applied the corresponding algorithm correctly and showed detailed working/reasoning (and therefore did not rely on ad-hoc methods) • read questions carefully and provided the answers that were requested • made efficient use of their calculator.	• did not give sufficient detail to demonstrate that an algorithm had been used correctly and completely • did not provide sufficient detail in questions that used the defined question 'command words'. used imprecise mathematical notation or incorrect language, especially for questions that required an explanation • did not check that the response provided the actual answer to the question set.

Question 1

1 A network has ten vertices, A to J. The table shows the distances between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I	J
A		12	6		7		8			2
B	12		5	14		9				
C	6	5					10			7
D		14						3		8
E	7					6			4	9
F		9			6					
G	8		10						5	
H				3						11
I					4		5			
J	2		7	8	9			11		

Apply the tabular form of Prim's algorithm to the network, starting at A, to find a minimum spanning tree for the network.

Your solution should contain the following.

- The order in which the arcs are selected
- The total length of the arcs in the minimum spanning tree **[4]**

This part was answered extremely well with most candidates correctly applying the tabular form of Prim's algorithm and stating all nine arcs (in the correct order) that formed the required MST. Occasionally, candidates only stated the nodes in order (so losing 1 mark) or simply drew the MST (which, as it was unclear whether a correct (or any) algorithm had been applied, scored no marks). Of those that had the correct arcs in the MST most correctly stated the corresponding total length.

Question 2 (a) (i)

2 The list below shows the sizes of eleven items.

28 25 19 32 18 22 3 12 20 7 5

(a) (i) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

This part was answered extremely well with most candidates giving the correct 4 bin solution. The most common errors were to place the 18 in Bin 3 rather than Bin 2 or to place the 5 in Bin 4 rather than Bin 3.

Question 2 (a) (ii)

(ii) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

Like part (a) (i), candidates were well prepared at performing first fit decreasing on a correctly sorted list. Occasionally examiners reported that some candidates did first fit increasing (and so scored no marks), other common mistakes were placing the 22 in Bin 3 or not placing the 18 in Bin 1.

Question 2 (b)

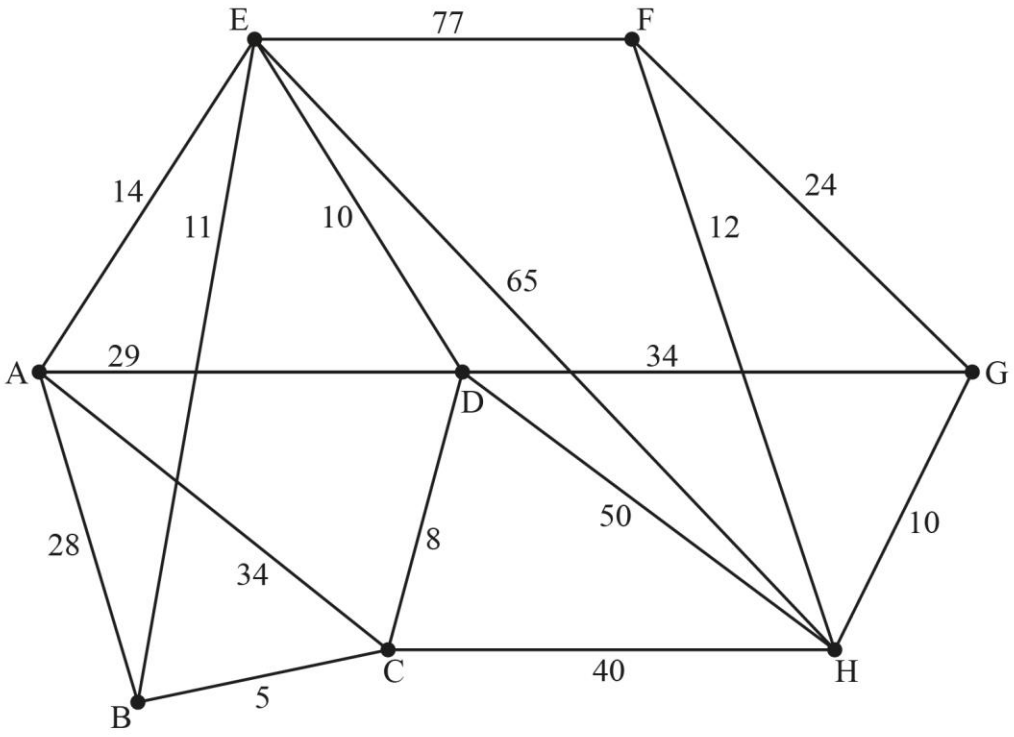
(b) A computer takes 4.7×10^{-8} seconds to pack 50 items with sizes 100, 99, 98, … , 53, 52, 51 into bins that have a capacity of 100 using the first fit decreasing algorithm.

