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Examiners' report

AS LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A

H235

For first teaching in 2017

Y535/01 Summer 2025 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y535 series overview

Y535 is one of the optional units for AS Further Mathematics A. Candidates generally seemed to be well prepared for the paper.

For some questions candidates produced strong responses, for instance Question 2 on finding a vector product and using it to calculate the area of a triangle, Question 3 on sections, contours and stationary points of a 3-D surface, parts of Question 4 on groups. Other questions, although less successful, had accessible marks, for example Question 5 on proof by induction and Question 6, the modelling question. There were also questions which candidates found challenging, like Question 4 (b) (ii) on finding subgroups of a group and explaining why there are no others, Question 4 (b) (iv) on showing that a group is abelian and Questions 7 (b) and (c) on number theory. Questions that require explanation, justification, supporting arguments, or include problem-solving elements tend to increase the challenge for candidates.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• carefully read the questions to check that they used the correct method and met all the requirements • applied their knowledge efficiently and effectively by choosing the simplest method to solve the problem • were able to use and apply standard techniques with confidence • used mathematical language correctly to communicate their reasoning clearly • understood the issues in the modelling question and knew how to use the INT function (Question 6) • used calculators appropriately and efficiently.	• did not gain marks due to arithmetic or algebraic mistakes • showed gaps in their knowledge of standard techniques • did not show sufficient working to justify their solutions in 'Determine', 'Show that' or 'Prove' questions • tried to prove general facts about groups by providing specific examples only.

Question 1

1 Use standard divisibility tests for small numbers to show that 2 573 208 is divisible by 792. **[4]**

Roughly half of the candidates gained full marks in this question. These candidates were familiar with divisibility tests by 8, 9 and 11 and were able to apply each rule accurately and efficiently, writing their reasoning clearly and logically, rather than just replying 'yes' or giving a result without explanation.

Some candidates did not know how to factorise 792 into useful factors (8, 9, and 11), but used instead 2 or 3, which were not sufficient on their own and did not lead to a complete solution. There was also some confusion between powers and multiples. A number of candidates believed that 9 was not a valid test unless it was written explicitly as 3^2.

Question 3 (b)

(b) Determine the coordinates of all stationary points of S. **[6]**

The vast majority of candidates gained at least two marks in this question part, with half of the responses earning full marks. Candidates who performed well followed a clear strategy. They understood that stationary points occur when both partial derivatives vanish and recognised this approach as part of multivariable calculus. They took care with their differentiation, writing down each step in a structured manner to reduce the chance of error and to allow for checking. After differentiating, they worked systematically, isolating one variable and substituting it into the other equation with care, taking the time to check their algebra at each stage and ensuring no critical steps were skipped. Importantly, they remembered that a quadratic equation could yield two roots, as was the case here. Finally, these candidates substituted their x and y values back into the original expression for z, clearly stating all coordinates of the stationary points.

Some candidates encountered difficulties when solving the system of equations, with the ensuing algebraic step proving challenging for many. Some struggled with rearranging the equations or squaring terms correctly, which led to incorrect or incomplete solutions. Another issue was that some candidates missed one of the valid solution pairs. In some cases, candidates gave additional, incorrect solutions, which meant that only 5 out of 6 marks could be awarded. Even when the working was mostly correct, some responses did not provide the full set of (x, y, z) coordinates of the stationary points, thereby not gaining the final A mark.

Question 3 (c)

(c) The contour C of S is given by $z = 37$.

Find the coordinates of the unique point on C where $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial y \partial x} = 0$. **[4]**

About a half of the candidates gained full marks in this part question. These correctly found the required second partial derivatives (B mark), substituted them in the given expression to find a relationship between x and y (first M mark), then used the latter to express one variable in terms of the other and substituted it in $z = 37$ to obtain a cubic in a single variable (second M mark). Finally, the A mark was for solving the cubic (by calculator was a valid method here) and providing the values of x and y (the absence of $z = 37$ was condoned here). Most candidates, despite obtaining the correct cubic equation, did not earn the final A mark due to errors in their algebraic manipulations. Other common reasons for not earning full marks were errors in the second partial derivatives, which lead to the incorrect equation (possibly not even a cubic one), thus resulting in the loss of the B and A marks and possibly also the second M mark.

