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Examiners' report

AS LEVEL

FURTHER MATHEMATICS A

OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A

H235

For first teaching in 2017

Y534/01 Summer 2025 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from Teach Cambridge.

Paper Y534 series overview

This is one of the option papers for AS Further Mathematics. It assesses Discrete Mathematics through graphs and networks, algorithms, linear programming and the application of these to problems including critical path analysis and game theory.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• drew accurate graphs and diagrams • explained their reasoning • worked neatly.	• did not use the algorithms given in the formulae booklet • laid their working in a less clear way • did not label their graphs (axes, scales and lines labelled) • did not read the questions carefully enough.

Question 1 (a)

1 A builder has some pieces of wood that are 6 m in length. The builder needs to cut smaller pieces to use in a project. The lengths of the smaller pieces required are shown below.

2.5 m 2 m 4.5 m 3 m 1.5 m 3.5 m 2 m

(a) Use the first-fit **decreasing** algorithm to determine the smallest number of 6 m lengths of wood needed to cut the seven smaller pieces. **[3]**

The majority of candidates carried out the algorithm correctly and stated that four pieces were needed.

Question 1 (b)

(b) First-fit decreasing is a **heuristic** algorithm. Explain what this means. **[1]**

Some candidates described other aspects of algorithms. A heuristic algorithm finds a good solution quickly, although it may not be optimal.

Question 1 (c)

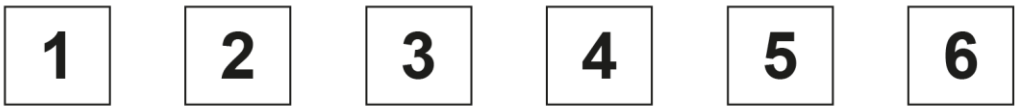
The first-fit decreasing algorithm is order $O(n^2)$.

(c) It takes a computer 3.2 seconds to solve an allocation problem involving 200 items of data using the first-fit decreasing algorithm. Calculate approximately how long it would take to solve an allocation problem involving 1000 items of data. **[2]**

This question part was usually answered correctly.

Question 2 (a)

2 A student has six cards numbered from 1–6. The cards are shown below.



(a) The student chooses four cards at random.

Use the Pigeonhole Principle to explain why they must have at least two cards whose numbers add to seven. **[2]**

Many candidates were able to identify how to pair cards to give a total of seven and some realised that this defined the three pigeonholes as {1, 6}, {2, 5}, {3, 4}. Some also explained why four cards cannot be put into three pigeonholes without at least one pigeonhole having two cards in it, and hence the result.

Question 2 (b) (i) and (ii)

(b) The student now chooses two cards and places them side by side to form a two-digit number.

(i) Determine the number of different two-digit numbers that the student can make. **[2]**

(ii) Determine the number of different **even** two-digit numbers that the student can make. **[2]**

Candidates who just wrote a numerical answer, with no evidence of how it had been obtained, did not get full marks.

Assessment for learning

'Determine' is a command word and means that justification should be given for any results.

Question 3 (a)

3 The table shows the lengths in km of footpaths between six places, A to F, in a town.

	A	B	C	D	E	F
A	–	5	3	5	8	10
B	5	–	6	2	7	5
C	3	6	–	1	10	8
D	5	2	1	–	9	4
E	8	7	10	9	–	3
F	10	5	8	4	3	–

(a) Apply the tabular form of Prim's algorithm, starting at A, to the copy of the table in the **Printed Answer Booklet** to construct a minimum spanning tree for the six places in the town. You should state the order in which the arcs are added to the tree.

Draw the minimum spanning tree and find its weight. **[4]**

The tabular form of Prim's algorithm is given in the formulae booklet.

Some candidates listed the order in which the vertices were added to the tree instead of the order in which the arcs were added.

Some candidates sorted the arcs into increasing order of weight and several just used an ad hoc method to construct the tree, or what they thought was the minimum spanning tree.