Calculate approximately how long it will take the same computer to pack 1 000 000 items with sizes 2 000 000, 1 999 999, 1 999 998, … , 1 000 003, 1 000 002, 1 000 001 into bins that have a capacity of 2 000 000 using the first fit decreasing algorithm. **[2]**

It was pleasing to note that most candidates recognised that in this worst case, the first fit decreasing algorithm has quadratic complexity (so scoring at least 1 mark in this part), and of those that did recognise this most then went on to correctly find the required time.

Question 3 (a)

3 The diagram shows a network of roads. The number on each arc represents the length, in miles, of the corresponding road.



Finley, who lives at F, needs to drive to a friend's house at A before they travel together to a campsite situated at C.

Finley decides to use Dijkstra's algorithm **once** to find the shortest route from F to C via A.

(a) Explain why vertex A should be chosen as the starting vertex for the algorithm. **[1]**

Almost all candidates recognised the fact that the shortest route from F to C via A was equivalent to finding the shortest path from A to C and also the shortest path from A to F, and as A appears in both paths then this should be the starting vertex for Dijkstra's algorithm.

Question 3 (b) (i)

(b) (i) On the copy of the network in the Printed Answer Booklet, apply Dijkstra's algorithm once to find the length of the shortest route from F to C via A. **[5]**

Many candidates scored full marks in this part. Centres should note that it is vital when applying Dijkstra's algorithm that candidates show all the working values at each node (in this case examiners had to see the correct working values of 29 and 24 at D to indicate that the algorithm was initially being applied correctly) and that the order of labelling is correct (a small number of candidates labelled two different nodes with the same label). It was slightly disappointing that several candidates correctly applied the algorithm but then did not state the shortest path from F to C via A (with some just stating the value of 80 which was the length from A to F only).

Assessment for learning

The Printed Answer Booklet provided the standard key for the completion of the boxes at each node together with a specific instruction to candidates that they should not cross out their temporary labels. Several candidates still did so. It is vital that examiners can read all the values at each node (so that they can check that the algorithm has been applied correctly) and it is strongly recommended that teachers' emphasis that no temporary labels are crossed out in classroom practice.

Question 3 (b) (ii)

(ii) State the corresponding shortest route from F to C via A. **[1]**

While many candidates correctly stated the shortest route from F to C via A, examiners noted that the incorrect route of FHGDEABC was seen quite often.

Question 4 (a)

4 (a) Determine the number of nodes in the complete graph with 1431 arcs. **[2]**

This part was answered well with many stating the correct equation $\frac{1}{2}n(n-1) = 1431$ and then solving it correctly to find the answer of 54. The two most common errors were either stating the incorrect equation $\frac{1}{2}n(n+1) = 1431$ or substituting 1431 for n.

Question 4 (b)

(b) In this question you must show detailed reasoning.

A simply connected graph G has eight nodes of orders 1, 1, 2, 3, 3, 4, 4 and x.

- Determine the **three** possible values of x.
- Use the nodes in the Printed Answer Booklet to draw an example of G for each of these **three** values of x. **[6]**

This part proved to be the most challenging on the paper. While some candidates realised that the possible values of x were 2, 4 and 6 (and could draw an example of each of these three graphs) only the most proficient were able to supply suitable detailed reasoning of why x could only take these three values.

