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Examiners' report

AS LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A

H235

For first teaching in 2017

Y533/01 Summer 2025 series

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y533/01 series overview

This is one of the optional modules for AS Further Maths, alongside the compulsory Pure Core, and the other optional modules in Statistics, Discrete Mathematics and Additional Pure Mathematics. This component complements the Mechanics content of the AS Mathematics course and requires candidates to solve practical problems using a variety of techniques including energy, power, momentum, connected particles, horizontal and vertical circles, and dimensional analysis.

To do well on this paper, candidates needed to be able to work systematically and accurately with the standard principles of Mechanics. They will often need to make sense of more complex situations, for example, correctly combining several forces and/or types of energy in a variety of situations, such as two or more sources of gravitational potential and/or kinetic energy and resistance. The ability to tackle multi-stage problems is also required for some questions, together with mathematical modelling and application of a derived model.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• made good use of diagrams to help them understand a problem before solving it • were consistent in defining the directions of different velocities and/or forces before attempting to solve a problem • recognised the different forces that were acting and when they needed resolving • understood the different quantities in mechanics, such as work, kinetic and potential energy, momentum, impulse, power, angular velocity, and the correct formula required to work with each of them, together with the associated dimensions • were able to use Newton's Second Law (F = ma) confidently together with the relationship between Force and energy • were efficient and effective in applying their knowledge, always choosing the simplest possible route to the solution • understood the concept of magnitude in the context of vector quantities • were able to explain concepts and models clearly relating them to real life.	• did not have an effective strategy for managing the different directions that objects might be travelling, especially in impulse/momentum and collisions questions, resulting, for example, in sign errors • did not identify all the relevant quantities required to solve a problem • did not recognise where a problem involved two separate stages in the same question • made avoidable errors in calculations and algebraic manipulation • were unable to communicate and explain concepts accurately.

Question 1 (a)

1 A cricket ball of mass 0.16 kg is at rest on rough horizontal ground when it is struck by a bat. Immediately after the ball is struck its speed is $15\,\mathrm{m\,s^{-1}}$ in a horizontal direction.

(a) Find the magnitude of the impulse exerted by the bat on the ball. **[1]**

Almost all candidates attempted this starter question on impulse and were able to find the correct answer. Generally, those that lost the mark did so due to slips and numerical errors rather than any other reason, for example, by using 16 kg or 0.6 kg instead of 0.16 kg. Others simply calculated the answer wrong, by entering 0.16 x 0.15 on the calculator and obtaining 0.024, having written 0.16 x 15 on their paper, or miscopied their own work with 1.6 becoming 0.6 on the next stage of working. Newton's 3rd Law was also seen incorrectly applied, with a response that claimed that it was 0 Ns in either direction. There were also cases where candidates did not use the correct units, but these were not penalised in this instance.

Good exam practice

Candidates are urged to double check their answers against both the information in the paper and their own written work to avoid losing marks unnecessarily.

Question 1 (b)

The ball remains in contact with the ground and the resistance to the motion of the ball is modelled as constant.

The ball comes to rest again after it has travelled 30 m along the ground.

(b) Use the work-energy principle to find the magnitude of the resistance to the motion of the ball. **[3]**

Again, this question on work and energy was attempted by almost all candidates. Most candidates obtained full marks, but a small proportion did not, instead generally scoring one mark only. Common issues were that candidates did not equate the work done against friction with the initial kinetic energy or equated it with the impulse instead, while some tried to account for a non-existent driving force. Some multiplied by 30 instead of dividing or calculated 30 ÷ 18 instead of 18 ÷ 30 after obtaining the correct equation. Some even included the impulse found in part (i) in their force-energy equation, while others started correctly but did not complete the subsequent calculations.

There were some candidates who did not follow the instruction to use an energy method and instead used their prior knowledge of *suvat*, for which was allowed a maximum of 1 mark. In very rare cases, candidates introduced considerations like an element of gravitational potential energy, or conflated resistance with energy.

Question 2 (a)

2 Two particles, A of mass m kg and B of mass $2m$ kg, are connected by a light inextensible string. The particles are initially placed together at rest on a smooth horizontal surface with the string slack.

At a certain instant, A and B are projected along the surface, directly away from each other, with speeds of $3\,\mathrm{m\,s^{-1}}$ and $u\,\mathrm{m\,s^{-1}}$ respectively. You are given that $u < 1.5$.

(a) Find, in terms of u, the speed of A immediately after the string becomes taut. **[2]**

For this question, candidates were required to use Newton's 3rd Law in the context of conservation of momentum to obtain a final speed. This question caused difficulties for some candidates who did not fully understand the setup with A and B moving apart but connected by a string. Therefore, they did not always realise that because the string was taut, the post-'collision' velocity would be the same (non-zero) value for both.

Some candidates initially assumed separate velocities, but then equated the two values, adding extra work and time that they could have used elsewhere, while some responded by assuming that the velocity before or after was zero. Others assumed that there were two different velocities and were unable to obtain an answer from their momentum equation. This led some candidates to formulate a second equation using elasticity or kinetic energy but this involved making assumptions about e or velocity. Others unsuccessfully attempted alternative algebraic methods to find a solution.

