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Examiners' report

AS LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A

H235

For first teaching in 2017

Y532/01 Summer 2025 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y532/01 series overview

Much of this paper was answered confidently and accurately. Parts that required conceptual understanding were often answered less successfully. Candidates had learned correct forms of words, but it was clear that they did not always know what they meant. This is particularly true for the 'constant average rate' assumption for the Poisson distribution in Question 2; this was widely misunderstood.

There was an unusual but widespread error in Question 6 (b), which raises quite deep questions about learning and understanding.

A small but not negligible number of candidates consistently gave numerical answers correct to only two significant figures. The examination instructions state that non-exact numerical answers should be given correct to **three** significant figures unless a different degree of accuracy is specified in the question.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• chose the correct form of null hypothesis for each of the different tests, and expressed the null hypothesis precisely • understood what the words 'constant average rate' really mean • gave conclusions to hypothesis tests with suitable acknowledgement of the uncertainty of the outcome • knew the difference between $E(X^2)$ and $[E(X)]^2$ and could confidently rearrange $Var(X) = E(X^2) - [E(X)]^2$.	• confused the different forms of null hypotheses for different tests, and stated them inaccurately • did not understand the meaning of 'constant average rate' • gave over-assertive conclusions to hypothesis tests • used $E(X^2) = [E(X)]^2$ when faced with an unfamiliar question style.

Question 1 (a)

1 A six-sided dice may or may not be equally likely to land on any one of its faces. It has two faces numbered 5 and the other four faces are numbered 1, 2, 3 and 4. The dice is thrown 60 times, and the outcome on each throw is recorded. The results are shown in the table.

Outcome	1	2	3	4	5
Frequency	7	13	11	15	14

It is required to carry out a test at the 5% significance level of whether the probabilities of the outcomes 1, 2, 3, 4 and 5 are in the ratio 1 : 1 : 1 : 1 : 2.

(a) State suitable hypotheses for the test. **[1]**

Many candidates stated hypotheses in a form suitable only for *conclusions* of tests, such as 'the data is consistent with …' or 'the data fits the model'. Some candidates gave the hypotheses the wrong way round, saying that the null hypothesis was that the probabilities were *not* in the given ratio.

Key points

Hypotheses refer to what the underlying distribution actually is, so a fully assertive statement is required. For example, 'H_0: the probabilities are in the ratio 1 : 1 : 1 : 1 : 2.' Hypotheses do not include phrases such as 'there is evidence that' or 'is consistent with'.

Exemplar 1

1(a)

H_0: the ratio 1:1:1:1:2 is a suitable fit

H_1: the ratio 1:1:1:1:2 is not a suitable fit

In this exemplar the candidate made an assertive statement but did not use appropriate language. There may be many different suitable fits. The question is, what is the one true probability distribution?

Assessment for learning

In this specification, the hypotheses for the different chi-squared and correlation tests take very different forms and can be easily confused.

Question 1 (b)

(b) Carry out the test. **[6]**

For the most part, candidates answered this question well and often got most of the marks. Common errors were:

- in the final term of ΣX^2, using a divisor of 10 instead of 20 (this gave $\Sigma X^2 = 8$)
- using a wrong critical value
- stating the conclusion too assertively. It is wrong to say 'There is significant evidence that the probabilities are in the ratio 1 : 1 : 1 : 1 : 2.'. A double negative is needed: 'There is *not* sufficient evidence that the probabilities are *not* in the ratio 1 : 1 : 1 : 1 : 2.'

Question 2 (a)

2 On any day, flights of helicopters and aeroplanes pass within sight of a certain building only during a daytime period of 12 hours, from 7.00 am to 7.00 pm.

The random variable H is the number of helicopters that pass within sight of the building during a randomly chosen daytime period of 12 hours.

(a) State **two** assumptions needed for H to be well modelled by a Poisson distribution. **[2]**

This was answered well by most candidates, although some tried to give one assumption in terms of probabilities. This can only be done by a complicated statement, such as 'The probability that a helicopter is seen in a very short interval of time δt depends only on the size of δt'. An alternative explanation is: 'for any given interval of time t, the expected number of helicopters is proportional to t, with the same constant of proportionality throughout'. (This 'constant of proportionality' is the average rate.)

Some candidates gave responses in terms of modelling *conditions*, such as 'events must occur in a fixed period of time or space', which is not an *assumption*.

Some candidates gave responses such as 'H appears independently.' This suggests that they think that H is merely an abbreviation for the word 'helicopters', instead of being the name of a numerical random variable.

