OCR
Oxford Cambridge and RSA

Friday 23 May 2025 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

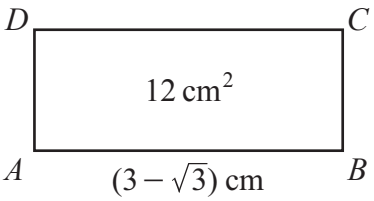
$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A
Pure Mathematics

1 Determine the equation of the line that passes through the point (1, 3) and is perpendicular to the line with equation $3x + 6y - 5 = 0$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers to be determined. **[3]**

2 **In this question you must show detailed reasoning.**



The diagram shows the rectangle $ABCD$ where $AB = (3 - \sqrt{3})$ cm.

The area of $ABCD$ is 12 cm^2.

Find the perimeter of $ABCD$. Give your answer in the form $(a + b\sqrt{3})$ cm, where a and b are integers to be determined. **[4]**

3 In a triangle ABC, $AB = 9$ cm, $BC = 7$ cm and $AC = 4$ cm.

(a) Show that $\cos CAB = \frac{2}{3}$. **[2]**

(b) Hence find the exact value of $\sin CAB$. **[2]**

(c) Find the exact area of triangle ABC. **[2]**

4 The cubic polynomial $2x^3 - kx^2 + 4x + k$, where k is a constant, is denoted by $f(x)$.
It is given that $f'(2) = 16$.

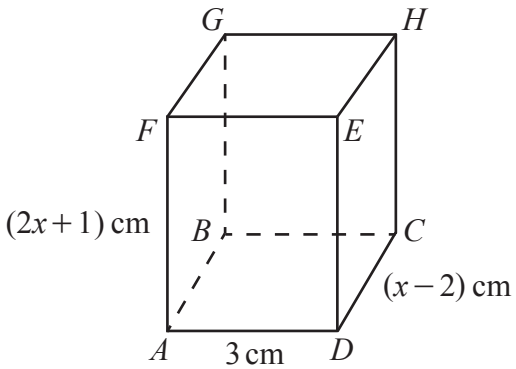
(a) Show that $k = 3$. **[4]**

For the remainder of the question, you should use this value of k.

(b) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$. **[2]**

(c) Hence show that the equation $f(x) = 0$ has only **one** real root. **[4]**

5 **In this question you must show detailed reasoning.**



The diagram shows the cuboid $ABCDEFGH$ where $AD = 3$ cm, $AF = (2x + 1)$ cm and $DC = (x - 2)$ cm.

The volume of the cuboid is at most $9\,\text{cm}^3$.

Find the range of possible values of x. Give your answer in interval notation. **[5]**

6 **(a)** The curve $y = \ln x$ is stretched by a scale factor 3 in the x-direction.

State the equation of the curve after it has been stretched. **[2]**

(b) Describe fully the **single** transformation that transforms the curve $y = x^2 + 1$ to the curve $y = x^2 - 6x + 8$. **[4]**

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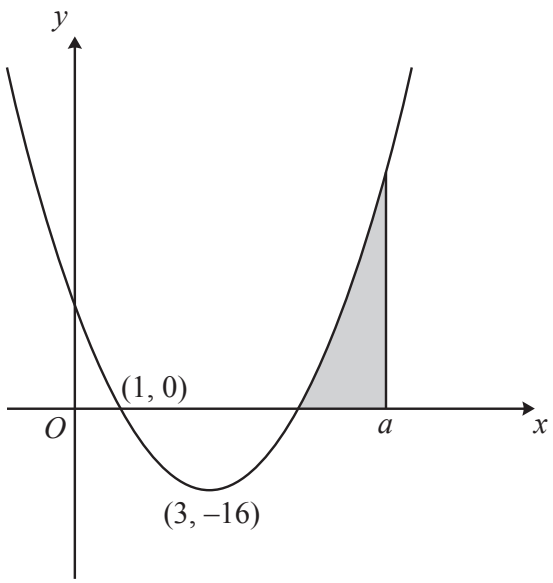

The diagram shows the curve with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function. The curve intersects the x-axis at the point $(1, 0)$ and has its minimum at the point $(3, -16)$.

(a) Show that $\mathrm{f}(x) = 4x^2 - 24x + 20$. **[3]**

The shaded region is enclosed by the curve, the x-axis, and the line $x = a$. The area of the shaded region is $\frac{100}{3}$ units^2.

(b) Determine the exact value of the constant a. **[6]**

8 The curves $\mathrm{e}^x - 2\mathrm{e}^y = 1$ and $2\mathrm{e}^x + 3\mathrm{e}^{2y} = 41$ intersect at the point P.

(a) Show that the y-coordinate of P satisfies the equation $3\mathrm{e}^{2y} + 4\mathrm{e}^y - 39 = 0$. **[1]**

(b) **In this question you must show detailed reasoning.**

Hence find the exact coordinates of P. **[5]**

Section B
Mechanics

9 An athlete runs with velocity $v\,\text{m s}^{-1}$ along a horizontal straight running track AB. At time t seconds after passing A, $v = 0.05t^2 - 0.4t + k$, where k is a constant.