Question 2 (b) (i)

(b) (i) State the direction of motion of B immediately after the string becomes taut. **[1]**

The majority of candidates gained this mark, realising that the greater momentum of A would outweigh the momentum of B and pull it in A's direction. A small minority did not answer the question, while others, having misinterpreted the setup, thought that A and B were moving away from each other, possibly somehow starting from an initial velocity of zero. Where candidates used the terms 'left' or 'right', these need to be qualified with a suitable diagram relating them to the directions of A and B. Interestingly, a not insignificant number of candidates relied on the value of v_A $(= v_B)$ to decide which way B was travelling.

Question 2 (b) (ii)

(ii) Find, in terms of m and u, the magnitude of the impulse exerted on B as a result of the string becoming taut. **[2]**

This question was found to be challenging to many candidates, despite it being the application of the formula for impulse, i.e. $\Delta mv = mv - mu$, or $m(v - u)$. A small proportion of candidates scored full marks.

A number of candidates managed to reverse the formula, calculating $-\Delta mv$ instead, which did not gain the accuracy mark unless there was also an explanation for why they needed to reverse the sign of their answer. A substantial minority also made individual sign errors, most commonly with the sign of u_B $(= -u)$ not being reversed when calculating the change of momentum, so effectively calculating $m(v_B + u_B)$ instead and losing the accuracy mark.

Another common error was to ignore the instruction to find the magnitude. Those candidates who assumed that A's direction was positive due to its greater momentum were at an advantage here, since the tension in the string naturally transmitted the impulse on B in the (positive) direction of A, while those who assumed B's direction was positive then had to find the magnitude separately, assuming that they obtained the correct negative answer. Aside from these issues, some candidates also experienced difficulties with simplifying the expression for the impulse once they had formulated it.

Application of the concept of impulse and magnitude

It is recommended that centres make sure candidates have a solid foundational knowledge of this basic formula and provide plenty of examples which include algebraic terms and negative velocities, to reinforce candidates skills in this key area.

Exemplar 1

2(b)(ii)	ΔmV
	$\cancel{2m}$ $2m$ u
	$3m$ $\frac{2}{3}u-1$
	$m \times -\frac{1}{3}u-1$
	$-\frac{m}{3}u-m$

This candidate demonstrates a very unusual approach to impulse, with little explanation, resulting in a method error. In 2 (a) they had previously found the correct value of $v_A = \frac{2}{3}u - 1$, based on B initially moving in the positive direction. The first line with 2m and u appears to refer to the mass and initial velocity of B, i.e. m_B and u_B, which is $2m$ and u, while the second line appears to refer to the total mass and velocity after the collision, i.e. $(2m + m) = m_A + m_B$ and $v_A = \frac{2}{3}u - 1 = v_B$. On the next line they have then stated the mass of $A\ (= m)$, and multiplied this by the post-collision velocity of A $\left(= \frac{2}{3}u - 1\right)$ minus the initial velocity of B $(= u)$, i.e. $\left(\frac{2}{3}u - 1\right) - u\ \left(= -\frac{1}{3}u - 1\right)$. This velocity is the same as $(v_A - u_B)$ or $(v_B - u_B)$ (since $v_A = v_B$), giving an expression for $m_A(v_B - u_B)$ or $m_A v_B - m_A u_B$. This is not a recognised quantity in mechanics and therefore scored no marks. If they had multiplied this by the mass of B instead, or subtracted the initial velocity of A, then they would have gained at least the first mark, followed by the second mark if they correctly obtaining the magnitude of the impulse on B.

Question 2 (b) (iii)

(iii) Write down an expression for the impulse exerted on A as a result of the string becoming taut. **[1]**

This question required a simple application of Newton's 3rd Law, which applies to impulses as well as to forces, since the impulse is equal to the product of the force and the time. While the contact time is inevitably the same for both objects, the forces, and therefore the impulses, are equal and opposite.

Full marks were given to any candidate that applied this to their answer to (b) (ii), although for those that had a negative impulse and positive magnitude, they had to be careful which sign to reverse.

However, a significant proportion of candidates decided instead to start again, usually calculating the change in momentum of particle A to get the impulse, which took considerably more time and frequently led to errors. As a result, a majority of candidates did not score this mark.

Displaying fractions

Candidates should be careful to place the minus sign clearly in front of the fraction line rather than in line with the numerator. Otherwise, it can easily be interpreted as applying only to the first term rather than the whole fraction, i.e. '$-\frac{a+b}{c}$' with the sign clearly ahead of the fraction is preferable to something that looks like it could be '$\frac{-a+b}{c}$'.

Question 2 (c)

(c) Show that the loss of kinetic energy as a result of the string becoming taut is $\frac{m(3+u)^2}{3}$ J. **[3]**

This question required candidates to find the absolute difference between the kinetic energy before and after the 'collision'. This proved to be a challenge to many candidates, with only a few candidates scoring full marks.

The first 2 marks were for substituting their final velocity (or possibly velocities) to get an expression for the final kinetic energy, and a dependent second mark for subtracting this from the initial kinetic energy to give an expression for the loss of kinetic energy.

Common errors up to this point included using incorrect masses, e.g. m for both masses instead of m and $2m$ for A and B respectively, substituting the wrong value(s) for the final velocity, including things like the impulse instead of the velocity, or just using v_A and/or v_B, and general sign errors. At this stage attempts to calculate the change in kinetic energy, which would have given a negative value, were not penalised.

For those who did manage to do the required substitutions and correctly gained the first 2 marks, the process of expanding brackets that included fractions and collecting the terms generally proved to be very challenging. Many of these responses included errors or were left incomplete, with only a few fully working through the process to a correct final answer. Many responses demonstrated inefficient algebraic manipulation, for example not combining the expressions for the final energy of each object into a single expression before multiplying out the brackets, thereby contributing twice as many terms to the expression, together with the usual sign errors and other numerical errors.

