OCR
Oxford Cambridge and RSA

Thursday 5 June 2025 – Morning

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2506

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ________________

Last name ________________

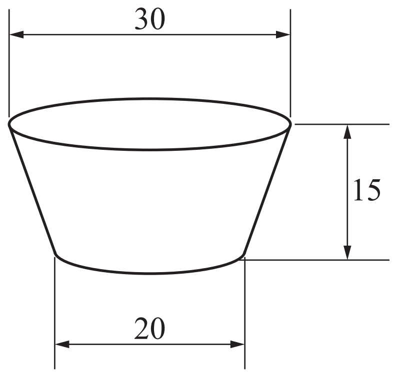
Date of birth | D | D | M | M | Y | Y | Y | Y |

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined page at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

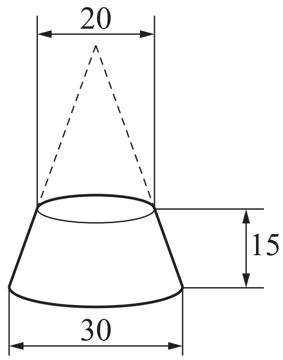
ADVICE
- Read each question carefully before you start your answer.

1 A garden plant tub has a circular cross-section with a solid base.



The tub has a diameter of 30 cm at its top and narrows uniformly to a diameter of 20 cm at its base. The depth of the tub is 15 cm.

The tub can be modelled as a truncated, inverted cone as shown below.



(a) Calculate the volume of the tub. The thickness of the sides and the base of the tub should be neglected.

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(b) Calculate the outside surface area of the tub including its base.

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2 Air pressure, P, air density, ρ, and air temperature, T, are related by the following formula

$P = \rho TR$, where

P N m^{-2} is the absolute pressure of the air,
ρ kg m^{-3} is the density of the air,
T °K (degrees Kelvin) is the absolute temperature of the air,
R is a gas constant associated with the air.

(a) Determine the SI units of R.

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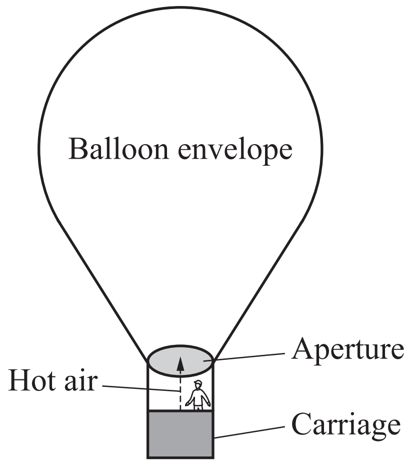
The diagram below shows a hot air balloon used to carry passengers for short, recreational sightseeing excursions. Passengers are accommodated in an open carriage suspended directly below the envelope of the balloon; this also contains a powerful burner which is used periodically to blast hot air into the envelope through an aperture at its base.



(b) The air in the envelope of the balloon is heated to a temperature of 373 °K while the pressure remains constant at 101 300 Pa. Given that $R = 287$, calculate the density of the air, ρ, in the balloon's envelope.

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According to the principle of buoyancy, a hot air balloon will remain floating in the air if its total mass, including the air in the balloon's envelope, the carriage with its passengers and any supporting equipment, is less than the mass of the air displaced by the balloon's envelope.

The mass of air **contained** in the balloon's envelope is the product of the density of that air and the volume of the envelope. The mass of the air **displaced** by the balloon's envelope is the product of the density of the surrounding air and the volume of the envelope.

(c) The volume, V, of the envelope of a particular hot air balloon remains constant at $3000\,\text{m}^3$. It is floating in air where the atmospheric air density is $1.2\,\text{kg}\,\text{m}^{-3}$.

Using your value for the density of air in the balloon's envelope, ρ, calculate in part **(b)**, calculate the maximum total mass of the balloon and the balloon's carriage with its passengers and equipment which will allow the balloon to remain floating.

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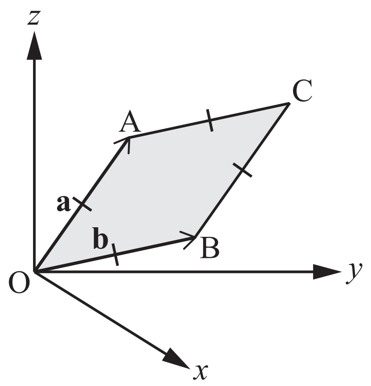
3 For this question you are given the following information.

If **a** and **b** are two vectors, the scalar product, **a.b**, is given by

$\mathbf{a}.\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$, where θ is the angle between the two vectors.

