Tuesday 17 January 2012 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Show that the equation

$$\tan x - \cos x = 1,$$

where x is in radians, has a root in the interval [0.9, 1.1] **[2]**

(ii) Use the bisection method to find an estimate of this root with maximum possible error 0.025. **[4]**

2 The function f(x) is a polynomial with exact values as shown in the table.

x	0	1	2	3	4
f(x)	–6	–7	–4	15	62

(i) Find the degree of f(x). **[3]**

(ii) Use Newton's forward difference formula to obtain an expression for f(x). (You are not required to simplify your answer.) **[3]**

(iii) Find f(2.5). **[2]**

3 In this question you should work entirely in radians.

(i) Write down the values of cos(0.11) and cos(0.12) correct to 4 significant figures. Use these values to write down a value for cos(0.11) – cos(0.12). **[3]**

(ii) You are now given that cos(0.11) – cos(0.12) = 2 sin(0.115) sin(0.005).

Write down values of sin(0.115) and sin(0.005) correct to 4 significant figures. Use these values to obtain another value for cos(0.11) – cos(0.12). **[2]**

(iii) Check your answer using the full accuracy of your calculator. Comment on your answers to parts **(i)** and **(ii)** indicating what gives rise to the outcome in part **(i)**. **[3]**

4 In an opinion poll, the percentage levels of support for the five political parties contesting an election were reported, to the nearest whole number, as follows.

Party	Red	Blue	Orange	Green	Yellow
% support	36	25	17	13	10

(i) These percentages do not sum to 100. Explain how this can occur. **[2]**

(ii) A second opinion poll is taken a month later. The five percentages are again reported to the nearest whole number. State the largest and smallest totals that the reported percentages could have, showing by means of an example how each could arise. **[4]**

5 Two estimates of the integral $\int_{0.2}^{1} \left(\frac{1}{2}\right)^x dx$ are given in the table below.

h	Mid-point	Trapezium
0.8	0.527 803 16	0.548 220 23
0.4		

(i) Obtain the two missing values in the table. **[4]**

(ii) Hence obtain two Simpson's rule estimates of the integral.

Give the value of the integral to the accuracy that you consider to be justified from your working. **[4]**

Section B (36 marks)

6 A function g(x) has the values, correct to 5 decimal places, shown in the table.

x	1.8	1.9	1.95	2	2.05	2.1	2.2
g(x)	2.359 60	2.485 71	2.551 17	2.618 28	2.687 10	2.757 65	2.904 13

(i) Find three estimates of g′(2), using the forward difference method with decreasing values of h.

Letting these three estimates be denoted by α, β, γ, respectively, find the value of $\frac{\gamma - \beta}{\beta - \alpha}$. What does this value indicate about the rate of convergence of the forward difference method? **[6]**

(ii) Find three estimates of g′(2), using the central difference method.

Show that the central difference method converges more quickly than the forward difference method. **[6]**

(iii) Suppose that D_1 and D_2 are estimates of a quantity d, and that the error in D_2 is approximately $\frac{1}{4}$ of the error in D_1. Show that

$$d \approx \frac{4D_2 - D_1}{3}.$$ **[3]**

(iv) Use the result in part **(iii)** to find two improved estimates of g′(2). Give a value for g′(2) to the accuracy that appears justified. **[4]**

7 **(i)** Show that the equation

$$x^6 - 3x + 1 = 0 \qquad (*)$$

has a root, α, in the interval [0, 1], and a root, β, in the interval [1, 2]. **[2]**

(ii) Determine how many turning points the curve $y = x^6 - 3x + 1$ has. What can you conclude about the number of real roots of (*)? **[4]**

(iii) Show that the iteration $x_{r+1} = \frac{1}{3}(x_r{}^6 + 1)$, with a suitable starting value, converges to α. Find α correct to 5 significant figures. **[4]**

(iv) Show, by considering an appropriate derivative, that the iteration in part **(iii)** will not converge to β. **[3]**

(v) Find another iteration, based on a rearrangement of (*), that converges to β. Find β correct to 5 significant figures. **[4]**