Question 3 (a) (i), (ii) and (iii)

3 A car of mass 900 kg has an engine that is capable of working at a maximum power of 12 kW.

When the car is travelling along a straight horizontal road, the maximum constant speed that it can maintain is $40\,\mathrm{m\,s^{-1}}$.

In an initial model of the motion of the car it is assumed that the total resistance to motion is constant.

(a) (i) Show that when the car is travelling at $15\,\mathrm{m\,s^{-1}}$ along the horizontal road with the engine working at a constant rate of 10 kW, the acceleration of the car is $0.407\,\mathrm{m\,s^{-2}}$ correct to **3** significant figures. **[3]**

(ii) Find the speed of the car when it is accelerating at $0.2\,\mathrm{m\,s^{-2}}$ along the horizontal road with the engine working at a constant rate of 9.6 kW. **[2]**

(iii) The car now starts to **descend** a straight road which is inclined at $1°$ to the horizontal.

Find the power produced by the engine of the car when it is travelling along this road at a constant speed of $40\,\mathrm{m\,s^{-1}}$. **[2]**

Question 3 (a) consisted of a series of linked scenarios requiring the use of Newton's 2nd Law, together with the connection between power, force and speed. Overall candidates responded well to this question.

Initially, resistance was modelled as a constant, with candidates required to find in turn the unknown acceleration, speed and power of the system. A large majority correctly obtained the given acceleration in part (a) (i) by using the formula $\frac{P}{v} - R = ma$, and similarly in part (a) (ii) to find the unknown speed.

The main issue in part (a) (i) was having to find the resistance from the information given in the initial description by equating it to the driving force at constant speed. Candidates who did not find the friction tended to omit this from their equation, equating the power to the resultant force multiplied by the speed to give an answer of 0.7047. On the other hand, a small number of candidates found the friction but did not continue to set up an equation using Newton's 2nd Law and $F = \frac{P}{v}$, and scored 1 mark only. A few candidates tried to work backwards from the given acceleration to deduce that the friction was 300 N.

In part (a) (ii), a small number of candidates did not start the question, unlike in part (a) (i), even though the method was similar, but this time with an unknown speed. Some gained the first mark by setting up the equation correctly, but then made errors such as not multiplying all terms by v, ending up with equations such as $9600 - 300v = 180$ instead of $9600 - 300v = 180v$, while others divided the mass by the acceleration instead of multiplying, or used the driving force and/or acceleration from part (a) (i). Others once again omitted the element of friction in the equation to give an answer of 53.3. There was also the very occasional case of a candidate not converting the power from kilowatts into watts before attempting to solve the problem.

Part (a) (iii) was less well done, with around half of responses gaining full marks. Many did not know that gravity was working in the same direction as the driving force, which caused them to subtract the force of gravity from the driving force on the left side of their equation, or to add it to the mass × acceleration on the other side, for which we allowed for the method mark only. This was a common problem throughout the paper where gravity was involved, and centres may find it useful to focus on helping candidates master this skill using a range of suitable problems and thinking activities.

Some candidates assumed that the force of gravity down the slope was the driving force or omitted the factor of g in the weight of the car or used cosθ instead of sinθ to resolve the weight in the direction of the slope, or divided by cos or sin when resolving instead of multiplying, or even tried to resolve the friction or driving force. Others equated the driving force with the power or the force of gravity down the slope or the friction, or used the acceleration from part (a) (ii). There were also cases where candidates used the wrong sign for the friction by not strictly following Newton's 2$^{\text{nd}}$ Law. It is good practice for candidates to write down *all* forces on the left side of the equation, and mass × acceleration or 0 (and nothing else) on the right side before rearranging the equation, to minimise the chances of such sign errors.

Since the question was framed in terms of kW, an answer of 5840 (Watts) instead of 5.84 (kW) required the correct units by way of explanation, so a small number of candidates were not awarded the final mark due to omitting the units or occasionally stating the wrong units, e.g. Newtons.

Use of gravity

Many candidates struggled to work with gravity and potential energy in their mechanical equations, often omitting it, resolving incorrectly, or having it act in the wrong direction in their equation. Where possible, centres could try to address these common weaknesses by including questions and activities that make candidates think about how to handle this tricky concept correctly. This could be, for example, via an activity where different scenarios are presented and candidates have to deduce and explain in what way gravity will affect the objects in the problems, and by being vigilant in generally reinforcing the correct concepts and rules.

Question 3 (b) (i)

In a refined model of the motion of the car it is assumed that the total resistance to motion is proportional to the speed of the car.

(b) **(i)** Show that the total resistance to the motion of the car is now given by $7.5v$ N where $v\,\mathrm{m\,s^{-1}}$ is the speed of the car. **[1]**

For this 'show that' question, candidates were required to employ a full and convincing argument to produce the given result $F = 7.5v$. At a minimum, candidates needed to divide the 300 N of friction found in part (a) (i) by the speed (= 40 m/s) to give $k = 7.5$, in conjunction with the statement that $R = 7.5v$ or $R = kv$.

Most candidates did this successfully. However, of the candidates who did not, most did not adequately communicate the result, such as not explicitly stating the result and/or not clearly saying that $k = 7.5$. Some made incorrect conclusions, e.g. equating the total resistance with k instead of kv, or even stating that $R \propto kv$, while others did not explain why $k = 7.5$, or simply verified it by stating that $40 \times 7.5 = 300$ (N).