A metal plate in the shape of a rhombus, OACB, is tilted in three-dimensional space about vertex O.



Treating vertex O as the origin of a Cartesian coordinate system, (x, y, z), the coordinates of vertex A are (2, 1, 3) and vertex B (3, 2, 1). These are associated with position vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ respectively.

(a) Calculate the coordinates of vertex C.

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(b) Use a dot product to calculate the angle between vectors **a** and **b**.

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(c) Calculate the vector product $\mathbf{a} \times \mathbf{b}$. Give your answer in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, where a, b and c are integers.

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(d) Calculate the area of the metal plate.

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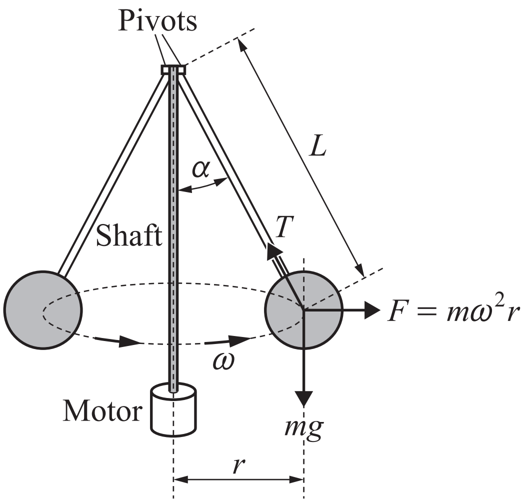
(a) A vertical shaft, driven by a motor, is rotating about its axis with an angular speed of ω radians per second and is used as part of a mechanical governor for regulating the motor's speed. Attached to the top of the shaft are two frictionless pivots which each support an arm of negligible mass. The angle between each arm and the vertical is α. At the end of each arm there is a steel ball of mass m kg whose centre is a distance of L m from the top of the shaft.



When the shaft rotates at sufficient, constant angular speed the balls are forced outwards by a centrifugal force F N and move in a horizontal circle with radius r m. The balls are held in vertical equilibrium by a horizontal force F, of magnitude $m\omega^2 r$, a downward force due to gravity, mg, and a tension force in each arm, T N, whose magnitude is equal to the magnitude of the resultant of the other two forces.

(i) Show that $\omega = \sqrt{\dfrac{g}{L\cos\alpha}}$.

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(ii) When $L = 0.15$ and the shaft rotates at 90 revolutions per minute, calculate angle α.

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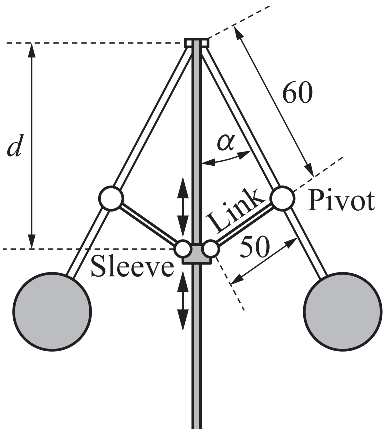
(b) The arrangement in part **(a)** is now extended and includes two links which are each 50 mm long as shown below.



Not to scale

The links are attached to the arms by pivots which are each 60 mm from the top of the shaft.
The other ends of the links are attached to sleeves which can slide freely up and down as the angle α changes. The vertical distance between the sleeves and the top of the shaft is d mm and their position is used by a servomechanism to regulate the motor's speed.

Calculate d when $\alpha = 45°$.

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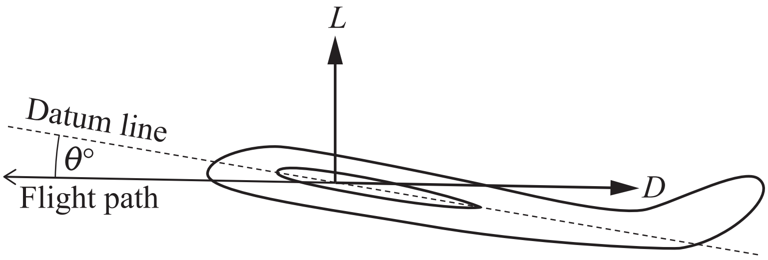
5 The diagram below shows a commercial aircraft travelling along a flight path which is inclined at a small angle, $\theta°$, relative to its datum line. The datum line passes through the aircraft's centre of mass and the angle $\theta°$ is known as the angle of attack.



The two forces indicated in the diagram are the lift force, L N, caused by the differential air pressure between the top and bottom of the wings and an aerodynamic drag force, D N, caused by the motion of the aircraft though the air. When an aircraft is flying at a constant altitude with a constant horizontal speed, both L and D may be modelled in terms of θ.