Exemplar 1

Simply connected : no loops

∴ Σ orders is even

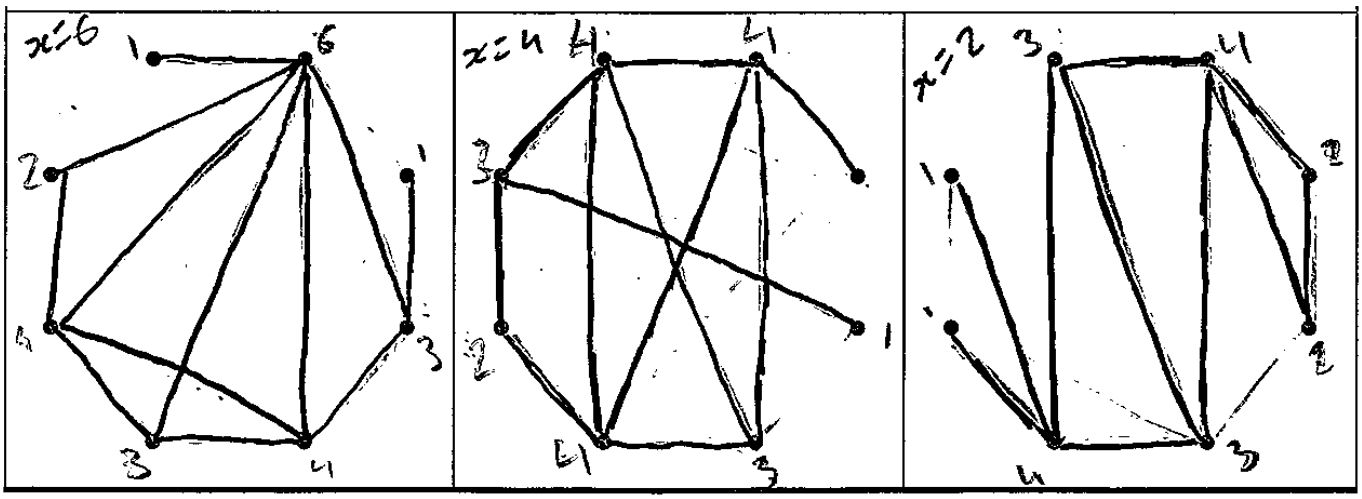
$1+1+2+3+3+4+4+x = 18+x$

∴ x is even

x cannot be greater than or equal to 7

$0 \leq x \leq 7$

∴ $x = 2, 4$ or 6

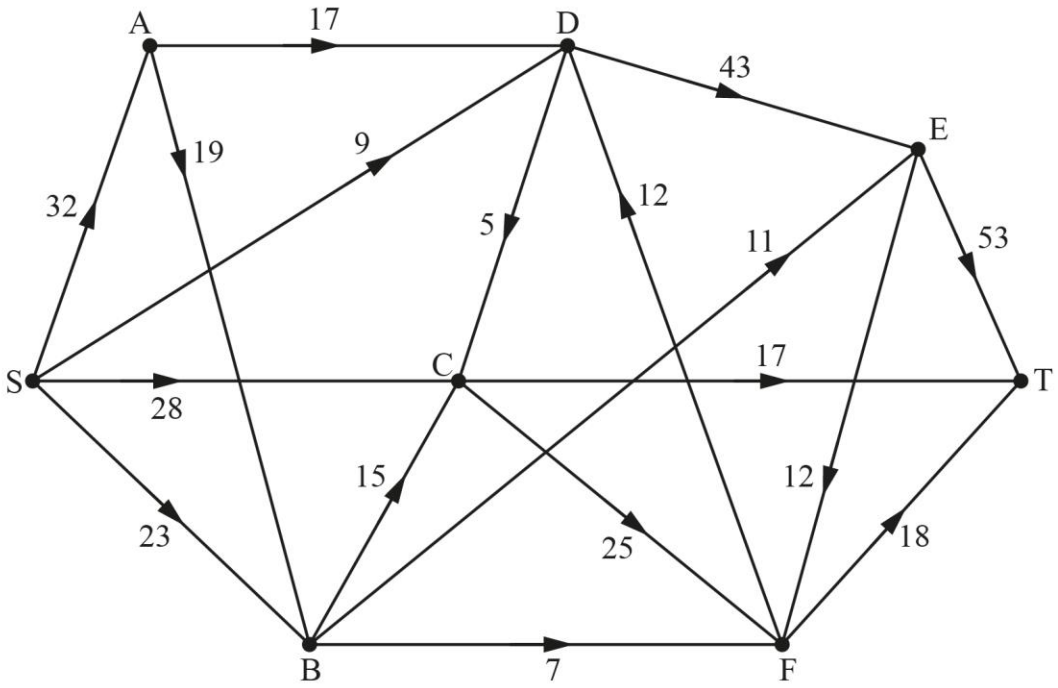


This question part has the command '**In this question you must show detailed reasoning**' so the examiner is expecting to see sufficient detail of a complete analytical method. This exemplar highlights that whilst this candidate has clearly stated why x must be even and gone on to state all three correct values of *x* with three corresponding correct graphs, they only scored 4 of the 6 marks available. The 2 marks that could not be awarded were for:

- no explicit justification given for why *x* could not be zero
- no clear explanation of why *x* must be less than or equal to 7.

Question 5 (a)

5 The diagram represents a system of pipes through which water flows continuously from a source S to a sink T. The weights on the arcs show the capacities of the pipes in litres per minute.



(a) Explain why the maximum possible flow along DE must be less than 43 litres per minute. **[1]**

Most candidates correctly stated that the maximum amount that could flow into D (or along DE) was 38 and therefore the maximum possibly flow along DE had to be less than 43.

Question 5 (b)

(b) The cut α partitions the vertices into the sets {S, A, B, C, D, F}, {E, T}.

Calculate the capacity of cut α. **[1]**

Almost all candidates correctly stated the capacity of the cut as 89.

Question 5 (c)

(c) Explain why partitioning the vertices into sets {S, A, D, E, T}, {B, C, F} does **not** give a cut. **[1]**

It was very pleasing in this part that most candidates realised that this was not an example of a cut due to the fact both the source (S) and sink (T) appeared in the same set (or argued this point in a similar manner, e.g. 'the cut doesn't separate the source and the sink').