Centres may find it useful to challenge candidates to think about how they convey results and impress on them the benefits of clear communication in all questions, not just those requiring detailed reasoning.

Assessment for learning

A simple activity might involve candidates working together to discuss the merits of a number of responses to a question such as the above, cutting out each one and sorting them into different categories such as adequate, inadequate or uncertain. Candidates could then compare their judgement with the actual mark given and discuss why each response did or did not gain the mark. Groups could then take it in turn to give their explanation of a selection of responses to the rest of the class.

Question 3 (b) (ii)

(ii) State **one** way in which the refined model is an improvement over the initial model. **[1]**

The key to this question was to identify the fact that the new model accounts for the fact that in real life, resistance increases with speed, which makes the model more realistic (as it makes the same prediction).

Only about a third of candidates were able to explain the new model adequately, with many instead using vague and imprecise statements, such as "in the new model the resistance is different at different speeds", without saying whether the relationship was positive or negative. Candidates should try to avoid using vague words such as 'different' and be encouraged to replace them with more meaningful comparisons such as 'more', 'greater', 'smaller', 'less', etc, depending on the level of detail required.

Some candidates confused 'resistance', which is a general term covering any impediment to motion such as air resistance, friction, drag, etc, with 'friction', which refers specifically to the effect of two surfaces in contact with each other resisting any motion along their respective surfaces. Interestingly, in real life, friction does not generally increase with speed as it depends on the contact force between the two surfaces, which is not usually affected by speed. If anything, friction tends to decrease slightly with increasing speed.

A very small number of candidates referred to the situation where the car is stationary. Since the question specifies that the resistances are to 'motion', such considerations are not valid in this case. However, an argument that the resistance in the second model tends to zero as the speed tends to zero and is therefore more realistic would have been acceptable.

Question 3 (b) (iii)

(iii) Find, according to the refined model, the speed of the car when it is accelerating at $0.2\,\mathrm{m\,s^{-2}}$ along the horizontal road with the engine working at a constant rate of $9.6\,\mathrm{kW}$. **[3]**

This question was similar to part (a) (ii) but required candidates to incorporate the new model of friction into their equation. Most candidates who managed 3 (a) (ii) were also able to set up the equation for the revised problem and solve the quadratic equation that followed to gain full marks. A few did not gain marks due to errors in the working such as not substituting for *P*, omitting the element of acceleration, not rejecting the negative solution, simple numerical errors, equating *F* with *P*, sign errors, or very occasionally including the gravitational force from 2 (a) (iii).

Candidates are reminded that in applied mathematics, where there are two possible solutions, it is very often the case that only one solution is valid, while the other one must be rejected. In this question, since candidates were only asked to "find" the correct solution, they were not required to use detailed reasoning, although many demonstrated good habits by doing so.

Question 4 (a)

4 Two particles, A of mass 4 kg and B of mass 2 kg, are free to move along the same straight line on a smooth horizontal surface. They are projected directly **towards** each other with speeds of $6\,\mathrm{m\,s^{-1}}$ and $5\,\mathrm{m\,s^{-1}}$ respectively.

After A and B collide the speed of A is $1\,\mathrm{m\,s^{-1}}$.

(a) Determine the possible values of the coefficient of restitution between A and B. **[6]**

This question required candidates to work out the coefficient of restitution for two slightly different scenarios within the concept of speed, where the final speed of particle A is given as 1 ms^{-1}. Because speed is just the magnitude of the velocity, for a given speed v there are two possible velocities ($+v$ and $-v$) when motion is in a straight line. While most candidates realised this and many gained full marks, a substantial minority only recognised one of the scenarios and could therefore only access the first 3 marks, which was the second most common score. Candidates are reminded to look carefully at the wording of the question, and also the number of marks allocated, to help gauge the overall number of steps required.

Of those candidates that attempted both solutions, some made the erroneous assumption that they could just change the sign elsewhere, for example the final velocity of B changing from 5 to -5, or even the coefficient of restitution becoming negative, or that only v_A would be changed, with v_B being the same in both cases. A small number of candidates ended up with four solutions by not linking related sign changes, thereby giving four possible options.

Other common problems included things like sign errors when substituting into the formula for e, using the wrong formula for e (e.g. $\frac{u_1-u_2}{v_2-v_1}$), as well as the usual numerical errors and slips.

Some candidates ended up with one or more answers outside of the range $0 \le e \le 1$. While some simply accepted such a result, others rejected the unfeasible solution. Since the question clearly states that both situations are possible, candidates should be aware that this indicates that they have made an error and attempt to correct their work to maximise their marks.

A small number of candidates opted to give their final answer(s) in decimals rather than as fractions. The guidance for this paper, as shown on the cover of the question paper, says that answers are to be given to at least 3 s.f. unless otherwise instructed. Some instead only gave the answer to 2 s.f. which meant they were not awarded the final A mark.

Use of momentum and restitution equations

A significant minority of candidates greatly increased the difficulty of the problem by using restitution to obtain an equation for v_B in terms of e, and then substituting this, together with their value(s) of v_A, into the conservation of momentum equation. They then often further complicated the working by using $\pm$ to combine the two possible equations. This technique is more complicated and therefore more prone to error than the conventional approach. Many of the candidates that used this approach struggled to handle the algebra correctly, as well as making incorrect assumptions.