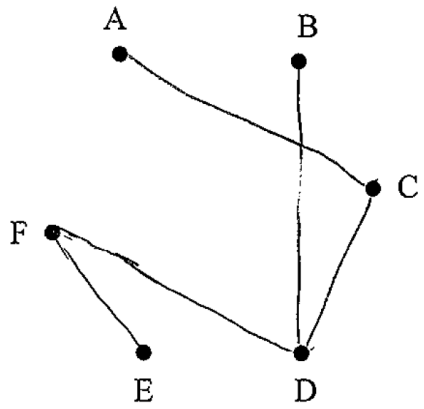
The question asked candidates to do four things: (1) apply the tabular form of Prim's algorithm starting at A; (2) state the order in which arcs are added to the tree; (3) draw the tree; (4) find its weight.

Exemplar 1

3(a)

Minimum spanning tree

	A	B	C	D	E	F
A	–	5	3	5	8	10
B	5	–	6	2	7	5
C	3	6	–	1	10	8
D	5	2	1	–	9	4
E	8	7	10	9	–	3
F	10	5	8	4	3	–



Arcs in order: CD, ~~DB~~, ~~AC~~ CD, BD, AC, EF, DF,

This candidate has not used the table to apply Prim's algorithm. They have then listed the arcs in increasing order of weight, suggesting that Kruskal's algorithm may have been used. The minimum spanning tree is drawn but its weight has not been given.

Question 3 (b)

(b) Sam needs to walk from A to F.

Give an example of a question Sam could ask which leads to an enumeration problem. **[1]**

Most candidates gave a suitable question, such as asking how many walks there are from A to F.

Some candidates gave an optimisation question, such as asking for the shortest walk from A to F.

Question 3 (c)

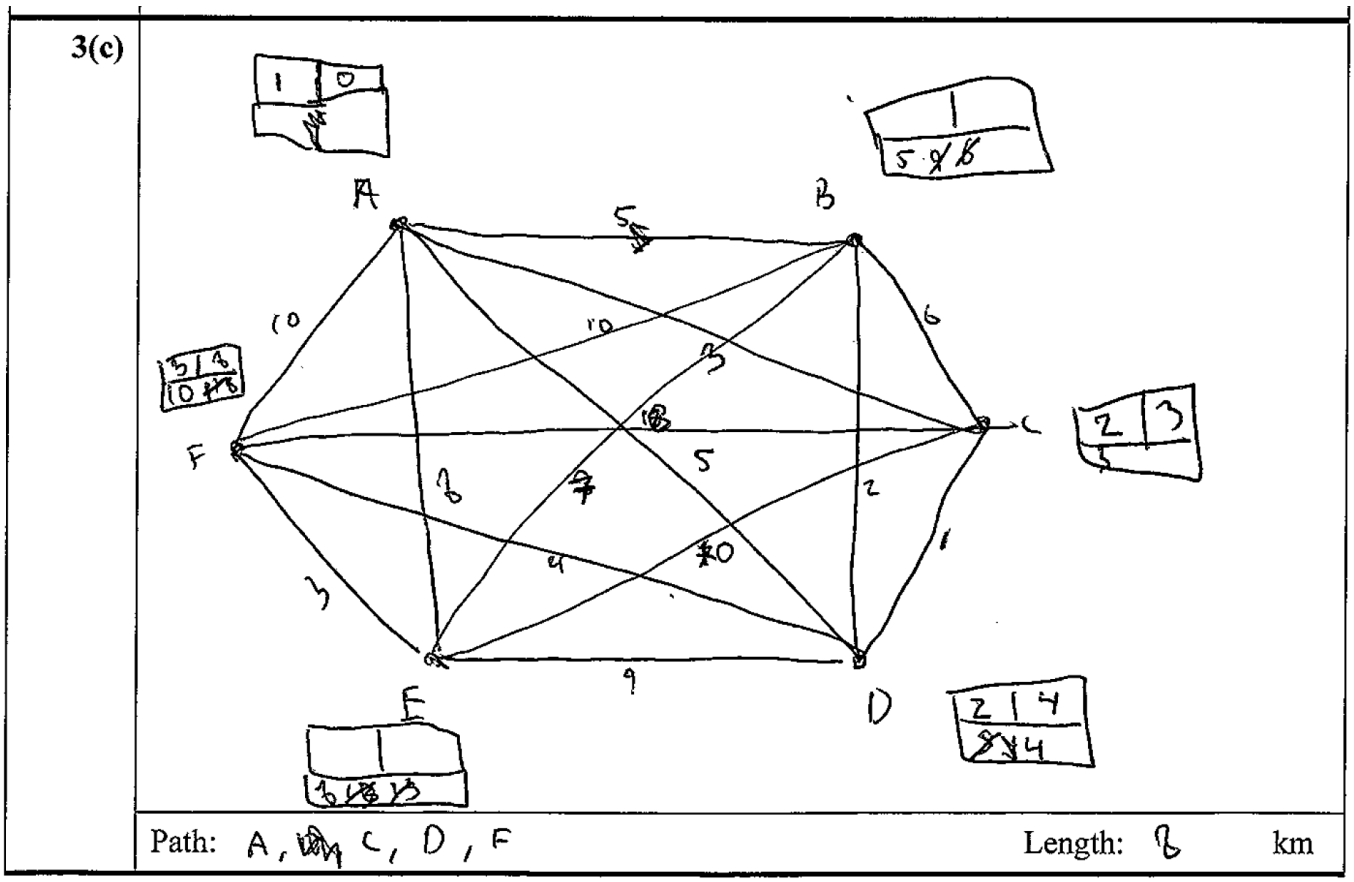
(c) Use an appropriate algorithm to find the shortest route from A to F. You must state its length. **[4]**

