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Examiners' report

AS LEVEL

FURTHER MATHEMATICS B (MEI)

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics B (MEI)

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y410/01 series overview

This proved to be an accessible paper with a mean mark of 45 and a median of 48 from a total 60 marks available. There were many excellent, well-presented scripts from well prepared candidates. All candidates completed the paper in the time allotted. Questions 6, 7 and 8 proved to discriminate well between candidates. There were very few scripts seen which scored less than 30 marks.

The instruction '**In this question you must show detailed reasoning**' features for the whole of Question 2 and Question 5, and in the final part (b) of Question 7 and Question 8. This is not a prohibition on calculator use but highlights to candidates that a full written solution is required that shows a detailed and complete analytical method.

OCR support

Full details of the command words used in questions can be found in the specification. A useful poster can be found on the qualification webpages.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• displayed a good knowledge of the whole specification • presented accurate work throughout • offered clear explanations of key concepts • used multiplication of complex numbers in modulus-argument form • showed good used of mathematical notation in Question 6 (b) • translated loci in the Argand diagram into cartesian equations accurately in 7 (b) • spotted the relevant information to solve Question 8 (b).	• displayed some gaps in their knowledge and understanding of the specification • made errors in algebra or numerical work • were unable to explain key concepts • scored poorly on Questions 6, 7 and 8.

Question 1 (a) and (b)

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}.$$

(a) Use **A** and **B** to show that matrix multiplication is **not**, in general, commutative. **[2]**

(b) Verify that **A** and **B** satisfy $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. **[3]**

This proved to be a straightforward starter question. Nearly all candidates gained full marks in part (a) by evaluating **AB** and **BA** and stating they were not equal. In part (b), we required a clear statement of the result to award the final B1 mark. Occasionally, candidates misunderstood the question by reproducing the proof of the result.

Question 2

2 **In this question you must show detailed reasoning.**

Find the acute angle between the vector $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and the normal vector to the plane $2x+3y+z=6$. **[4]**

This question was answered correctly by most of candidates. Detailed reasoning was found in all of these solutions. This required evidence of the formula for the angle between vectors, and correct calculation of the modulus of each vector.

Question 3 (a) and (b)

3 The matrices **M** and **N** are given by

$$\mathbf{M} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix} \text{ where } a \text{ and } b \text{ are positive constants.}$$

(a) Given that $\mathbf{M}^2 = \mathbf{N}$, determine the exact values of a and b. **[4]**

(b) Hence state the transformations of the plane associated with matrices **M** and **N**. **[3]**

This routine problem was answered well by most candidates. Provided part (a) was correct, the matrices were usually correctly identified as rotations.

Question 4 (a)

4 **(a)** The transformation T is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix}$.

A shape S_1 is mapped to a shape S_2 by the transformation T.

Show that volume of S_1 is the same as the volume of S_2. **[2]**

Nearly all candidates calculated the determinant of the given matrix, but not all candidates went on to 'show that' the volumes were the same. Reference needed to be made to the volume scale factor to earn the second mark.

Question 4 (b) (i)

(b) Three planes have equations

$$\begin{aligned} x - 2y + 2z &= \lambda, \\ 2x + y \phantom{{}+2z} &= 2, \\ x + 2y - z &= 0, \end{aligned}$$

where λ is a constant.

(i) Explain why the three planes intersect at a point for any value of λ. **[2]**

Some candidates pointed out that the planes were not parallel. However, this was not sufficient to make sure of a unique solution. Most recognised the connection with part (a). They were then required either to state that the determinant was non-zero, the matrix non-singular, or that the inverse matrix existed.

Question 4 (b) (ii)

(ii) Use a matrix method to determine, in terms of λ, the coordinates of this point. **[4]**

This question was extremely well done, using matrix operations on a calculator. It was pleasing to see that there were very few attempts which did not use matrix methods.

Question 5 (a) (i)

5 In this question you must show detailed reasoning.

The complex number w is given by $w = -4\sqrt{2} + (4\sqrt{2})\mathrm{i}$.

(a) (i) Find $|w|$. **[2]**

This was well answered, with the correct method for calculating the modulus shown.

Question 5 (a) (ii)

(ii) Find $\arg(w)$. **[2]**

Most understood the argument as the arctan of Im(*w*)/Re(*w*), but some were unable to deduce the correct second quadrant angle from the Argand diagram.

Question 5 (b) (i)

The complex numbers z_1 and z_2 are given by $z_1 = a + \mathrm{i}$ and $z_2 = 4(\cos\theta + \mathrm{i}\sin\theta)$, where a is a positive real constant and $-\pi < \theta \leqslant \pi$.