Exemplar 2

4(a)	momentum: $4 \times 6 - 2 \times 5 = 14\text{Ns}$
	$e = \frac{v_B \pm 1}{11}$ $\quad 11e = v_B \pm 1$
	$v_B = 11e \pm 1$
	$4v_A + 2v_B = 14$
	$4v_A + 11e \pm 1 = 14$
	$+1: \; 4 + 1 + 11e = 14 \Rightarrow e = 9/11$
	~~$-1: \; 4 \mp 11e + 1 = 14$~~

In the above response, the candidate has started by finding the total momentum from the initial velocities. They have then formed a dual equation for v_B in terms of *e*. After obtaining an equation for conservation of momentum in terms of v_A and v_B, they have attempted to substitute the restitution equation, together with one of the values of v_A, into their equation for conservation of momentum. For some reason, possibly due to the extra complexity of the work, they did not multiply the velocity of B by the mass, resulting in an error, although M mark was awarded since they eventually substituted for v_A as required, having already obtained the momentum equation.

Notice that the candidate has combined the two possible restitution equations by using a $\pm$ sign to give $11e = v_B \pm 1$, but it is not clear that $v_A = +1$ corresponds to $11e = v_B - 1$ and that $v_A = -1$ corresponds to $11e = v_B + 1$, meaning that the candidate has to guess which one to use, or may have to expend considerable effort to make sure that they don't get it the wrong way round. They could very well have ended up with four equations in total, since they have $v_A = \pm 1$ and $v_B = 11e \pm 1$.

They have then made an error in the next line since the $11e = v_B \pm 1$ should really rearrange to $v_B = 11e \mp 1$. On the penultimate line (immediately before the crossed out working), they have then assumed, with no explanation, that $v_A = 1$ corresponds to $11e + 1$ rather than $11e - 1$, which happens to be correct. This could potentially have given the right answers if substituted correctly into the momentum equation and carried through to find both values of *e*.

In any case, the method used is far more complicated, risky and time-consuming compared to using the conventional approach of first finding two separate values of v_B from two separate values of v_A and then substituting each v_A, v_B pair into the restitution equation.

Question 4 (b)

(b) State whether the collision between A and B is perfectly elastic. Justify your answer. **[1]**

This question tested candidates' conceptual understanding of restitution, in particular their knowledge of the meaning of the term 'perfectly elastic', which means that $e = 1$ or equivalently that kinetic energy is conserved in the collision.

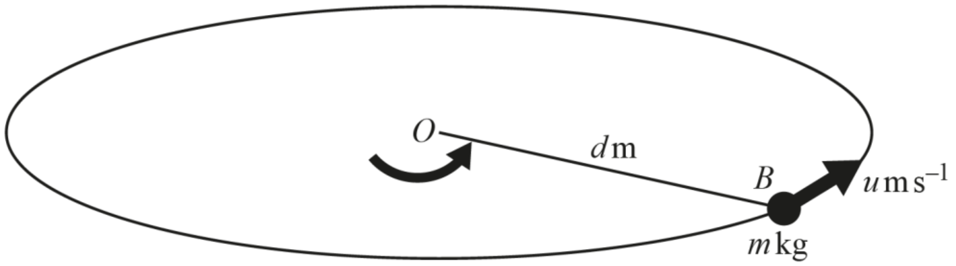
Most candidates realised that, since they had already obtained one or more value(s) of e in part (a), the simplest approach would be to compare the value(s) they already had to $e = 1$ to show that the collision was not perfectly elastic. However, a significant minority used an energy argument, but only the very small number who calculated the actual values from all of their results in part (a) were able to score the mark.

There were occasionally other incorrect responses, such as the conceptually incorrect statement that e was not equal to 1 or -1, incorrect claims that the collision was (perfectly) inelastic (i.e. $e = 0$), or that it was not perfectly elastic because the particles don't coalesce.

Question 5 (a)

5 One end of a light inextensible string of length d m is attached to a particle B of mass m kg. The other end of the string is attached to a fixed point O on a smooth horizontal plane.

Particle B moves on the plane with constant speed $u\,\mathrm{m\,s^{-1}}$ in a horizontal circle with centre O (see diagram).



The period, which is the time taken for B to complete one revolution, is denoted by p seconds and the tension in the string is F N.

(a) Use dimensional analysis to derive a formula for F in terms of d, m and p. You should define any other symbols used in your formula for F. **[4]**

In this question, candidates were expected to carry out a complete dimensional analysis (DA) on a proposed formula. Most were able to quote or work out the correct units of *F* in terms of M, L and T to gain the method mark. However, for most candidates the unknown constant is very poorly understood and mostly ignored, as is the general equation that is being analysed. On this occasion, the initial general equation should be $F = kd^{\alpha}m^{\beta}p^{\gamma}$, and k should be explained as an unknown constant. Most candidates omitted the initial equation altogether and just started with the units of *F* and sometimes the dimensional equation. This was usually followed by answer of dmp^{-2} without mentioning k at all. For this we awarded a maximum of 2 marks, making this the most common mark by a considerable margin.

Instead of using dimensional analysis (DA) as instructed, some candidates approached the problem by using Newton's 2nd Law which, while mathematically correct, was the subject of part (c) and not part (a). Conversely, those same candidates who used Newton's 2nd Law in part (a) often tried to use DA to complete part (c), which again achieved no marks.