On a particular test flight, the drag force was recorded for three values of θ as shown in the following table.

Angle of attack, $\theta°$	Drag force, D N
0°	70 000 N
5°	120 000 N
10°	250 000 N

(a) D is modelled by a quadratic formula $a\theta^2 + b\theta + c$ for $0 \leqslant \theta \leqslant 10$, where a, b and c are constants.

Use the values in the table to calculate the values of a, b and c.

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For the same test flight, the lift force is modelled by $L = 1\,000\,000 + 400\,000\,\theta$.

(b) In aircraft design the ratio $\frac{L}{D}$ is an important measure of aerodynamic efficiency and is maximised when θ is a small positive angle.

Using the formulae for L and D, together with your values for the constants a, b and c found in part **(a)**, calculate the value of θ for which $\frac{L}{D}$ is a maximum.

(You are **not** required to prove mathematically that your answer provides a maximum value rather than a minimum.)

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6 For this question you are given that the coordinates, $\overline{x}$ and $\overline{y}$, of the centroid of a lamina bounded by the curves $y_1 = f_1(x)$ and $y_2 = f_2(x)$ between x_1 and x_2 are given by

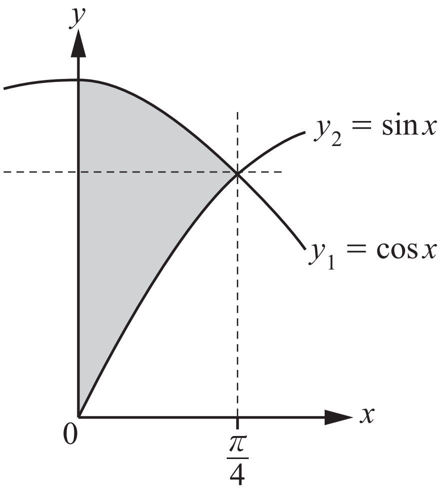
$$\overline{x} = \frac{1}{A}\int_{x_1}^{x_2} x(y_1 - y_2)\,dx, \qquad \overline{y} = \frac{1}{2A}\int_{x_1}^{x_2}\left(y_1^2 - y_2^2\right)dx,$$

where A is the total area of the lamina.

You may also use the following trigonometric identities

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}.$$

The shaded section of the diagram below shows a lamina bounded by the curves $y_1 = \cos x$ and $y_2 = \sin x$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.



(a) Show that the area, A, of this lamina is given by $A = \sqrt{2} - 1$.

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(b) Calculate $\overline{x}$.

You may leave your final answer as an expression involving $\sqrt{2}$ and π.

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(c) Calculate $\overline{y}$.

You may leave your final answer as an expression involving $\sqrt{2}$.

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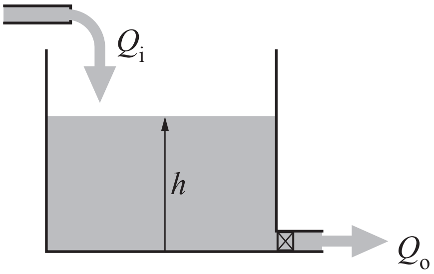
7 A water tank in the shape of a cube has an open top and a maximum capacity of 1000 L (litres). The four sides of the tank, and the base, are each squares with side-length 1 m. At the base of the tank there is an outlet pipe containing a valve that regulates the rate, Q_o L s^{-1}, at which water will flow. Q_o is directly proportional to the height, h m, of water in the tank so that $Q_o = ch$, where c is a constant.



Water is allowed to flow continually into the tank at a constant rate of Q_i L s^{-1} and at the same time water escapes through the valve in the outlet pipe. While water continues to flow the height of water in the tank, h, is modelled by the equation

$$A\frac{dh}{dt} = Q_i - ch,$$

where A is a constant and t s is time.

(a) Given that the tank is empty when the water starts to flow, show that

$$h = \frac{Q_i}{c}(1 - e^{-B}), \text{ where } B = \frac{ct}{A}.$$

You may use the result $\int \frac{1}{A - Bx}dx = -\frac{1}{B}\ln(A - Bx)$.

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(b) The tank is exactly half full when $Q_o = 0.15$.

Calculate the value of c.

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(c) Starting with an empty tank, water is allowed to flow continually into the tank at a rate of $0.25\ \mathrm{L\,s^{-1}}$. Using the value of c calculated in part **(b)** determine whether the water will overflow from the tank.

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(d) Given that $A = 1000$, calculate the time when $h = 0.4$.

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END OF QUESTION PAPER

EXTRA ANSWER SPACE

If you need extra space use this lined page. You must write the question numbers clearly in the margin.