Question 2 (b)

Assume now that H can be well modelled by the distribution Po(1.8).

(b) Find the probability that, during a randomly chosen daytime period of 12 hours, the number of helicopters that pass within sight of the building is 2, 3 or 4. **[2]**

Candidates almost always answered this correctly, although some candidates gave their answer as three separate probabilities without adding them.

Question 2 (c)

(c) During a period of t hours (within the daytime period of 12 hours), the probability that no helicopters pass within sight of the building is 0.95.

Use an algebraic method to determine the value of t. **[3]**

Most candidates were able to use an equation of the form $e^{-\lambda} = 0.95$. Many could solve this using logarithms, but turning the value of λ into a value of t was found to be harder. Nevertheless, many candidates answered correctly.

Question 2 (d)

(d) During a randomly chosen daytime period of t hours (where t is the value found in part **(c)**) two helicopters pass within sight of the building.

Explain whether this casts doubt on the validity of the model. You do not need to carry out any calculations. **[1]**

Few candidates made the principal point, which is that a single observation is poor evidence on which to reject a model. Other explanations seen included 'no because it could happen by chance' and 'no because conclusions based on a short time interval may not be reliable'. Most gave responses such as 'yes because the probability of 2 passing is very low.' It is worth noting that any probability calculation would need to involve the probability of two *or more*.

Question 2 (e)

Throughout the daytime period of 12 hours on any day, aeroplanes pass within sight of the building at a fixed constant rate of 1 aeroplane every 4 minutes.

(e) Explain whether the number of aeroplanes that pass within sight of the building on any day is likely to be well modelled by a Poisson distribution. **[1]**

Some candidates pointed out that 'constant average rate' is very different from 'fixed constant rate'. Generally, candidate responses showed that there is a lot of misunderstanding of these concepts.

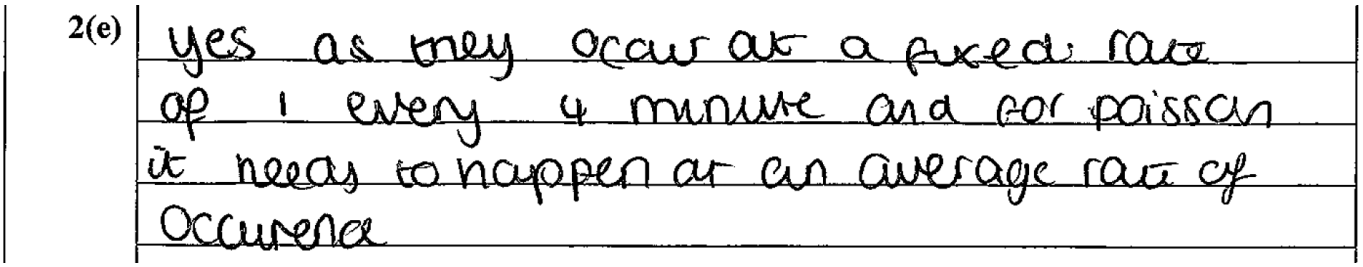
Misconception

A 'fixed constant rate' means that the events are exactly predictable, so there is no need for a probability distribution at all; the number of planes passing is totally non-random. This misconception has been noted in previous examination series.

In terms of modelling assumptions, 'constant average rate' does indeed hold, but the events are not independent – they could not be more dependent.

Probability distributions cannot be used for non-random events.

Exemplar 2



In this exemplar 'fixed rate' has been assumed to be the same as 'constant average rate'. The use of 'an average rate of occurrence' is also incorrect.

Question 2 (f)

(f) Find the expected value of the total number of helicopters and aeroplanes that pass within sight of the building on a randomly chosen day. **[1]**

Some candidates seemed to misread the question and used only the figure for aeroplanes. Some considered a 24-hour period rather than a 12-hour period.

Question 3 (a)

3 **(a)** It is required to test whether there is a relationship between two numerical variables.

State why in certain circumstances it would be better to use a test based on Spearman's rank correlation coefficient rather than a test based on the product-moment correlation coefficient.

[1]

It is not enough to say 'use Spearman when the data is ranked' (Spearman is the pmcc applied to rankings.). Candidates who attempted to distinguish between correlation and association needed to explain what the difference was. Most candidates knew that Spearman is likely to be more useful when a relationship is non-linear.

Some candidates said that Spearman is preferable when one judge is much stricter than another, but this is not necessarily true. The pmcc is independent of different sample means and variances.