Question 4 (a) (i)

4 The binary operation * is defined on the set $A = \{1, 3, 5, 7, 9\}$ by $x * y = x + y + 3 \pmod{10}$ for all $x, y \in A$.

(a) **(i)** Show that * is associative on the elements of A. **[2]**

In this question, candidate performance was very mixed, with an even spread of candidates gaining no, one or two marks.

Some candidates struggled to show associativity clearly and correctly (AO2.1 - construct rigorous mathematical arguments, including proofs). A common issue was using only one numerical example, without general reasoning, which earned one mark.

Successful candidates used algebra to demonstrate that the operation was associative; they carefully grouped the expressions, added the constant at the correct stage and showed clearly that both groupings produced the same result after applying the modulo operation.

Those who used a numerical example and earned one mark, did so with clear step-by-step working, computing both sides of the expression separately and showing that the results were the same. They made sure to include the modulo reduction at the right stage, confirming that the operation behaved as expected.

Question 4 (a) (ii) and (iii)

(ii) Complete the Cayley table for $(A, *)$ given in the Printed Answer Booklet. **[2]**

(iii) Hence show that $(A, *)$ forms a group, G. **[4]**

This part question assessed three strands of AO2, Reason, interpret and communicate mathematically:

AO2.4 - explain their reasoning

AO2.2a - make deductions

AO 2.1 - construct rigorous mathematical arguments, including proofs.

The vast majority of responses gained either three or four marks in this question part. Candidates who performed well followed a structured approach, beginning by showing that the operation was closed because only elements from the original set appeared in the Cayley table. They then correctly identified the identity element and also found the correct inverse of each element. Those who had already shown associativity in part (a) (i) referred to it, ensuring their group structure was complete. They avoided adding irrelevant comments about commutativity or the order of the group, staying focused on the four group properties.

Among the responses which gained fewer than three marks, a common error was to vaguely refer to 'the table' without actually using it to show closure. The mark scheme made it clear that just saying 'it is closed from the table' was not enough—candidates needed to explain *why* the table showed closure. Moreover, a significant number of candidates generically referred to the existence of inverses for each element, without stating the inverse pairs.

Some candidates mistakenly discussed commutativity or order, which were irrelevant and not credited.

Question 4 (b) (i), (ii), (iii) and (iv)

(b) **(i)** State the order of each non-identity element of G. **[1]**

(ii) List all subgroups of G, and explain why there are no others. **[2]**

(iii) Explain whether G is cyclic. **[1]**

(iv) Explain whether G is abelian. **[1]**

Candidates generally performed very well in part (b) (i), with most of them stating that all non-identity elements of G have order 5. On the other hand, candidate performed less well in part (b) (ii). Many candidates were able to gain at least one by listing the two possible subgroups of G ($\{e\}$ and G itself). The second B mark, for explaining why there were no other subgroups, was something candidates found difficult and was accomplished by relatively few of them. Interestingly, a significant proportion of these gave an explanation based on Lagrange's theorem on the order of a subgroup of a finite group, which is Stage 2 subject content and therefore not required at AS Level. In part (b) (iii), about two thirds of the candidates gained the available mark, by stating that group G is cyclic and explaining why, as instructed in the question. In general, the third of the responses which did not gain the mark did state that G was cyclic, but only vaguely referred to the presence of a generator, without specifying which or of what order (AO 2.2a – make deductions).