Candidates needed tell us that they were using Dijkstra's algorithm and then apply it. This meant starting Dijkstra's algorithm from vertex *A* and updating at *D* (or starting at vertex *F* and updating at *C*). Some candidates showed all the temporary labels, whether they were improvements or not.

Most candidates used the box notation, either by listing the vertices with their boxes or by showing them on a network that they had drawn.

If the algorithm is applied as given in the formula booklet, the labelling should have stopped as soon as *F* was given a permanent label.

Exemplar 2



This candidate has drawn the network and started to apply Dijkstra's algorithm. They have updated the temporary label at vertex D (this would have been easier to read if they had not crossed out the value 5). They have then slipped into an ad hoc approach so the upper boxes at B have been left blank and the order of labelling box is wrong at D and F.

Question 4 (a)

4 Ryan and Casey play a game.
They can each choose to play one of three options.
Ryan can play options A, B and C and Casey can play options X, Y and Z.

The scores for each player for each combination of choices are shown in the pay-off matrices below:

Scores for Ryan

Ryan's Options	Casey's Options: X	Y	Z
A	1	4	3
B	5	3	4
C	3	1	3

Scores for Casey

Ryan's Options	Casey's Options: X	Y	Z
A	5	2	3
B	1	3	2
C	3	5	3

(a) Explain how the game between Ryan and Casey can be expressed as the zero-sum game with the pay-off matrix below (the entries represent pay-offs for Ryan). **[1]**

Ryan's Options	Casey's Options: X	Y	Z
A	–2	1	0
B	2	0	1
C	0	–2	0

For a game to be able to be expressed as a zero-sum game the sum of the scores for corresponding cells must be constant (in this case 6). Candidates needed to say this as well as explaining how the entries in the zero-sum pay-off matrix were calculated. Some candidates answered the question the wrong way round, using the zero-sum pay-off matrix to calculate the matrix showing Ryan's original scores.

Question 4 (b)

(b) If Casey knows that Ryan will play option A, state the option that Casey should play. **[1]**

Nearly all candidates were able to identify X as the appropriate choice, either from the zero-sum matrix or from using the original scores.

Question 4 (c)

(c) Find the play-safe strategies for both players. **[3]**

By calculating the row minima (-2, 0, -2) and identifying the row maximin value as 0, Ryan's play-safe strategy can be seen to be B.

By calculating the column maxima (2, 1, 1) and identifying the column minimax value as 1, Casey's play-safe strategies can be seen to be Y and Z.

Some candidates did not seem to like the idea of Casey having two play-safe strategies and came up with reasons for preferring one over the other. There is no reason why a player should not have multiple play-safe strategies.