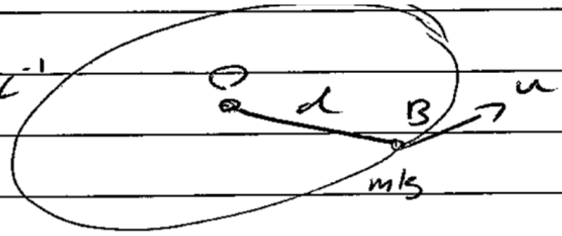
Handling of Dimensional Analysis questions

In general, candidates struggled to fully demonstrate an understanding of the dimensional analysis process in this paper. Centres may find it useful to clearly explain the steps involved, backed up with appropriate directed practice, and impress on future candidates the importance of rigorously following all of the steps, i.e.:

- Formulating the initial equation from the given terms with an unknown constant and unknown exponents;
- Establishing the dimensions of all the given terms involved in terms of M, L and T;
- Setting up a dimensional equation by replacing the original terms with their equivalent dimensional terms;
- Forming balanced equations for the exponents of each individual dimension;
- Solving the equations and resubstituting the values back into the original equation, while reinstating the constant (usually k).

Exemplar 3

5(a)



$\cancel{\omega = RPS \times 2\pi}$

$\cancel{a = r\omega^2 = \frac{v^2}{r}} = L^2 T^{-2} L^{-1} = LT^{-2}$

$\cancel{v = r\omega}$

$\cancel{p = \text{time} \quad \omega = \frac{2\pi}{p}}$

$\cancel{F = ma = m(r\omega^2) = m\left(d\left(\frac{2\pi}{p}\right)^2\right) = \frac{4md\pi^2}{p^2}}$

m = mass of B

d = distance from O to B = circle radius $= \cancel{M \times L \times T^{-2}} = |F|$ ✓

p = time taken for 1 revolution

$|\omega| = T^{-1}$ $\qquad |F| = MLT^{-2}$

$\cancel{|a| = LT^{-2}}$

$|m| = M$ $\qquad \cancel{|m| \times |a| = MLT^{-1}}$

$|u| = LT^{-1}$ $\qquad \cancel{\text{so } ma = F}$

$|d| = L$ $\qquad \frac{|m| \times |d|}{|p^2|} = MLT^{-2} = |F|$ ✓

$|p| = T$

$|p^2| = T^2$

$\frac{md}{p^2}$

In this response, the candidate has initially attempted to use Newton's 2nd Law but then realised that they should be using dimensional analysis. While there is a clear statement of the dimensions of *F*, gaining the M mark, there is no initial equation or any mention of a (dimensionless) constant term. As they have gained the M mark and then stated $F = \frac{md}{p^2}$, this gains the SC mark for an answer omitting k.

Question 5 (b)

(b) Explain why it was **not** necessary to consider the quantity u in the analysis in part **(a)**. **[1]**

Of all the questions in this exam, this was the one (along with 5(d)) that candidates found to be the most challenging. The key to this question was to understand that *u*, which is the tangential speed, can be written in terms of *d* and *p* using the formula $u = \frac{2\pi d}{p}$, which is the circumferential distance divided by the time taken for 1 revolution. Therefore, *u* is already contained in the terms *d* and *p* and does not need to be included as a separate term.

Only a small minority of candidates realised this. The most common incorrect explanations were that *u* is constant and therefore had no effect on *F*, that *u* is tangential and therefore perpendicular to *F*, that *u* is dimensionless, there is more than one way to calculate *F* (which is true but insufficient), that you can define acceleration by ω, which is defined by *p* (with no mention of *u*), or that ω can be written in terms of *p*, again with no mention of *u*.

Candidates should be aware that the motion of an object moving at a tangent is perpendicular to the centripetal force and therefore does not do any work in that direction. However, this does not mean that it has no effect on the perpendicular force.

Question 5 (c)

(c) Instead, use Newton's second law to derive a formula for F in terms of d, m and p. **[3]**

In this question, candidates were asked to use Newton's 2nd Law instead of dimensional analysis to derive the formula for *F*. This was not often completed successfully. A notable number of responses gained no credit, while a smaller proportion were awarded full marks. Some candidates, especially those that used Newton's 2nd Law in part (a), tried to use dimensional analysis, which did not merit any marks.

Candidates using Newton's 2nd law often used the wrong formula for ω and/or for acceleration, e.g. $\omega = \frac{1}{p}$, leading to the incorrect formula $a = \frac{d}{p^2}$. Other errors seen were candidates finding the momentum instead of the force by not squaring ω, omitting constants, or not substituting the terms from the question into the formula. One or two decided to use the formula $a = \frac{v}{t} = \frac{\frac{2\pi d}{p}}{p} = \frac{2\pi d}{p^2}$, with or without *m*. A reasonable minority gained the first 2 marks but did not substitute for *u* or ω (depending on which formula they used), thereby not gaining the last mark.

Question 5 (d)

(d) Considering your answers to part **(a)** and part **(c)**, state one **disadvantage** of using dimensional analysis to derive a formula for F. **[1]**

This question required candidates to recognise that while it is possible to deduce an exact formula from Newton's 2nd Law, dimensional analysis will, by its nature, not allow you to derive the value of any dimensionless constants without further information. Along with 5 (b), this also proved challenging for a substantial majority of candidates, who commonly stated that dimensional analysis (DA) does not take account of constants or words to that effect, which is incorrect as DA simply does not tell you the value of constants rather than ignoring them.