In part (b) (iv), approximately a quarter of the candidates earned the available mark (also AO 2.2a – make deductions) for stating that G was abelian and using algebra to demonstrate that the operation was commutative (not forgetting to apply the modulo operation). Most responses provided a numerical example, which did not earn the available mark. Very few candidates used the fact that all cyclic groups are abelian or that all elements of a cyclic group are powers of a generator and therefore commute, which gained the B mark.

Misconception

?

A certain number of students were not entirely familiar with the definition of a group (OCR Ref. 8.03c) and included comments on order of elements/commutativity among the requisites to show that the given structure was a group.

OCR support

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For support material on groups, see [Delivery Guide: Section 8.03 Groups].

Question 5

5 Let $f(n) = 3^n + 4^n$ for all positive integers n.

Prove by induction that $f(n)$ is a multiple of 7 for all **odd** integers $n \geqslant 1$. **[5]**

In this question, which assessed a variety of reasoning and problem-solving AOs, approximately a half of the candidates obtained between three and five marks.

Successful candidates began by evaluating the given expression for $n = 1$ and showed explicitly that it produced 7, which is divisible by 7. They clearly stated that this confirmed the base case (AO2.5 – use mathematical language and notation correctly). On the other hand, some candidates did not show that the result was divisible by 7, thereby not gaining the first B mark.

Some issues arose in the inductive step (AO2.1 construct rigorous mathematical arguments, including proofs), which was required for the second B mark. Some candidates attempted to show the result directly for the next odd number without connecting it properly to the assumption that the statement held for a previous odd number. There was also confusion between whether k was increasing by 1 or by 2. Since the question dealt specifically with odd positive integers n, the correct inductive step required assuming that if the statement was true for one odd positive integer $(2k + 1)$, it must be true for the next odd positive integer $(2k + 3)$. Some candidates mistakenly worked with k and $k + 1$, which did not align with the logic required.

Candidates who earned the M mark and the first A mark showed careful manipulation of indices (AO3.1a - translate problems in mathematical contexts into mathematical processes), using the inductive assumption to replace or factorise parts of the expression. Some worked modulo 7 and used the fact that $3^2 \equiv 2$ and $4^2 \equiv 2$ mod 7 to simplify the algebra and prove the result more efficiently. Others rewrote the expression using known identities or factorisations, while yet others used substitution effectively.

For the final A mark, responses needed to have a clear statement that the result was a multiple of 7, completing the inductive step (AO2.4 - explain their reasoning).

Exemplar 1

5 $f(n) = 3^n + 4^n$

If n is odd $n = 2K+1$

When $n=1$ $f(1) = 3^1 + 4^1 = 7$

$7|7$ thus this is true for $n=1$

If true for $n = 2K+1$

such that $3^{2K+1} + 4^{2K+1} = 7m$

then is it true for $n = 2(K+1)+1$
$n = 2K+3$?

$3^{2K+3} + 4^{2K+3} = 9 \cdot 3^{2K+1} + 16 \cdot 4^{2K+1}$

$= (3^{2K+1} + 4^{2K+1}) \times 9 + 7 \times 4^{2K+1}$

$= 7m \times 9 + 7 \times 4^{2K+1}$

$= 7(9m + 4^{2K+1})$

Thus $7 | 3^{2K+3} + 4^{2K+3}$ if
$7 | 3^{2K+1} + 4^{2K+1}$

As this is true for $n=1$ $K=0$ then by induction it must be true for all $n \geq 1$ where $n = 2K+1$ meaning it is odd.

In this exemplar the candidate lays out a very clear, well-reasoned response until the conclusion. They start by proving that the base case holds, explicitly stating that 7|7 (first B mark). This is followed by the inductive hypothesis, where $3^n + 4^n$ is assumed to be equal to $7m$ (and not just declared to be 'a multiple of 7') for $n = 2k + 1$, which gains the second B mark. The next step is to consider $3^n + 4^n$ for $n = 2k + 3$ and to start showing that it is also a multiple of 7 by carrying out some index work and isolating 9 and 16 (the M mark). The first A mark is for showing the required result by factorising 7. This response does not achieve the second A mark as the conclusion is incorrect.

