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Examiners' report

AS LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A

H235

For first teaching in 2017

Y531/01 Summer 2025 series

Contents

Introduction 3
Paper Y531/01 series overview 4
Question 1 (a) 5
Question 1 (b) 5
Question 2 (a) 5
Question 2 (b) 6
Question 3 (a) 6
Question 3 (b) 6
Question 4 (a) (i) 7
Question 4 (a) (ii) 7
Question 4 (b) 7
Question 4 (c) 7
Question 5 (a) 8
Question 5 (b) 8
Question 5 (c) 9
Question 6 (a) 10
Question 6 (b) 10
Question 7 10
Question 8 11
Question 9 (a) 12
Question 9 (b) 12
Question 9 (c) 12
Question 9 (d) 12

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y531/01 series overview

Y531 is the mandatory paper for AS Further Mathematics. It tests knowledge of proof, complex numbers, matrices, vectors and further algebra as well as testing the understanding of the overarching themes of mathematical argument, problem solving and modelling. To do well on this paper candidates need a thorough understanding of the techniques covered and they need to support their answers with detailed working. They also need to have good algebraic and numerical manipulation skills.

Candidates found this paper more approachable as compared to ones from recent years, and there were good answers to all the questions, especially ones requiring more routine methods. The questions that were found to be particularly difficult were the complex loci geometry problems in Questions **5(b)** and **5(c)** and the last part of Question **9** which required candidates to link together the ideas from the rest of the question. For some questions, including the routine Question **1(a)**, a sizeable number of candidates choose lengthy and less efficient methods. Sometimes these methods were successful, but often candidates who chose to use them did not gain full credit for the question part.

Candidates' layout and presentation has improved, but there are still some cases where it can be difficult to decipher what a candidate has written. Many candidates had difficulties with algebraic manipulation, in particular omitting necessary brackets or making sign errors. In particular, candidates must make sure in proof questions that all of their algebra is correct and that all necessary brackets are present.

Candidates must take careful note of any command words given in the question. In particular, if a question asks for 'detailed reasoning', candidates must show all of their working. Calculators can be used as a check, but the solutions must be fully detailed in order to gain full credit.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• set out their working logically and made good use of space • read the questions carefully, checking to see if any methods were prescribed by the question • ensured that they supported their answers with sufficient working, especially when answering detailed reasoning questions • had a good understanding of the OCR command words • had fluent algebraic manipulation skills • drew clear diagrams to help determine how to solve a problem, or to check that their answer was reasonable.	• made algebraic manipulation mistakes and sign errors • did not always include relevant brackets • did not provide sufficient detail in 'show that' or proof questions, often running several steps together resulting in a response which was not fully convincing • did not provide sufficient justifications in 'detailed reasoning' questions • did not read questions carefully and make sure that they were using the method prescribed by the question or not giving their answer in the required form.

Question 1 (a)

1 **(a)** The complex number z is such that $|z| = 7$ and $\arg(z) = 2.2$ radians.
Express z in cartesian form. **[3]**

There were some correct responses to this question, but a lot of candidates did not understand the requitement to express z in cartesian form and instead left their answer as 7(cos 2.2. + i sin 2.2).

Some candidates chose a more difficult and inefficient method by solving the simultaneous equations $x^2 + y^2 = 49$ and $\frac{x}{y} = \tan 2.2$. These candidates rarely gained full marks as they often did not pick the correct sign for x when taking the square root, or sometimes due to lack of accuracy of their final answer. A clear diagram would have helped these candidates realise that the x value had to be negative.

Misconception

? A lot of candidates expressed z in modulus argument form rather than cartesian form.

Question 1 (b)

(b) Use an algebraic method to determine the exact square roots of $1 + (4\sqrt{3})\mathrm{i}$. **[5]**

This part was answered more successfully than the previous part with some efficient and logical solutions. Almost all candidates understood what was required by the question.

Some candidates did not gain marks as they had not expanded $(a + \mathrm{i}b)^2$ correctly, some used both the positive and negative values for a^2 or b^2, some made sign errors and a few erroneously write their answer as $\pm 2 \pm \sqrt{3}\,\mathrm{i}$, which represents four different complex numbers rather than the two intended.

Question 2 (a)

2 Two vectors, **a** and **b**, are given by $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 13 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 6 \\ p \end{pmatrix}$ where p is a constant.

(a) Find expressions in terms of p for each of the following.

- **a.b**
- $\mathbf{a} \times \mathbf{b}$

[3]

The majority of candidates gained full credit for this part of the question. The most common mistakes made were sign errors when finding the vector product and writing the scalar product as a vector (usually by multiplying together the x coordinates, y coordinates and z coordinates).

Question 2 (b)

(b) Hence or otherwise find the value of p in each of the following cases.

- **a** and **b** are perpendicular
- **a** and **b** are parallel

[2]

Most candidates knew what was expected from this question, and for the parallel case the method preferred by those candidates who gained full marks was scaling, despite the question pointing towards using the vector product.

The most common mistakes made were to not identify which value of p belonged to which case, mixing the two cases up, and by assuming that parallel meant that $\mathbf{a} \cdot \mathbf{b} = 1$.