Question 4 (d)

(d) Determine whether or not the game is stable. **[1]**

Mostly done well, usually by comparing the values of the play-safe strategies ($0 \neq 1$) and deducing that the game is not stable.

A few candidates claimed that the play-safe strategies were different (rather than the values of the play-safe strategies) and some thought, incorrectly, that the game could not be stable because Casey had two play-safe choices.

Question 4 (e)

(e) Use a dominance argument to reduce the zero-sum game to a 2×3 game. **[1]**

A dominance argument involves giving the appropriate comparisons and stating which option is dominated by which other option. In this case 2 > 0, 0 > -2, 1 > 0 so row B strictly dominates row C and the game can be reduced by deleting row C.

Question 4 (f)

(f) Let p be the probability that Ryan plays option A on the reduced game from part **(e)**.

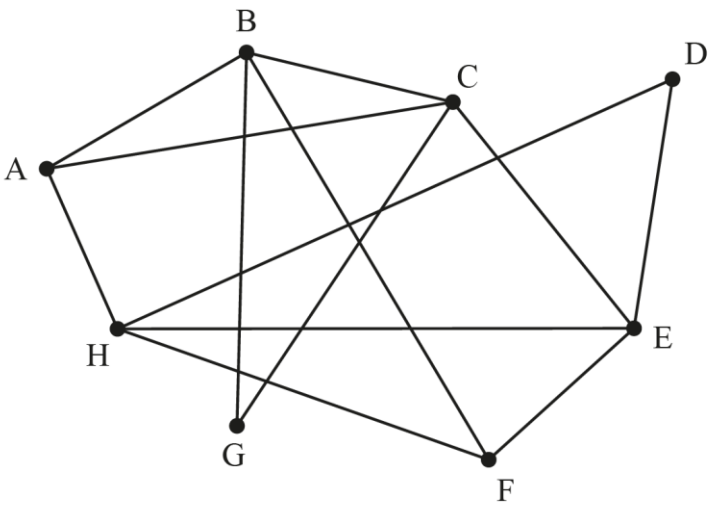
Use a graphical method to determine the optimal mixed strategy for Ryan. **[4]**

Most candidates knew how to calculate, for each of Casey's choices, the expected pay-off for Ryan as a linear function of p. Many then assumed that the line $E = p$ and the p-axis were the same thing and ended up with a graph that had only two lines rather than three. Some candidates drew the graph but did not calculate a p-value and some chose the wrong intersection.

Most candidates drew reasonably scaled labelled graphs with ruled lines, but some chose scales that made the part of the graph we needed to see ($0 \leq p \leq 1$) too small.

Question 5 (a)

5 Graph X is shown below.



(a) Explain whether graph X is Eulerian, semi-Eulerian or neither. **[1]**

Candidates needed to explain that the graph is semi-Eulerian because there are <u>exactly</u> two odd nodes.

Question 5 (b)

(b) Give an example of a cycle on the graph above. **[1]**

A cycle needs to be a path that has no repeated arcs and no repeated nodes and then joins back to the start, so that it starts and ends at the same node, e.g. ABCEHA or BCGB or ABGCEDHA but not ABCGBFHA which repeats vertex B.

Assessment for learning

'State' is a command word and means that the answer may be written down with no working or justification needed.

Apart from questions that ask, 'give a reason' (or similar) the command word 'give' is usually just asking for an example to be written down.

Other command words (determine, prove, show that, etc.) require some evidence.

More information about these command words are given in Section 2b of the specification. OCR have also produced posters that can be used in the classroom.

Question 5 (c)

The adjacency matrix for a graph Y is shown below.

	I	J	K	L	M	N	O	P
I	0	1	1	1	0	0	0	1
J	1	0	1	1	1	0	0	0
K	1	1	0	1	0	0	0	0
L	1	1	1	0	1	0	0	0
M	0	1	0	1	0	0	0	1
N	0	0	0	0	0	0	1	1
O	0	0	0	0	0	1	0	1
P	1	0	0	0	1	1	1	0

(c) Show that X and Y are **not** isomorphic. **[3]**

Several candidates just wrote out the vertex degrees and stopped or assumed that the order of the vertices needed to match (A to I, B to J, etc.).

