OCR
Oxford Cambridge and RSA

# Friday 6 June 2025 – Afternoon

## AS Level Further Mathematics A

### Y535/01 Additional Pure Mathematics

**Time allowed: 1 hour 15 minutes**



**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

**ADVICE**
- Read each question carefully before you start your answer.

**1** Use standard divisibility tests for small numbers to show that $2\,573\,208$ is divisible by 792. **[4]**

**2** The vectors **p**, **q** and **r** are such that $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 2 \\ -4 \\ 9 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 11 \\ 1 \\ -2 \end{pmatrix}$.

**(a) (i)** Determine the value of the integer $k$ for which $\mathbf{p} \times \mathbf{q} = k\mathbf{r}$. **[2]**

**(ii)** Use the result of part **(a)(i)** to explain **geometrically** why $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r} = \mathbf{0}$. **[1]**

Relative to the origin $O$, the points $P$ and $Q$ have position vectors **p** and **q** respectively.

**(b)** Show that the area of triangle $OPQ$ can be written in the form $\frac{1}{2}n\sqrt{n-1}$ where $n$ is a positive integer to be determined. **[3]**

**3** The surface $S$ has equation $z = 2x^2y - 6y^3 + 3x + 4$ for all real values of $x$ and $y$.

**(a) (i)** State the equation of the section of $S$ cut by the plane $y = 1$. **[1]**

**(ii)** Sketch the section of $S$ cut by the plane $y = 1$. Give the coordinates of the points of intersection with the axes. **[2]**

**(b)** Determine the coordinates of all stationary points of $S$. **[6]**

**(c)** The contour $C$ of $S$ is given by $z = 37$.

Find the coordinates of the unique point on $C$ where $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial y \partial x} = 0$. **[4]**

**4** The binary operation * is defined on the set $A = \{1, 3, 5, 7, 9\}$ by $x * y = x + y + 3 \pmod{10}$ for all $x, y \in A$.

**(a) (i)** Show that * is associative on the elements of $A$. **[2]**

**(ii)** Complete the Cayley table for $(A, *)$ given in the Printed Answer Booklet. **[2]**

**(iii)** Hence show that $(A, *)$ forms a group, $G$. **[4]**

**(b) (i)** State the order of each non-identity element of $G$. **[1]**

**(ii)** List all subgroups of $G$, and explain why there are no others. **[2]**

**(iii)** Explain whether $G$ is cyclic. **[1]**

**(iv)** Explain whether $G$ is abelian. **[1]**

**5** Let $f(n) = 3^n + 4^n$ for all positive integers $n$.

Prove by induction that $f(n)$ is a multiple of 7 for all **odd** integers $n \geqslant 1$. **[5]**

**6** An investment company offers customers a three-year investment scheme.

The scheme is modelled by the recurrence system

$I_0 = \alpha$ and $I_{n+1} = 0.000625\, I_n^2 - 0.625 I_n$ for $n \geqslant 0$.

$I_n$ is the value in pounds of the scheme $n$ years after its start, and £$\alpha$ is the **integer** value of the initial sum invested.

**(a)** Determine the minimum initial sum invested that guarantees that the value of the scheme increases every year. **[3]**

**(b)** The investment company decides to make the following change in the scheme.

At the end of each year, the value of $I_n$ is always rounded **down** to the nearest pound.

**(i)** Write down a modified recurrence formula that takes this change into account. **[1]**

**(ii)** A customer invests £2700 in the three-year investment scheme.

Work out how much less the customer's investment would be worth after three years under the new scheme compared to the old scheme. **[6]**

**7** Consider the two arithmetic sequences $A_n = \{2n+7 : n \in \mathbb{N}\}$ and $B_n = \{3n+1 : n \in \mathbb{N}\}$.
Let $h = \text{hcf}(A_n, B_n)$ for each chosen value of $n$.

**(a)** **(i)** Find a value of $n$ for which $h = 1$. **[1]**

**(ii)** Find a value of $n$ for which $h = 19$. **[2]**

**(b)** Determine all the values of $n$ for which $h = 19$. Give your answer as an expression in terms of an integer $k$. **[2]**

**(c)** By considering $3A_n - 2B_n$, show that $A_n$ and $B_n$ are co-prime for all values of $n$ that are not given by the expression found in part **(b)**. **[4]**

**END OF QUESTION PAPER**