Question 3 (a)

3 The roots of the equation $2x^2 + 3x + 5 = 0$ are denoted by α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. **[2]**

Almost all candidates scored full credit for this question part. The small number who lost a mark usually did so by mislaying a negative sign.

Question 3 (b)

(b) Using the answers to part **(a)** determine the value of each of the following.

- $\alpha^2 + \beta^2$
- $\frac{1}{\alpha} + \frac{1}{\beta}$

[4]

Candidates did well on this question part. Some candidates did not follow the instruction to use the answers to the previous part and instead used a substitution to find a related quadratic – this was most commonly seen in the $\frac{1}{\alpha} + \frac{1}{\beta}$ case. A few candidates made sign errors.

Question 4 (a) (i)

4 Two transformations, T_A and T_B, are represented by matrices $\mathbf{A}$ and $\mathbf{B}$ respectively.

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) **(i)** Describe the transformation T_A. **[1]**

Almost all candidates gained the mark here.

Question 4 (a) (ii)

(ii) Explain geometrically why $\mathbf{A}^{-1} = \mathbf{A}$. **[1]**

This part was found to be more difficult with approximately half of the candidates scoring 0 marks. Most that did not gain the mark did not provide a geometrical explanation, instead finding the inverse matrix explicitly. Some of those who attempted a geometrical explanation said that the reason that the transformation was self-inverse was that it had a line of invariant points.

Question 4 (b)

The matrix $\mathbf{B}$ is given by $\mathbf{B} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(b) Describe the transformation T_B. **[2]**

This was generally done well, but a few candidates wrote down the wrong angle of rotation or described the transformation as a combination of two transformations (usually a rotation and an enlargement).

Question 4 (c)

The transformation T_C is equivalent to T_A followed by T_B.

(c) Determine the single matrix which represents T_C. **[2]**

Many candidates gained full credit here with a minority performing the multiplication in the wrong order and a few making a sign error in their calculation.

Question 5 (a)

5 The locus L is defined by $L = \{z : z \in \mathbb{C}, |z - (20 + 15\text{i})| \leqslant 7\}$.

(a) On the Argand diagram in the Printed Answer Booklet, sketch and label L. **[2]**

Almost all candidates knew what was expected, although a sizeable number lost a mark through not indicating that the interior of the circle was part of the set, not clearing showing that the radius was equal to 7, or by making a sign error with the coordinates of the centre of the circle.

Question 5 (b)

(b) Determine the value of $z \in L$ for which the value of $|z|$ is smallest. Give your answer in cartesian form. **[3]**

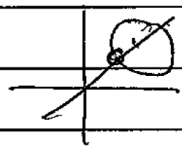
This question part was found to be difficult, although not quite as difficult as Question **5 (c)**. A lot of candidates were not sure how to solve this problem.

Some candidates correctly found the smallest value of $|z|$ but then stopped, not realising that they had to find the value of z where this minimum was obtained. Several candidates choose to set up simultaneous equations for the line joining the origin and the centre of the circle and the equation of the circle and attempted to solve this. Some were successful, but this was a less efficient and more error prone approach than the main solution in the mark scheme.

Exemplar 1

5(b)

$|z|$ is smallest



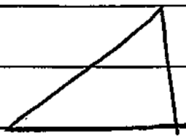
$(x-20)^2+(y-15)^2=49$

$x^2-40x+400+y^2-30y+225=49$

~~[illegible]~~

~~[illegible]~~ $x^2-40x+y^2-30y=-576$

$x(x-40)+y(y-15)=-576$



15

20

Gradient $=\frac{15}{20}=\frac{3}{4}$

$y=\frac{3}{4}x$

$x(x-40)+\frac{9}{16}x^2-\frac{45}{2}x=-576$

$x=\frac{128}{5}, \frac{72}{5}$

smaller

$\frac{25}{16}x^2-\frac{125}{2}x+576=0$

$x=14.4$

$25x^2-1000x+9216=0$

$y=10.8$

$=14.4+10.8i$

This candidate used an alternate method effectively to solve the problem, but it is a lengthier method with more opportunity for making a mistake. Most candidates who attempted this method gained the first mark for finding the equation of the straight line, but then either made no further process or made mistakes when solving the simultaneous equations.

Question 5 (c)

(c) Determine the largest value of $\arg(z)$ for $z \in L$. **[3]**

This was the question part was skipped by many with some candidates omitting a response. Some candidates thought that the point with greatest argument must be the one at 13+15i. Some other candidates drew a useful diagram but then placed the right angle of their triangle between the radius and the line joining the origin to the centre of the circle, rather than correctly between the radius and the tangent.

Several candidates used the fact that the line with greatest argument is a tangent along with simultaneous equations to set up a quadratic equation with only one root. These candidates often managed to complete the argument successfully to find the correct solution, but this method was longer and more complicated than considering the relevant right angled triangle.

Question 6 (a)

6 The equations of two lines, l_1 and l_2, are $l_1 : \mathbf{r} = \begin{pmatrix} 16 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -27 \\ -19 \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 10 \\ -10 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 10 \\ 10 \end{pmatrix}$.