There were many other attempts at criticisms which did not merit any credit including that DA is misleading or inaccurate, can only deal with 3 variables (true but not relevant in this example), does not consider resistance, you cannot derive the units of a formula using DA or DA does not take into account the units as much. Less common incorrect answers included statements such as *u* just gets cancelled out, DA is time-consuming, is a longer and more confusing formula, or that *k* could only be found from experimental results.

Question 6

6 A disc of mass 2 kg is projected, with initial speed $16.4\,\text{m}\,\text{s}^{-1}$, directly up a rough plane which is inclined at an angle of $15°$ to the horizontal. The coefficient of friction between the plane and the disc is 0.2.

This question required knowledge of the coefficient of friction from the A Level Maths specification which is outside the assumed knowledge for this paper. We are very sorry for the confusion and distress this caused. Many candidates appeared to have been taught this content, despite it not being part of the assumed knowledge for the paper and therefore produced solutions to this question across the mark range. Some candidates assumed that the coefficient of restitution was intended (giving, e.g. 16.4 x ±0.2 = ±3.28 for the final velocity) or that 0.2 was the frictional force. Other candidates left the question unanswered.

To reward candidates who were able to answer the question while not disadvantaging those who could not, the question was marked as usual, but candidates' marks for Question 6 were replaced with a proportion of the total marks available based on their performance on Questions 1 to 5 if higher.

We have provided a commentary on the question to support A Level teachers using this question at the appropriate point of the course.

Question 6 (a) (i)

(a) (i) Determine the average power generated by the frictional force in the first 3 seconds of the disc's motion. **[6]**

To answer this question, candidates were required to use in turn Newton's 2nd Law, *suvat*, and finally an energy/power calculation, to find the average power generated by a frictional force.

For those that attempted the question, there were elements of the question that they found challenging. The first mark was for resolving perpendicular to the surface to find the normal (with an allowance for sin/cos confusion), which was carried out successfully by many candidates that attempted the question, but not all. Those that managed this usually multiplied by the coefficient of friction to get the second mark. A small number of candidates attempted to resolve vertically and horizontally, which was invalid as they didn't include the relevant component of acceleration in either or both equations, while others divided by sin or cos instead of multiplying when resolving.

The third and fourth marks were for using Newton's 2nd Law to find the acceleration and then use *suvat* with the given time to find the distance travelled as the particle moved up the slope. Candidates commonly did not realise that they needed to include both the friction *and* the component of weight down the slope to find the acceleration, often missing out one of these and gaining no further marks. Some assumed that $a = -9.8$ as if in freefall or tried to use an energy argument instead of *suvat*, often assuming that the final velocity was zero or 3.28 and sometimes missing out the unknown friction to get an answer.

The final 2 marks were for finding the energy lost due to friction and dividing by the given time of 3 seconds to find the average power generated by the frictional force. For the responses that reached this stage, the simplest way to do this was to multiply the frictional force by the distance and divide by the time taken. An alternative was to use *suvat* to calculate the final speed instead of the distance travelled, find the average speed and multiply by the frictional force. Some candidates decided to use initial – final energy instead, which was a more complicated calculation therefore often led to errors, as it involved accounting for the GPE as well.

Question 6 (a) (ii)

(ii) Explain the significance of the sign of your answer to part **(a)(i)**. **[1]**

For this question, a more in-depth answer was expected, rather than just stating that the power or the friction was opposing the motion. Successful candidates needed to have first obtained a negative value in 6 (a) (i) or give a suitable explanation of why it should be negative. Some corrected the sign as a result of the prompt. Only a few candidates were able to explain that the effect of the negative power was to reduce the energy of the object.

Question 6 (b)

(b) **Use an energy method** to find the distance the disc travels up the plane from the point where it is projected to the point where it comes to instantaneous rest. **[4]**

This question was attempted by most candidates and resulted in a good spread of results. Those who made reasonable progress in part (a) (i) generally did well in this part as well.

The key to this question was to set up an energy equation with a final KE of zero, together with terms for gravitational potential energy and work done against friction, both in terms of d or h. This was sufficient for the first 3 marks if using d, or 2 marks if using h until this was converted into a distance along the slope by dividing by sin15°. The final mark was an accuracy mark, which required the correct answer. Therefore, this was only open to those who had previously calculated the correct friction.

Many candidates missed out at least one term, usually the friction, even if they had obtained a value for it in (a) (i). This limited them to a maximum of 2 marks, which became the most common score, or 1 mark if using h only, with a relatively small number achieving 3 or 4 marks.

As is commonly the case in energy equations, a small number of candidates opted to use *suvat* approach which scored no marks, as the question specified an energy method.

Question 7 (a)