Many said that a hypothesis test for the pmcc is not valid unless the data has a bivariate normal distribution. This is not on the specification but gained credit, provided it was correctly stated.

Question 3 (b) (i)

(b) A publisher wants to test whether there is any association between the ratings given by two readers, A and B, to books. The publisher selects 7 books at random and gives a copy of each book to each reader. Each reader gives a rating out of 20, where 20 is high and 0 is low, to each of the 7 books. The ratings are shown in the table.

Book	P	Q	R	S	T	U	V
Reader A	3	7	8	12	13	18	19
Reader B	17	15	16	14	4	5	13

(i) In the Printed Answer Booklet, complete the table giving the ranks of the ratings given by Reader A and Reader B. **[2]**

Most candidates answered this correctly.

Question 3 (b) (ii)

(ii) Use Spearman's rank correlation coefficient to test at the 5% significance level whether there is any association between the readers' ratings of books. **[7]**

Hypotheses for tests of correlation should be stated in terms of the value of the population parameter ρ or, as here, ρ_s. As always, hypotheses concern the distribution of the whole population and have nothing to do with samples, so candidates were wrong to say 'ρ_s is the *sample* rank correlation coefficient'.

The value of r_s, the sample rank correlation coefficient, –0.8214, was almost always correct. Some candidates were confused by its negative value and wrote –0.8214 < 0.7857. This then led to the wrong conclusion. It was necessary to compare r_s with –0.7857, or, as most candidates did, compare $|r_s|$ with 0.7857.

Question 4 (a)

4 In a science experiment, the values of a variable W are selected by the scientist. The experiment involves measuring the corresponding values of the variable V. The results are summarised as follows.

$$n = 10 \quad \sum v = 176.2 \quad \sum w = 87.5 \quad \sum v^2 = 4077.64 \quad \sum w^2 = 951.25 \quad \sum vw = 1963.4$$

(a) Calculate the value of the product-moment correlation coefficient. **[2]**

Almost all candidates answered this correctly.

Question 4 (b)

(b) By identifying the controlled variable in the experiment, calculate the equation of the appropriate least squares regression line. **[3]**

Many candidates realised that W was the controlled variable and deduced that the appropriate line was v on w. Candidates who used w on v could still get 10 of the 11 marks in Question 4 due to follow through.

Calculations were often done well, although the value of a was often inaccurate due to premature rounding. Some candidates gave answers in terms of y and x rather than v and u.

Question 4 (c) (i)

It is required to obtain an estimate of the value of the response variable when the value of the controlled variable is 10.0.

(c) (i) Find the value of the required estimate. **[1]**

A few candidates incorrectly tried to use the *w*-on-*v* regression line 'backwards', solving 10 = 0.433*v* + 1.114 (In this case it gives almost the same answer, but that is only because the two lines are very similar, as there is such strong correlation.). Inverse regression is a difficult topic and is not on this specification.

Question 4 (c) (ii)

(ii) Use the appropriate mean and standard deviation to determine whether 10.0 is likely to be within the range of the data for the controlled variable. **[2]**

Some candidates confidently calculated the relevant mean and standard deviation and then made statements such as '10 is within one standard deviation of the mean, so it is very likely to be within the range of the data'. There were many errors in calculating the standard deviation. Many simply used $\sqrt{S_{ww}}$. Some wrongly tried to check whether their ***estimate*** (their answer to part (c) (i)) was within the range of the dependent variable.

Assessment for learning

To help develop understanding, candidates might find it useful to explore on a spreadsheet whether it is possible to obtain a set of ten data values with these properties:

- all the data values are less than 10
- the mean is 8.75
- the standard deviation is greater than 4.30.

Question 4 (c) (iii)

(iii) Explain whether the estimate found in part **(c)(i)** is likely to be reliable. **[2]**

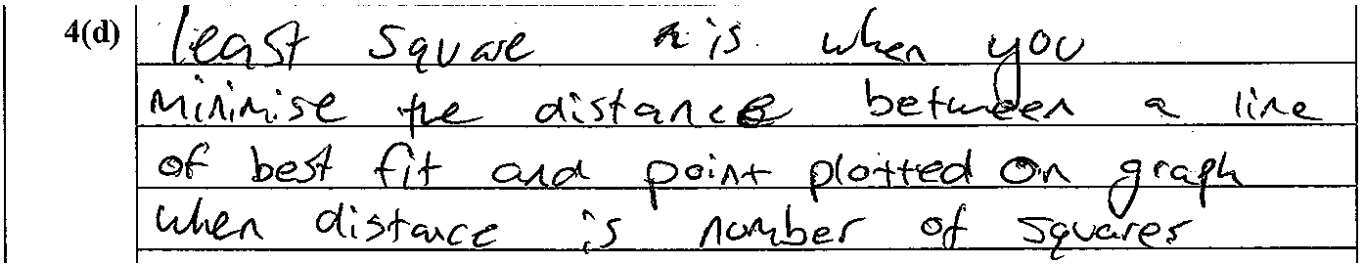
Many candidates gave one reason for reliability but often omitted a second reason. The reason 'high value of the pmcc' was seen less often. Candidates that gave two reasons almost always gave a single overarching conclusion, typically 'therefore it is reliable', as was required.

