

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $x^2 \tan 2x$. [3]

2 The functions $f(x)$ and $g(x)$ are defined as follows.

$$f(x) = \ln x, \quad x > 0$$

$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions $fg(x)$ and $gf(x)$, and state whether these functions are odd, even or neither. [4]

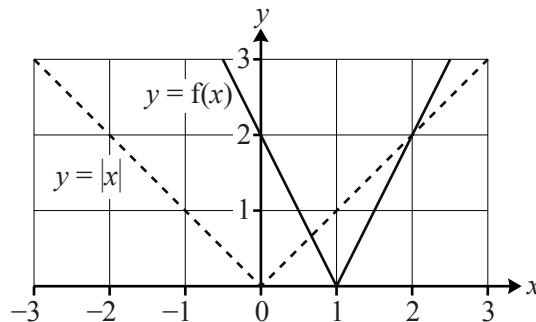
3 Show that $\int_0^{\frac{\pi}{2}} x \cos^{\frac{1}{2}} x \, dx = \frac{\sqrt{2}}{2} \pi + 2\sqrt{2} - 4$. [5]

4 Prove or disprove the following statement:

‘No cube of an integer has 2 as its units digit.’ [2]

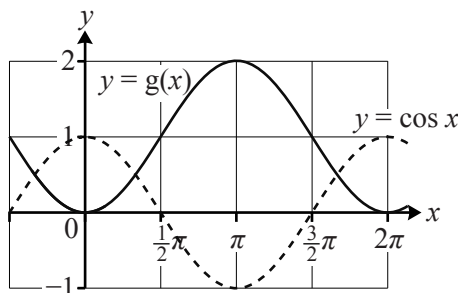
5 Each of the graphs of $y=f(x)$ and $y=g(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for $f(x)$ and $g(x)$.

(i)



[3]

(ii)



[3]

- 6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. [4]

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. [4]

- 7 Fig. 7 shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.

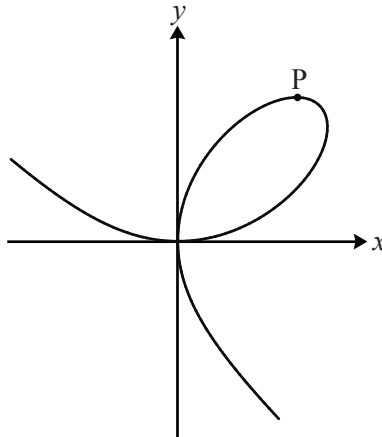


Fig. 7

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. [4]

(ii) Hence find the exact x -coordinate of P. [4]

Section B (36 marks)

- 8 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$. The curve meets these lines at P and Q respectively. R is the point (11, 11).

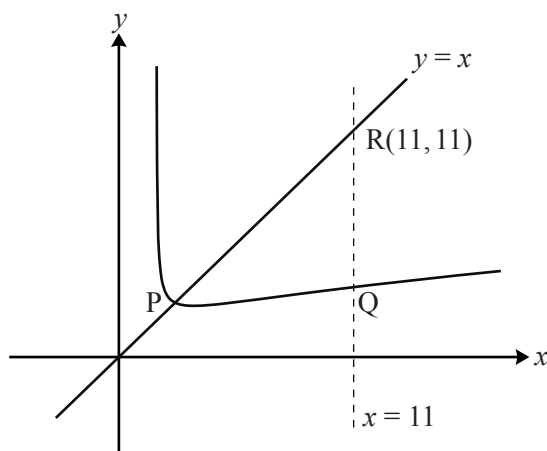


Fig. 8

- (i) Verify that the x -coordinate of P is 3. [2]
- (ii) Show that, for the curve, $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$.
Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. [7]
- (iii) Using the substitution $u = x - 2$, show that $\int_3^{11} \frac{x}{\sqrt{x-2}} dx = 25\frac{1}{3}$.
Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. [9]

- 9 Fig. 9 shows the curves $y = f(x)$ and $y = g(x)$. The function $y = f(x)$ is given by

$$f(x) = \ln \left(\frac{2x}{1+x} \right), \quad x > 0.$$

The curve $y = f(x)$ crosses the x -axis at P, and the line $x = 2$ at Q.

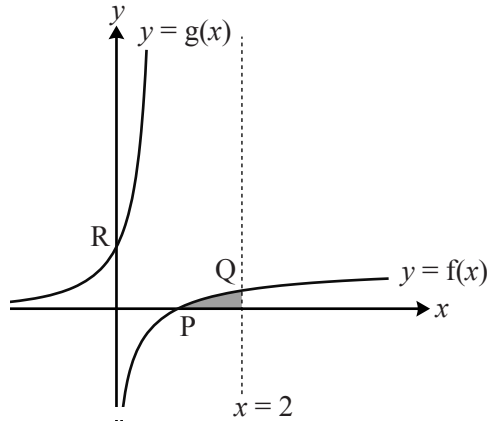


Fig. 9

- (i) Verify that the x -coordinate of P is 1.

Find the exact y -coordinate of Q.

[2]

- (ii) Find the gradient of the curve at P. [Hint: use $\ln \frac{a}{b} = \ln a - \ln b$.]

[4]

The function $g(x)$ is given by

$$g(x) = \frac{e^x}{2 - e^x}, \quad x < \ln 2.$$

The curve $y = g(x)$ crosses the y -axis at the point R.

- (iii) Show that $g(x)$ is the inverse function of $f(x)$.

Write down the gradient of $y = g(x)$ at R.

[5]

- (iv) Show, using the substitution $u = 2 - e^x$ or otherwise, that $\int_0^{\ln \frac{4}{3}} g(x) dx = \ln \frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$.
[Hint: consider its reflection in $y = x$.]

[7]

BLANK PAGE

BLANK PAGE

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.