Question 5 (d) (i)

An LP formulation is set up to find the maximum flow through the system.

(d) (i) Write down the required constraint in the LP formulation regarding the flow through vertex D. **[1]**

This part was answered extremely well although some candidates did not realise the subtle difference in an LP formulation between the correct answer of 'SD + AD + FD – DC – DE = 0' and an incorrect answer of 'SD + AD + FD – CD – ED = 0'.

Misconception

In any LP formulation it is vital that the order in which the letters are used is correct and consistent with any information provided in the question. Some candidates had no hesitation in using CD even though in this context it had to be stated as DC (as dictated by the direction of the flow through the corresponding pipe).

Question 5 (d) (ii)

(ii) Write down the required constraint in the LP formulation regarding the flow along arc AD. **[1]**

This part was answered extremely well too, although some candidates incorrectly used a strict inequality or included zero as a lower bound (which is not required in an LP formulation).

Question 5 (e) (i)

The LP formulation for the system was run in a solver and some of the output is shown in the table.

Variable	Value
SA	32.000 000
SC	28.000 000
SB	15.000 000
BC	12.000 000
DC	0.000 000
DE	38.000 000
ET	49.000 000

(e) (i) Complete the diagram in the Printed Answer Booklet to show the maximum flow through the system. **[2]**

Although this part was answered well, future candidates are reminded of the two conditions when tackling this type of question. They are the feasibility condition (which says that the flow along each arc must not exceed the capacity of that arc) and the conservation condition (which says that apart from the source (S) and sink (T) nodes the total flow into a node must equal the total flow out of that node).