Assessment for learning

Many responses offered an insufficient written conclusion, with some degree of confusion between n and k, $n = 1$ and $n = k$, etc. It is important that candidates understand and are able to formulate a correct conclusion in proofs by induction.

Question 6 (a)

6 An investment company offers customers a three-year investment scheme.

The scheme is modelled by the recurrence system

$I_0 = \alpha$ and $I_{n+1} = 0.000625\, I_n^{\,2} - 0.625 I_n$ for $n \geqslant 0$.

I_n is the value in pounds of the scheme n years after its start, and £α is the **integer** value of the initial sum invested.

(a) Determine the minimum initial sum invested that guarantees that the value of the scheme increases every year. **[3]**

Candidates generally found this question part challenging, with roughly half of them gaining one or more marks. The first M mark was for considering $I_{n+1} > I_n$ (and not just $I_1 > I_0$) and tested AO3.3 (translate situations in context into mathematical models). The second M mark was for solving the inequality and obtaining $I_n > 2600$ and assessed AO3.1a (translate problems in mathematical contexts into mathematical processes). Finally, the A mark was for interpreting the solution in context (AO3.2a) and stating that the minimum initial investment sum had to be £2601, given that it was an integer. Very few responses gained all three marks. Furthermore, a significant number of candidates derived 2600 by setting $I_1 = I_0$ in the recurrence relation or by finding the 'fixed point' ($I_{n+1} = I_n = L$), thereby earning the special case B mark, and then went on to state the minimum value of £2601, gaining also the A mark.

Exemplar 2

6(a)

$I_{n+1} = 0.000625 I_n^2 - 0.625 I_n$

let $I_{n+1} = I_n = L$

$L = 0.000625 L^2 - 0.625 L$

$L(1.625) = L^2(0.000625)$

$L = \frac{1.625}{0.000625} = 2600$

if $\alpha = 2600$ then it stays constant

$\therefore$ let $\alpha = 2601$

$\alpha = 2601 \rightarrow$ converges to 2602.6

all values converge

This exemplar demonstrates a commonly observed response, where candidates look for the 'fixed point' of the recurrence relation by setting $I_{n+1} = I_n = L$, obtaining a quadratic equation and solving for L. The absence of rejection of the solution $L = 0$ was not penalised. Although this response provides the 'correct' value of 2600, this method is incomplete as it does not address the fact that the sequence is increasing and therefore gains the special B mark but not the two M marks. The final A mark is then given for the figure of £2601 as the minimum amount to be invested.

Assessment for learning

Students should be reminded that the command word **Determine** signposts that justification should be given for any results found, including working where appropriate.

More information about command words can be found in Section 2b of the specification. OCR have also produced a poster along with some blogs to help understand these command words.

Question 6 (b) (i) and (ii)

(b) The investment company decides to make the following change in the scheme.

At the end of each year, the value of I_n is always rounded **down** to the nearest pound.

(i) Write down a modified recurrence formula that takes this change into account. **[1]**

(ii) A customer invests £2700 in the three-year investment scheme.

Work out how much less the customer's investment would be worth after three years under the new scheme compared to the old scheme. **[6]**

Part (b) (i) had quite a high omit rate, despite similar questions on the INT function often featuring in Y535 past papers. Slightly more than a half of the candidates gained the available mark here. A commonly observed, incorrect response was to write the correct expression with INT and then subtract 1, misinterpreting the information on the value of I_n being rounded down to the nearest pound.

Many candidates who attempted part (b) (ii) managed to obtain at least between one and four marks, with slightly fewer than half of the responses earning all the six marks available. The omit rate was also relatively high here. Marks were typically lost for arithmetic errors when using either the original recurrence relation or the modified one with INT (or both) to find the value of the investment after three years. Substitution of £2700 in an incorrect modified recurrence relation from part (b) (i) could still gain a method mark but would result in the loss of an accuracy mark. A small proportion of candidates only calculated I_3 under the original scheme, thereby gaining only the first two marks. This part question tested AO 3.1a (translate problems in mathematical contexts into mathematical processes) and AO3.4 (use mathematical models).