Question 4 (d)

(d) Explain what is meant by the words "least squares" in the phrase "least squares regression line". **[1]**

Some candidates gave confident and accurate responses, but there were also many partial ones. Many omitted to mention the *sum* of the squared differences. A few responses seemed to suggest that 'squares' referred to the squares on graph paper, for example, 'if the points are drawn on a squared grid, then it minimises the distance of the points from the line.'

Exemplar 3



In this exemplar, the use of 'distance is the number of squares' suggests the misconception described above.

Question 5 (a)

5 The probability distribution of a discrete random variable U is given in the table.

u	0	1	2	3
$P(U = u)$	r	0.3	p	$0.5 - p$

(a) Find the value of the constant r. **[1]**

Almost all candidates answered this correctly.

Question 5 (b)

(b) Determine an expression for $\text{Var}(U)$ in terms of p. Give your answer in the form $a - (p + b)^2$, where a and b are constants. **[4]**

Most candidates were able to obtain correct expressions for E(U) and E(U^2) in terms of p, and to use E(U^2) – [E(U)]^2. There were frequent algebraic errors in converting to completing-the-square form, probably because the p^2 coefficient was negative.

Question 5 (c)

(c) Use your expression in part **(b)** to determine the smallest possible value of $\mathrm{Var}(U)$. **[3]**

Correct answers that used $p = 0.5$ in the previous expression were reasonably common. Many candidates did not consider the limitations on the value of p. Some tried to make $(p + b)$ zero, and others tried to find a value of p for which $\mathrm{Var}(U)$ was zero, or to use differentiation. All of these give impossible values of p.

Question 5 (d)

(d) Find the value of $\mathrm{E}(U)$ that corresponds to the smallest possible value of $\mathrm{Var}(U)$. **[1]**

Whether or not candidates answered correctly depended only on using the correct value of p, namely 0.5.

Question 6 (a)

6 A spinner has probability 0.2 of landing on an edge coloured blue. The spinner is spun repeatedly. The number of spins required, up to and including the first spin on which the spinner lands on an edge coloured blue, is denoted by X.

(a) Find $\mathrm{P}(X \leqslant 9)$. **[3]**

Almost all candidates answered this correctly.

Question 6 (b)

(b) Determine the value of $\mathrm{E}(X^2)$. **[4]**

Very many candidates incorrectly wrote down 'E(X) = 5, so E(X^2) = 25'. This was seen much more often than the correct answer.

Assessment for learning

Candidates will have learned that $\mathrm{E}(X^2)$ and $[\mathrm{E}(X)]^2$ are different, but it is likely that the unfamiliar question style in an exam setting has led to the prevalence of the use of $\mathrm{E}(X^2) = [\mathrm{E}(X)]^2$.

Opportunities should be given to candidates to work with E(X) and Var(X) in unfamiliar contexts.

Question 6 (c)

(c) In this question you must show detailed reasoning.

The spinner is spun n times. The probability that X takes the value n is 0.041 943 correct to 6 decimal places. In these n spins, the number of times the spinner lands on an edge coloured blue is denoted by Y.

Find the variance of Y. **[6]**

Candidates often answered this question correctly. Less successful candidates struggled with the different variables and probabilities in the question, typically using Geo(0.041943) or B(n, 0.041943).

The 'detailed reasoning' command was ignored or misunderstood by many candidates who went straight from $0.8^{n-1} = 0.209715$ to $n - 1 = 7$ without showing any working. To obtain full marks it was necessary to use logarithms. Most of those who obtained $n = 8$ could then use B(8, 0.2) to obtain the correct answer.

Command words: detailed reasoning

The **detailed reasoning** instruction requires candidates to 'give a solution which leads to a conclusion showing a detailed and complete analytical method'. This means that trial and improvement or direct calculator methods should not be used as they will not get full marks.

Further information and examples can be found in this OCR blog.

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