Misconception

An arc that has no flow passing through it, such as DC or EF in this part, should be labelled with a 0 and not simply left blank.

Question 5 (e) (ii)

(ii) State the maximum value of the flow through the system. **[1]**

Many candidates correctly stated the maximum flow as 84.

Question 5 (f)

(f) Use a suitable cut to prove that this is the maximum flow. **[2]**

The responses to this part were mixed and while many candidates did give a correct proof, many candidates did not either state a correct cut with capacity 84 or they did not give sufficient detail in proving that this flow was indeed maximal.

Assessment for learning

Proving a flow is maximal:

Step 1: state a cut, based on the capacity of saturated arcs directed from source to sink and arcs with zero flow directed from sink to source, as either a set of nodes or as a list of arcs which the cut passes through, and conclude that therefore the minimum cut is less than or equal to this value.

Step 2: state the value of the current flow through the network and conclude that therefore the maximum flow is greater than or equal to this value.

Step 3: if these two values are the same then by the maximum flow-minimum cut theorem a maximum flow for the network has been found.

Misconception

When calculating the capacity of a cut candidates must use the capacity of the arcs and not the current flow which is passing through those arcs (otherwise all cuts for the network would have the same capacity).

Question 5 (g) (i)

The capacity of pipe SD is increased to x litres per minute. In doing so, the maximum flow through the system is increased as much as possible. The capacities of the other pipes in the system remain unchanged.

(g) (i) Find the least possible value of x. **[1]**

Generally, only those candidates in the upper quartile of the grade distribution found the correct answer (of 13) in this part with both 5 and 19 being common incorrect answers.

Question 5 (g) (ii)

(ii) State the corresponding maximum possible flow through the system. **[1]**

Many more candidates correctly stated the maximum possible flow (88) in this part than found the least possible value of x in the previous part.

Question 6 (a)

6 Consider the following LP problem in X and Y.

Maximise $Q = -8X + 9Y$

Subject to

$$-5X + 6Y \leqslant 87$$
$$2X + Y \leqslant 6$$
$$-3X - Y \leqslant 16$$
$$X \leqslant -2, Y \geqslant 0$$

(a) Explain why the simplex algorithm cannot be used to solve this LP problem. **[1]**

This part was answered well with many candidates either stating that the origin was not in the feasible region (or that $X = Y = 0$ was not a basic feasible solution of the feasible region) or mentioning that X could take negative values.

Question 6 (b)

The LP problem is reformulated into standard form. The reformulated problem consists of a linear objective function P (that is to be maximised) and constraints in terms of x and y.

The tableau **after** the first iteration of the simplex method, is given in **Table 6.1**.

Table 6.1

P	x	y	s_1	s_2	s_3	RHS
1	−26	0	0	9	0	90
0	17	0	1	−6	0	17
0	−2	1	0	1	0	10
0	1	0	0	1	1	20

(b) State the column that contains the pivot value for the first iteration. You must give a reason for your answer. **[2]**

It was clear than many candidates had not read this part carefully and gave an answer of x together with an explanation that made some mention of 'negative value in the objective row' – these candidates had clearly thought that the question was asking about which column the next pivot value should come from and not which column the pivot value for the first iteration came from. Of those that did read the question carefully most gave the correct answer of y together with a correct explanation that y was now a basic variable.

Question 6 (c)

(c) Perform the second iteration of the simplex method. Give each entry as an exact value. **[3]**

Candidates overall were very proficient in carrying out the simplex method and many identified the correct pivot value (the 17 in the *x*-column) and used this to carry out the iteration as requested. The most common errors were sign errors on certain values or difficulty in dealing with fractions. There were a significant minority who pivoting on an incorrect value and some who used a negative value as a pivot.