To show that the graphs are **not** isomorphic meant finding something that was structurally different and dealing with all such cases. For example, each graph has two vertices of degree 2 (D and G in X and N and O in Y) but N and O are directly connected by an arc whereas D and G are not.

Question 6 (a)

6 The table shows the activities involved in a project, together with their immediate predecessors and the duration of each activity in hours.

Activity	Duration (Hours)	Immediate Predecessors
A	3	–
B	1	–
C	2	A
D	2	A, B
E	5	C
F	2	C, D
G	4	E, F

(a) Model the project using an activity network. **[3]**

Activity networks should have the activities on the arcs. They should have a single start and finish point and the arcs should all be directed. In this case there also needed to be two directed dummy activities to deal with the precedence for activities D and F.

Question 6 (b) (i) and (ii)

(b) (i) Carry out a forward pass to find the early event times and a backward pass to find the late event times. **[3]**

(ii) Write down the critical activities. **[1]**

The early and late event times may be shown on the diagram (at the vertices) but they need to be legible.

A few candidates assumed that the early event times and late event times are always equal, and some could not deal with merges on the forward path (for example claiming that the early event times at the start of D is 1).

Most candidates were able to write down the critical activities, A, C, E, G, although a few also claimed that D was critical.

Question 6 (c)

(c) Determine the effect on the minimum project completion time if the duration of activity F is 4 hours longer than planned. **[2]**

Some candidates thought that the delay was the new duration of activity F, rather than it now taking 2+4 = 6 hours.

Many candidates calculated the float of activity F and used this to deduce that the minimum project completion time would increase by 1 hour.

Some candidates identified that F would become a critical activity, instead of E, and deduced that the minimum project completion time increased from 14 hours to 15 hours. (D also became critical but the increase in the duration of F is what affected the time needed to complete the project).

Question 7 (a)

7 A shop sells three different arrangements of flowers, X, Y and Z. The number of roses, lilies and carnations in each arrangement are shown in the table below.
The shop will sell all of the arrangements that they make.

Arrangement	Roses	Lilies	Carnations
X	5	3	5
Y	4	7	2
Z	9	6	7

Arrangement X sells for £20, arrangement Y sells for £15 and arrangement Z sells for £24.

The shopkeeper has a maximum of 40 roses, 80 lilies and 30 carnations that they can use to make the flower arrangements.

The shopkeeper wants to make as much money as possible from selling the flower arrangements.

The shopkeeper decides that **no more** than 50% of the total number of arrangements can be arrangement Y.

(a) Formulate a linear programming problem to represent the problem. You should define the variables that you are using in your formulation. **[5]**

Several candidates were able to formulate the linear programming problem although some did not identify appropriate variables and some could not deal with the inequality from no more than 50% of the total number of arrangements being of type Y.

Question 7 (b) (i)

The shopkeeper decides to make the same number of arrangement Z as arrangement X.

(b) (i) Use a graphical method to determine how many of each arrangement should be made. **[6]**

Many candidates realised that they needed to replace z with x in their constraints. A few just dropped all the z terms, but then the number of type X and the number of type Z are not the same.

Candidates often drew graphs with no scales or labels and sometimes the feasible region was so small that it could not be seen properly.

Some candidates found the optimum point as (2, 3) but assumed that this means 1 each of X and Z with 3 of Y, rather than 2 each of X and Z.

There was also some evidence of candidates running out of time for this part and not showing how they had identified the optimum point.

Question 7 (b) (ii)

(ii) State the maximum amount of money that the shopkeeper can make. **[1]**

Several candidates did not attempt this part, including some who had found the maximum correctly in part (b) (i).

Question 7 (c)

The maximum number of lilies available changes from 80 to k. The number of arrangement Z is still the same as the number of arrangement X.

(c) Determine the range of values of k for which the new optimal solution is the same as in part **(b)(i)**. **[2]**

Only a minority of candidates made any progress with this part, but there were some good solutions seen.

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