(a) Show that l_1 and l_2 intersect at a single point, P, giving the coordinates of P. **[5]**

A sizeable majority of candidates gained full credit for this part. Those that lost a mark usually did not show clearly that their values of λ and μ satisfied all three coordinate equations, sometimes as it was not possible to tell which two equations they had used to find λ and μ.

Candidates should make sure that they clearly show which equations they are solving simultaneously and which one they are substituting into to check that the lines do indeed intersect.

Question 6 (b)

O is the origin of the coordinate system. The point Q lies on the line segment OP.

(b) Comment on the claim that the distance OQ is less than 100. **[2]**

Most candidates gained the mark for finding the distance OP. The majority wrote down a correct comment on the stated claim, but some candidates rounded OP to 100 and then stated that this meant that the distance OQ must be less than 100. Some other candidates made erroneous statements such as 'since OP >100 then we must have OQ <100'.

Question 7

7 Prove by induction that $n! > 20^n$ for all integers $n \geqslant 52$. **[5]**

Most candidates gained the marks for considering the base case and for stating the assumption when n = 52.

When it came to completing the proof by induction, many candidates did not gain credit as their logic was not clear enough or because their manipulation of inequalities was not fluent. The majority of candidates started by writing down the result they were aiming for, i.e. $(k+1)! > 20^{k+1}$, and then trying to manipulate this into the kth case, not always entirely logically or convincingly. A lot of candidates mislaid brackets in their proof, usually by writing $(k+1)! = k + 1 \times k!$ which was penalised as not being a formal, complete and correct proof.

The best candidate solutions started by writing down the $n = k$ case, i.e. . $k! > 20^k$, and then multiplying throughout by $(k + 1)$.

Exemplar 2

Assume it is true for $n = k$:

$k! > 20^k$

$(k+1)! > (k+1) \times 20^k$

Since $k \geqslant 52$ $k+1$ is greater than 20

$\therefore (k+1) \times 20^k > 20^k \times 20$

$\therefore (k+1) \times 20^k > 20^{k+1}$

Since $(k+1)! > (k+1) \times 20^k$,

the $(k+1)!$ must always be ~~great~~ greater than 20^{k+1}

$\therefore$ it is true for $n = k+1$

This response has written out the induction step in a clear and logical way, starting with the $n = k$ case and multiplying throughout. This was a much clearer method than starting with $(k+1)! > 20^{k+1}$ and trying to justify that this was true.

Question 8

8 Matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 5+a & 5 & 1 \\ 3 & 13+a & 6 \\ a-4 & -20 & -9 \end{pmatrix}$ where a is a constant and all entries of $\mathbf{A}$ are integers.

The transformation represented by $\mathbf{A}$ is applied to a shape of volume 8 units.
The image shape has volume 40 units and the orientation of the image is reversed.

Determine the image under $\mathbf{A}$ of the point $(1, 2, 3)$. **[7]**

There were a lot of good responses to this question but many candidates struggled with the algebraic manipulation required to simplify their expression for the determinant which mean that they did not obtain the correct quadratic expression. A reluctance to include brackets and sign errors were what generally led to algebraic manipulation errors. Most candidates could state that det $\mathbf{A}$ = -5 but some missed the negative sign.

Question 9 (a)

9 **(a)** Prove that if z is any complex number then $z(z^*) = |z|^2$. **[1]**

Most candidates gained the mark here, but some mislaid brackets and some claimed that $|z|^2 = a^2 + (ib)^2$

Question 9 (b)

(b) Prove that if w and z are any complex numbers then $(wz)^* = w^* z^*$. **[2]**

Many candidates gained full credit here, but again some had missing brackets which meant that their proof was not convincing.

Question 9 (c)

The complex number v is defined by $v = (10 + 3\text{i})(9 + 4\text{i})$.

(c) **In this question you must show detailed reasoning.**

Express v in the form $a + b\text{i}$ where a and b are real. **[2]**

This question part seemed to be one of the most accessible in the paper with just a few candidates omitting or losing a mark through arithmetical slips.

Question 9 (d)

(d) **In this question you must show detailed reasoning.**

By considering $v(v^*)$, write 10573 as the product of two prime factors. **[3]**

Despite the scaffolding from the previous three parts, candidates found this question part to be difficult. The vast majority scored one mark for calculating $v(v^*)$ but made no further credit. A small number of candidates managed to complete the question convincingly.

Exemplar 3

9(d)	$(78+67i)(78-67i) = 6084 - 5226i + 5226i + 4489$
	$= \cancel{1595}\ 10573$
	$\cancel{V(wz)} = V^* = \cancel{(4)}(10+3i)(9-4i)$ as V^* $(wz)^* = w^* z^*$
	$(10-3i)(9-4i)(10+3i)(9+4i) = V(V^*)$
	$(10-3i)(10+3i) = 100+30i-30i-9i = 109$
	$(9-4i)(9+4i) = 81+36i-36i-16i = 97$
	$10573 = 109 \times 97$

This candidate successfully applied the ideas from the previous parts, especially Question **9 (b)**, to solve this problem in this question part.

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