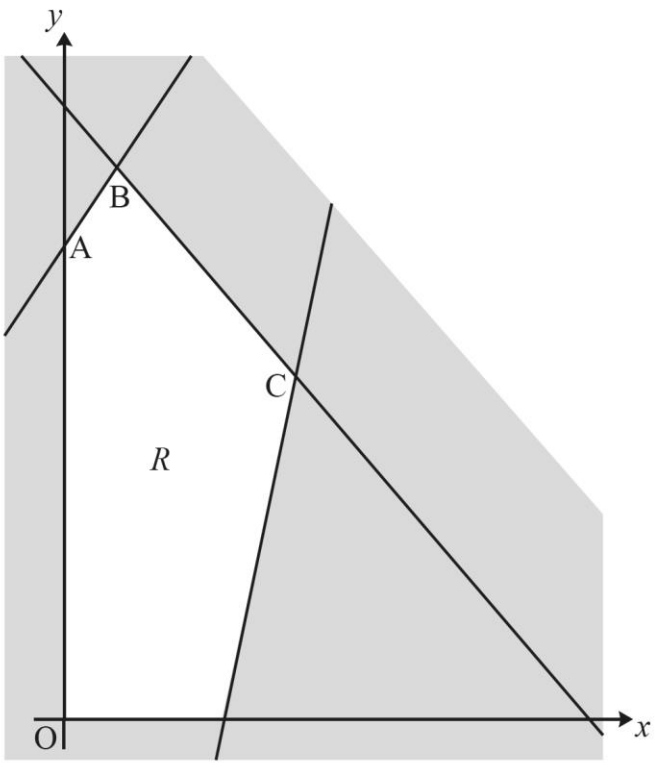
Question 6 (d)

The tableau after a third iteration of the simplex method, is given in **Table 6.2**.

Table 6.2

P	x	y	s_1	s_2	s_3	RHS
1	0	0	$\frac{35}{23}$	0	$\frac{3}{23}$	$\frac{2725}{23}$
0	1	0	$\frac{1}{23}$	0	$\frac{6}{23}$	$\frac{137}{23}$
0	0	1	$\frac{3}{23}$	0	$-\frac{5}{23}$	$\frac{181}{23}$
0	0	0	$-\frac{1}{23}$	1	$\frac{17}{23}$	$\frac{323}{23}$

The diagram shows the constraints of the reformulated LP problem. The feasible region, R, is the unshaded region together with its boundaries. Three vertices, A, B, and C, of R are also shown.



(d) Write down the coordinates of A, B and C. **[3]**

This part proved challenging with few candidates realising that the solutions after each of the three iterations of simplex provided the coordinates of A, B and C (in that order).

Assessment for learning

Students should consider the geometrical nature of simplex alongside its tabular form.

Question 6 (e)