Question 7 (a) (i) and (ii)

7 Consider the two arithmetic sequences $A_n = \{2n+7 : n \in \mathbb{N}\}$ and $B_n = \{3n+1 : n \in \mathbb{N}\}$.
Let $h = \text{hcf}(A_n, B_n)$ for each chosen value of n.

(a) **(i)** Find a value of n for which $h = 1$. **[1]**

(ii) Find a value of n for which $h = 19$. **[2]**

Almost all candidates gained the available mark for finding a value of n for which $h = 1$ in part (a) (i). In part (a) (ii), performance was more mixed, with the majority of candidates gaining full marks. Of those who didn't, most gained no marks. Given that the command work was '**Find**', it was enough to state for example $n = 6$ (or $n =$ any other value for which $h = 19$) to receive both marks, and indeed this was a common recurrence.

Assessment for learning

Candidates should be reminded that the command word '**Find**' entails that no justification needs to be given for any results found, even though working may be necessary to answer the question.

Question 7 (b)

(b) Determine all the values of n for which $h = 19$. Give your answer as an expression in terms of an integer k. **[2]**

This question tested AO3.1a (translate problems in mathematical contexts into mathematical processes), which candidates found challenging. Many candidates did not succeed in achieving any mark. About a quarter gained the first mark for either considering either $2n + 7 \equiv 0$ or $3n + 1 \equiv 0$ mod 19 or for stating $n = 19k + 6$ without proof. The A mark required candidates to either fully solve the simultaneous linear congruences or to verify that $n = 19k + 6$, obtained from one linear congruence, was also a solution of the other. This was accomplished by only a small proportion of candidates.

Question 7 (c)

(c) By considering $3A_n - 2B_n$, show that A_n and B_n are co-prime for all values of n that are not given by the expression found in part **(b)**. **[4]**

This final question part on number theory tested both problem-solving and reasoning:

AO3.1a - translate problems in mathematical contexts into mathematical processes

AO2.2a – make deductions

About one quarter of responses gained no mark here. One half of responses gained the first M mark for evaluating $3A_n - 2B_n$ as 19, as instructed by the question. After that, about a quarter of candidates were able to make the connection with h, namely that $h|19$ thus gaining the second M mark. Some responses then concluded that either $h = 1$ or $h = 19$ (for the first A mark) and therefore, since h was not 19, it followed that $h = 1$

Exemplar 3

7(c)

$h | 3A_n - 2B_n$

$h | 3(2n+7) - 2(3n+1)$

$h | 6n+21 - 6n - 2$

$h | 19$

$h | 19.$

19 is prime so h is either 1 or 19.

$1 | 19$ or $19 | 19$.

This response encapsulates what was required to gain the first three marks and does that in a very precise and concise manner. Many candidates provided lengthy explanations which, however, missed the necessary points. The candidate obtains the first M mark for the evaluation of $3A_n - 2B_n$ as 19. They also receive the second M mark for stating that h must divide $3A_n - 2B_n$, namely 19. They then conclude that h can only be 1 or 19, given that 19 is prime, and in so doing they obtain the first A mark. Only the second A mark is withheld as the conclusion that A_n and B_n are coprime for $h = 1$ (since $h \neq 19$) is missing.

Misconception

A commonly observed misconception was that, since $h|19$ and 19 is prime, then $h = 1$. A second misconception was that candidates, having found that $3A_n - 2B_n = 19$, concluded that $19|3A_n - 2B_n$ and then tried to prove that $19|A_n \Leftrightarrow 19|B_n$, rather than considering their hcf.

OCR support

For support material on number theory, see Delivery Guide: Section 8.02 Number theory.

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