(a) Find the acceleration of the athlete when $t = 6$. **[2]**

(b) Given that the minimum velocity of the athlete is $6.6\,\text{m s}^{-1}$, show that $k = 7.4$. **[3]**

The athlete runs in a straight line from A to B in 13.1 seconds.

(c) Calculate the distance AB. **[2]**

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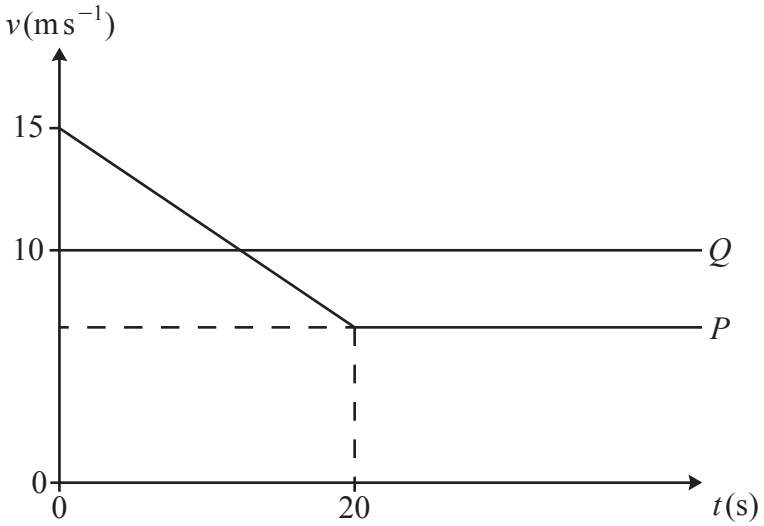

The velocity-time graph represents the motion of two cyclists P and Q who are travelling along a straight, horizontal road. At time $t = 0$, the cyclists pass the same point on the road with P cycling at $15\,\text{m s}^{-1}$ and Q cycling at $10\,\text{m s}^{-1}$.

From time $t = 0$, P cycles with constant deceleration $0.4\,\text{m s}^{-2}$ for 20 seconds and Q cycles at a constant velocity of $10\,\text{m s}^{-1}$. From $t = 20$ onwards, P's velocity remains constant.

(a) Find the value of t when P and Q have the same velocity. **[2]**

(b) Determine the value of t when Q overtakes P. **[4]**

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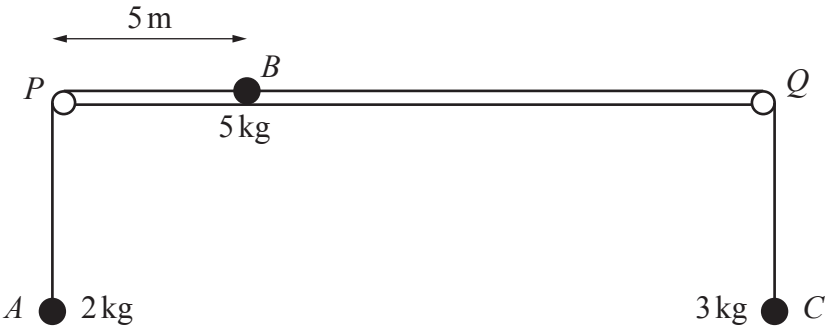

One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 5 kg. Particle B is in contact with a smooth horizontal surface. The string passes over a small smooth pulley P which is fixed at one edge of the surface. Particle A hangs vertically below P.

A second light inextensible string is attached to B. The other end of this second string is attached to a third particle C of mass 3 kg. This string passes over a small smooth pulley Q which is fixed at the other edge of the surface. Particle C hangs vertically below Q (see diagram).

The system is released from rest with the strings taut and B a distance 5 m from P. In the subsequent motion A does not reach P.

(a) Determine, in terms of g, the tension in the string connecting B and C. **[4]**

When the particles have been in motion for 3 seconds the string connecting A and B breaks. Particle B reaches Q after a further 2 seconds.

(b) Determine the distance PQ. **[6]**

(c) State what it would mean about the tension in the string connecting B and C if pulley Q could **not** be assumed to be smooth. **[1]**

It is given that in fact the horizontal surface is **not** smooth. When the system, as shown in the diagram, is released, it remains at rest. (You may assume that pulley Q **is** smooth.)

(d) Complete the diagram in the Printed Answer Booklet showing all the forces acting on B. **[1]**

(e) State the magnitude of the frictional force acting on B. Give a brief justification for your answer. **[1]**

END OF QUESTION PAPER