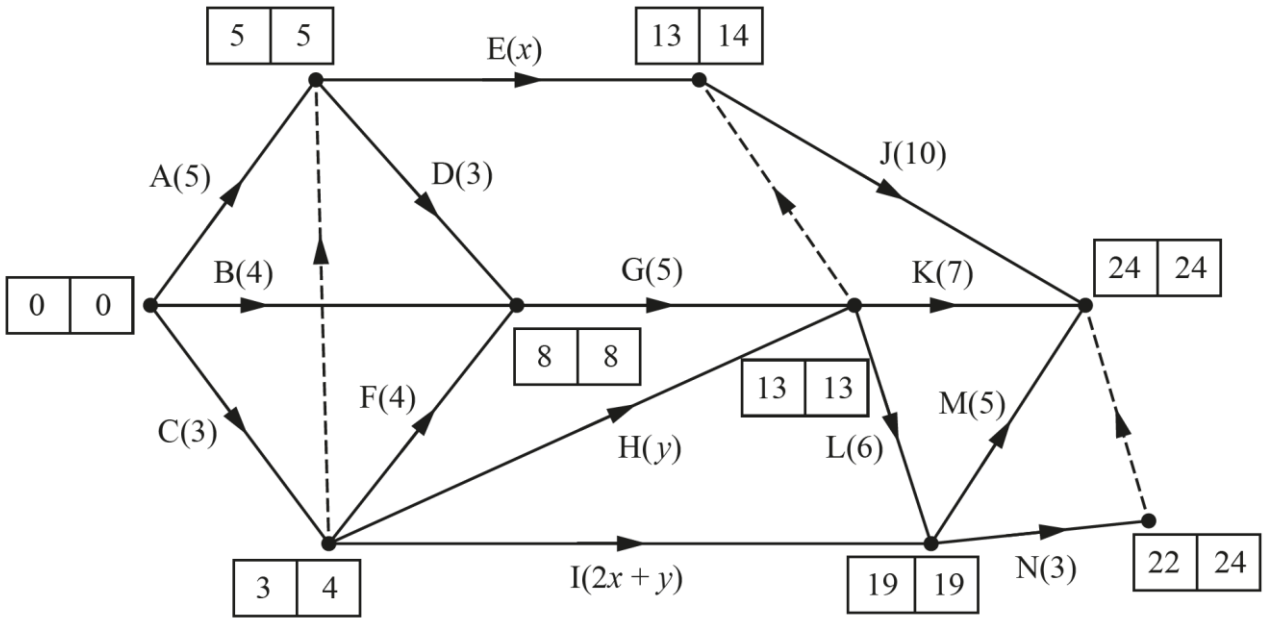
You are given that the reformulation of the original LP into standard form was done using the substitutions $X = -2 - x$, $Y = y$ and $Q = P + 16$.

(e) Determine the maximum value of Q, and the corresponding values of X and Y. **[2]**

However, it was pleasing that even though most candidates struggled or left part (d) blank, most persevered and used the final tableau in Table 6.2 to correctly calculated the three required values in this part.

Question 7 (a)

7 The diagram shows an activity network for a project. Each activity is represented by an arc. The expressions in brackets show the duration of these activities in hours. The early event times and late event times are shown at each vertex.



(a) Complete the tables in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

This part was answered extremely well with nearly all candidates scoring at least 1 mark for correctly completing six rows of the table. The most common error was due to the dummy activity at the end of G (and so falling to include G and/or H in the immediate predecessors for activity J).

Question 7 (b) (i)

(b) (i) State the minimum completion time for the project. **[1]**

Almost all candidates correctly stated the minimum completion time as 24 hours.

Question 7 (b) (ii)

(ii) State the critical activities of the project. **[1]**

While many candidates correctly stated the critical activities as A, D, G, L and M, examiners commented that several candidates also included B (and occasionally K) too.

Question 7 (c)

A student is tasked with finding the durations of activities E, H and I. The student knows the following information regarding the activities in the project.

- The sum of the durations of all the activities in the project is at least 79 hours.
- The durations of activities E, H and I must satisfy the early event times and late event times shown in the activity network.
- The combined duration, in hours, of activities E and H must be as small as possible.

(c) Formulate the student's task as an LP problem in x and y. **[4]**

The responses to this unfamiliar part were mixed; some candidates provided a perfect LP formulation of the problem while others seemed to struggle and were unsure about where to begin. Many although correctly used the sum of the durations being at least 79 hours to derive the constraint $3x + 2y \geq 24$. While many correctly realised that the objective was given by the expression $x + y$ not all stated that this expression needed to be minimised. Finally, while most candidates who attempted this part considered the forward and backward passes for activities E, H and I, only around 20% of the cohort successfully considered both passes correctly and obtained the three constraints $x \leq 8,\ \ y \leq 9,\ \ 2x + y \leq 15$.

Question 7 (d)

(d) By representing the feasible region for x and y graphically, find the durations of activities E, H and I. **[4]**

While most candidates could represent graphically one (or possibly two) of the correct constraints from part (c), only a small proportion of the cohort had all four correct lines and corresponding correct feasible region in this part. The final mark for the correct durations of activities E, H and I was dependent on both a complete LP formulation in part (c) and a correct graph in this part.

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