(b) You are given that $z_1 z_2 = w$.

(i) Find the exact value of a. **[3]**

This was usually answered correctly using the modulus-argument forms of z_1 and z_2. Otherwise, there were occasional attempts to equate real and imaginary parts as shown in the alternative method in the mark scheme, but this required a lot more work and rarely gained much credit.

Question 5 (b) (ii)

(ii) Find the value of θ. Give your answer as an exact multiple of π. **[3]**

Again, knowledge of multiplication on modulus-argument form was important to answering this question– we allowed some follow through from their answer to 5 (a) (ii). Other methods rarely succeeded.

Question 6 (a)

6 **(a)** Express $\dfrac{1}{(r-1)^2} - \dfrac{1}{(r+1)^2}$ as a single simplified fraction. **[2]**

Most candidates combined the fractions correctly. However, some expanded the denominator incorrectly (and needlessly) and did not gain an accuracy mark. The connection with part (b) was also lost.

Question 6 (b)

(b) Hence determine the limit which $\displaystyle\sum_{r=2}^{n} \frac{r}{(r-1)^2(r+1)^2}$ converges to as $n \to \infty$. **[5]**

This question proved to be a bit more demanding, with some candidates getting it fully correct. Most recognised the method of differences but omitted the factor of ¼ required. There were some notational errors, for example using '*r*' instead of '*n*', or using 'infinity' signs. The limit argument was sometimes missing, often because students combined the fractions in *n* into a single fraction, whose limit was less obvious.

Assessment for learning

When using sigma notation, it is important to emphasise the difference between the variables *r* for the *r*th term and *n* for the last term.

Misconception

The use of the 'infinity' symbol, rather than a valid limit argument, did not gain marks in this question.

Question 7 (a)

7 The region R of the Argand diagram consists of the set of points representing complex numbers z which satisfy the following inequalities.

$\operatorname{Im}(z) \geqslant 0 \qquad \arg(z+2) \leqslant \frac{1}{4}\pi \qquad |z| \leqslant |z-4-2\mathrm{i}|$

(a) Sketch and clearly label the region R on an Argand diagram. **[4]**

It was pleasing to see many neat and accurate sketches of these loci. The second locus sometimes showed the full line rather than a half line, and some circles were seen for the third inequality. Some of the shading of the intersection was unclear – shading out the points outside the loci is perhaps clearer.

Misconception

? Some students confused the third locus with that of a circle such as $|z| < a$.

Question 7 (b)

(b) **In this question you must show detailed reasoning.**

Find the largest value of $\operatorname{Im}(z)$ in the region R. **[7]**

This question discriminated the better candidates, but there were plenty of fully correct solutions. Some good solutions did not gain a mark by referring the maximum value of Im(z) as 3i rather than 3.

Question 8 (a)

8 The three distinct roots of the equation $z^3 - 4z^2 + pz + q = 0$, where p and q are real, are drawn on an Argand diagram. The three points which represent these roots do not lie on a straight line but instead form a triangle T.

(a) Show that T is isosceles. **[3]**

Most solutions mentioned the two complex conjugate roots for a mark, but to gain the first mark we needed to see the case of three real roots ruled out as they would not form a triangle, and the third mark required a convincing argument why the triangle is isosceles, for example saying the two complex conjugate roots are equidistant from the real axis and the real root lies on the real axis. However, we accepted sketches which implied this.

Exemplar 1

8(a) ~~two roots will be~~
two roots will be imaginary with one being z and the other being z^*, it's conjugate. They would have the same real value but one of them would have a negative imaginary value and the other positive causing them to be ~~an equalt~~ equidistant from the third root.

In this exemplar script, no mention is made of the three real roots case, so the first B1 mark cannot be awarded. A diagram would have helped to secure the final A mark, but there is no mention of the other point lying on the real axis, or of the complex conjugate roots being equidistant from the real axis, so the A1 cannot be awarded on this script.

Question 8 (b)

(b) In this question you must show detailed reasoning.

You are given the following information.

- The area of T is 10 square units.
- One of the roots of the equation $z^3 - 4z^2 + pz + q = 0$ is $z = -2$.

Find the other roots of the equation. **[5]**

The factor theorem on *z* = -2 is not particularly helpful in this case, and neither are the product of the roots, in pairs or all three, as these yield equations in *p* and *q*. So, success here relied on picking out the relevant equations, using the complex conjugate pair and the sum of roots, and then the area of the triangle. This proved to be a good discriminator of more able candidates.

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