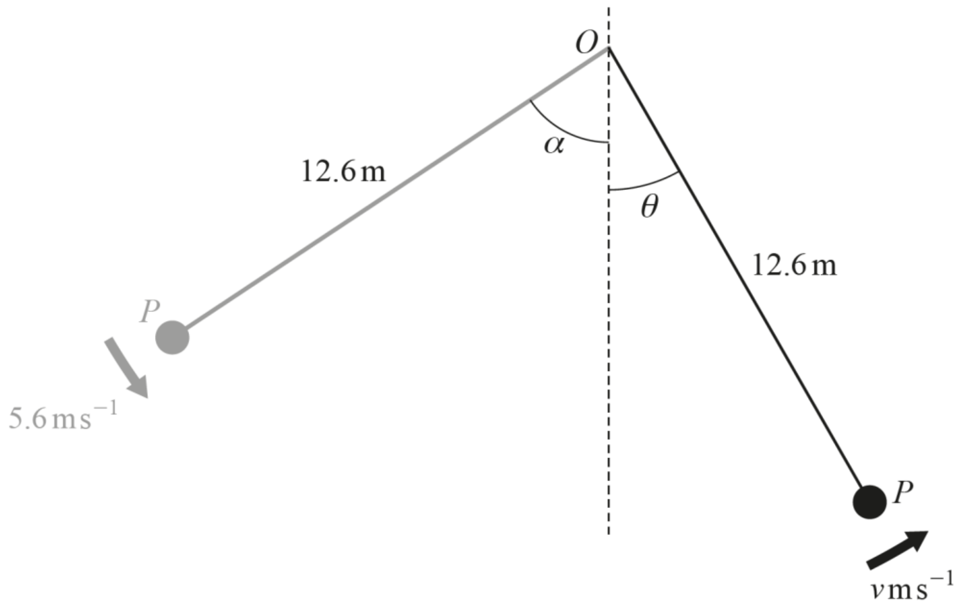
7 In this question you must show detailed reasoning.

A particle P of mass m kg is attached to one end of a light rod of length 12.6 m. The other end of the rod is attached to a fixed point O.

Particle P is initially held at a point so that OP makes an angle of α to the downward vertical through O, where $\cos\alpha = \frac{5}{9}$.

Particle P is then projected so that it instantaneously starts moving downwards at right angles to the rod with a speed of 5.6 m s^{-1} in a vertical circular path around O.

When the rod makes an angle of θ with the downward vertical, P has speed v m s^{-1} (see diagram).



(a) At a certain instant in P's motion, $v = 8.4\cos\theta$.

Determine the value of v at this instant. **[6]**

Despite this appearing to be quite a challenging question, the great majority of candidates attempted it, usually gaining at least the first 1 or 2 marks for the initial energy of particle P, with more achieving 2 than 1 mark. A small number decided to work with the approximate value of α rather than just using the given value of cosα = $^5/_9$, which created extra work and room for error. A small number were not able to calculate the height for the GPE. This depended on the point of reference, which for most candidates was the lowest point of the motion. Using O as the reference point would have simplified the calculation to -12.6gcosα for the initial potential energy and -12.6gsinθ for the final potential energy instead of 12.6g(1 – cosα) and 12.6g(1 – cosθ) respectively, both of which give a difference of -12.6gsinθ – - 12.6gsinα, or 7g - 12.6gsinθ. Some candidates calculated 11.8 ms^{-1} as the maximum speed at the lowest point to simplify their calculations later on, although they only needed the total energy.

For the next stage, candidates generally opted to equate the initial and final energies, using the equation $v = 8.4\cos\theta$ to substitute for v and then simplify to a quadratic in $\cos\theta$, A small number obtained an equation in v instead by substituting for $\cos\theta$, which gives a more direct route to the solution. Quite a few candidates made errors here, for example by missing out one of the potential energy terms or not handling the potential energy correctly. Some candidates simplified the equation prematurely instead of writing it down in full first, often making sign errors or missing out terms like g in one or more places. Those who attempted to form an equation for the change of potential energy usually made the critical mistake of equating it with the change of kinetic energy instead of the negative of this. As a result they were then unable to get a viable equation. Credit was given for reasonable attempts to obtain a three-term quadratic equated to zero from a reasonable attempt to equate the energies, although some candidates stopped before this point and only gained 3 marks.

The last part involved solving the quadratic equation and obtaining the correct value of v. Since this question had the detailed reasoning requirement, candidates needed to justify each step and decision made throughout their working, including justifying their choice of solution. For those finding $\cos\theta$, this meant pointing out that $\cos\theta = -3$ did not fulfil the condition that $-1 \le \cos\theta \le 1$, or possibly that the answer was an imaginary number. For v, this had to be ≤ 8.4 from $8.4\cos\theta$, or at least ≤ 11.8 (the maximum speed at any time from the total initial energy). Unsubstantiated statements such as 'out of range', or 'not possible' did not gain the fifth mark. However, we did allow the final mark for a correct value of v at the end, except for those who made errors in their equations, and a small number that chose the wrong value of v.

Question 7 (b)

(b) Show that P does **not** describe complete circles around O. **[2]**

This final question was most easily solved by considering the energy at the top of the circle (usually $2 \times 12.6m$g), which gained the first mark. After that, a comparison with the initial energy from part (a) (usually $70.56m$), showed that it does not have sufficient energy to reach the top as $70.56m < 25.2m$g ($= 246.96m$, depending on the initial reference height). Several alternatives were possible, such as comparing the maximum height of P with the height at the top of the circle (usually $7.2m$ versus $2 \times 12.6m$), or showing that v^2 would have to be a negative number, or even calculating the maximum angle of P from the vertical and showing that it was less than 180° or π.

A substantial minority of candidates did not attempt this question. Many candidates gained no marks, for example by not realising that $\theta = 180°$ at the top, or assuming that the highest point was when $\theta = 90°$.

Most of those who gained the first mark also gained the second mark by using a suitable method, but some did not as they did not explicitly state the correct conclusion, which had to be supported by a suitable explanation such as $5.6m$g < $25.2m$g or the worded equivalent. Others lost the A mark by using incorrect values, for example a total energy of $70m$ instead of $70.56m$, only including the kinetic energy element in the comparison, or even calculating the wrong angle of P, for example using 90 – α for the maximum angle of P.

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